Mathematical model of interaction of milk thistle seed with grooved working surfaces

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Abstract. The interaction of milk thistle seed with grooved working surfaces is revealed. The dependences of curvature of the ellipse of contact on the step between grooves and pressure distribution on the area of contact of normal force of compression are identified. The achieved dependences make it possible to justify optimal geometrical parameters of grooved working surfaces required for peeling milk thistle seed coat.

Key words: seeds of milk thistle grooved working surface, area of contact.

SETTING OF THE PROBLEM

Under contemporary market conditions small farms should grow and process minor oilseeds (flax, mustard, milk thistle, and radish) and avoid competition from powerful agricultural producers. To obtain high-quality oil, meal and oilcake it is necessary to introduce new technology and processing equipment, their adaptation to small production volumes. The task will include milk thistle to one of the most profitable crops [14, 2].

Depending on the physical and mechanical properties and morphological structure of the seed we use peeling method that provides the best technological effect. The initial condition for the justification of technical parameters of means for peeling the seeds of milk thistle are such mechanical indications of seeds as their tangential and normal stress and deformation in which the elastic deformation develops in the plastic one and shell of seeds is peeled [13].

LAST PUBLICATIONS

Analysis of published works [1, 3–6] showed that at this stage there is a need for equipment and implementation of complex processing technologies of minor oilseeds.

In industry, the most widely used are such machines that use a blow, compression, shift, friction and more rarely - cut, shear and combinations of them. The existing shelling equipment should not be used for thistle seeds as thistle seed coat is much stronger than the core, with almost complete absence of space between the shell and the core. Usage of such machines would lead to the destruction of not only fetal membranes but also the core of milk thistle, which is highly undesirable effect.

Theoretical studies of membrane destruction process of oilseeds presented in published papers [8−12] are devoted to the main oilseeds - sunflower, canola, soybean, mustard, radish. Therefore, the justification of parameters of shelling equipment for milk thistle seeds processing need to be theoretically explored: the nature of the interaction of milk thistle seed with grooved working surfaces, set the value of the normal force and tangential load on fruit shell in which it suffers its destruction.

OBJECTIVES

Create a mathematical model of the interaction of milk thistle seed with grooved working surfaces. Justify the grooves' parameters that ensure the destruction of fetal membranes in the longitudinal area of the seed and prevent from cranking around its axis. This will ensure the stability of the peeling process, while maintaining the integrity of the core and increase the quality indicators of milk thistle meal [7].

MAIN PRESENTATION

We believe that the seeds of milk thistle has the shape of an ellipsoid of revolution. Then the equation of the surface of the ellipsoid in the principal central axes of inertia can be written [15–16]:

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}+\frac{z^2}{a^2}=1, \tag{1}$$

where: b and a – semiaxis of ellipsoid of revolution. As a result of the force F seed turns at an angle α and will contact with protrusion at points B and B_1 (Figure 1). The equation of the surface of the turned seed will be:

$$\frac{(x\cos\alpha + z\sin\alpha)^2}{b^2} + \frac{y^2}{a^2} + \frac{(z\cos\alpha - x\sin\alpha)^2}{a^2} = 1. \quad (2)$$

We will find the angle α substituting in equation (2) coordinate of point B $(t, 0, a)$:

$$\frac{(t\cos\alpha + a\sin\alpha)^2}{b^2} + \frac{(a\cos\alpha - t\sin\alpha)^2}{a^2} = 1. \quad (3)$$

The squaring:

$$\frac{t^2}{b^2}\cos^2\alpha + 2\frac{at}{b^2}\cos\alpha\sin\alpha + \frac{a^2}{b^2}\sin^2\alpha +$$

$$+\cos^2\alpha - 2\frac{at}{a^2}\cos\alpha\sin\alpha + \frac{t^2}{a^2}\sin^2\alpha = 1.$$

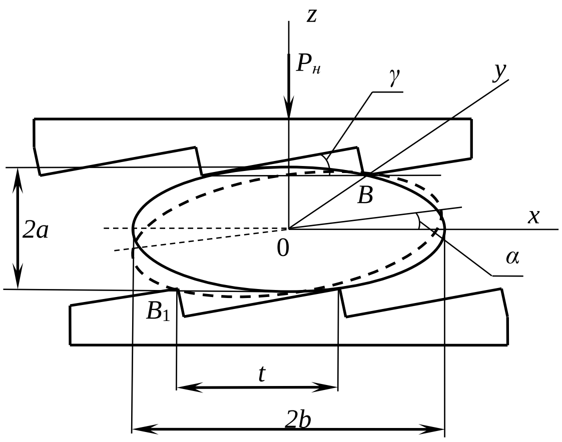

Fig. 1. Scheme of thistle seed orientation in space when interacting with the working surfaces

After transformations we obtain a quadratic equation for tga:

$$\left(1 - \frac{a^2}{b^2} - \frac{t^2}{a^2}\right)tg^2\alpha + 2at\left(\frac{1}{a^2} - \frac{1}{b^2}\right)tg\,\alpha - \frac{t^2}{b^2} = 0. \quad (4)$$

Hence we find:

$$tg\,\alpha = \frac{-2at\left(\frac{1}{a^2} - \frac{1}{b^2}\right) + \sqrt{4a^2t^2\left(\frac{1}{a^2} - \frac{1}{b^2}\right)^2 + 4\frac{t^2}{b^2}\left(1 - \frac{a^2}{b^2} - \frac{t^2}{a^2}\right)}}{2\left(1 - \frac{a^2}{b^2} - \frac{t^2}{a^2}\right)}. \quad (5)$$

Then:

$$\sin\alpha = \frac{tg\,\alpha}{\sqrt{1 + tg^2\alpha}}, \quad \cos\alpha = \frac{1}{\sqrt{1 + tg^2\alpha}}. \quad (6)$$

Find the slope of the tangent to the ellipse (y = 0) at point B in the area $(x\,0\,z)$. To do this, find the derivative $\frac{dz}{dx}$ of equation (2) at y = 0:

$$\frac{2}{b^2}(x\cos\alpha + z\sin\alpha)\left(\cos\alpha + \frac{dz}{dx}\sin\alpha\right) + \frac{2}{a^2}(z\cos\alpha - x\sin\alpha)\left(\frac{dz}{dx}\cos\alpha - \sin\alpha\right) = 0 \quad (7)$$

Hence, solving relatively $\frac{dz}{dx}$, we obtain:

$$\frac{dz}{dx} = \frac{-\left[x\left(\frac{\cos^2\alpha}{b^2} + \frac{\sin^2\alpha}{a^2}\right) - z\sin\alpha\cos\alpha\left(\frac{1}{a^2} - \frac{1}{b^2}\right)\right]}{z\left(\frac{\sin^2\alpha}{b^2} + \frac{\cos^2\alpha}{a^2}\right) - x\cos\alpha\sin\alpha\left(\frac{1}{a^2} - \frac{1}{b^2}\right)}. \quad (8)$$

Then the tangent of the slope at point B equals:

$$tg\,\gamma = -\left(\frac{dz}{dx}\right)_{\substack{z=a\\x=h}} = \frac{t\left(\frac{\cos^2\alpha}{b^2} + \frac{\sin^2\alpha}{a^2}\right) - a\sin\alpha\cos\alpha\left(\frac{1}{a^2} - \frac{1}{b^2}\right)}{a\left(\frac{\sin^2\alpha}{b^2} + \frac{\cos^2\alpha}{a^2}\right) - t\cos\alpha\sin\alpha\left(\frac{1}{a^2} - \frac{1}{b^2}\right)}. \quad (9)$$

Accordingly:

$$\sin\gamma = \frac{tg\,\gamma}{\sqrt{1 + tg^2\gamma}}, \quad \cos\gamma = \frac{1}{\sqrt{1 + tg^2\gamma}}. \quad (10)$$

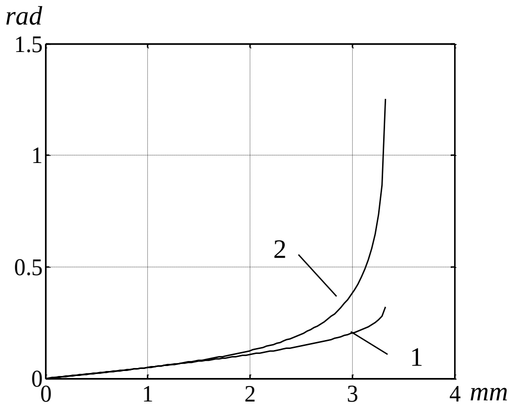

Fig. 2. Dependence of the angle γ (1) The slope of the tangent at the point of contact and angle of rotation of seed α (2) the step between the grooves t

Figure 2 shows that an increase of step between grooves t ranging from 0 to 2 mm, a twist of ellipsoidshape seed will not exceed 11º, it will enable us to stabilize its position at the moment of contact with the groove.

Let us identify the main ellipsoid surface curvatures at the point of contact. To do this, again take the derivative of equation (7) and solve relatively $\frac{d^2z}{dx^2}$, that is:

$$\frac{d^2z}{dx^2} = -\frac{\frac{1}{b^2}\left(\cos\alpha + \frac{dz}{dx}\sin\alpha\right)^2 + \left(\frac{dz}{dx}\cos\alpha - \sin\alpha\right)\cdot\frac{1}{a^2}}{z\left(\frac{\sin^2\alpha}{b^2} + \frac{\cos^2\alpha}{a^2}\right) - x\cos\alpha\sin\alpha\left(\frac{1}{a^2} - \frac{1}{b^2}\right)}. \quad (11)$$

Then find the main curvature at the point of contact in the area xOz:

$$K_1 = \left. \frac{\left| \dfrac{d^2 z}{dx^2} \right|}{\left(1 + \left(\dfrac{dz}{dx}\right)^2\right)^{3/2}} \right|_{\substack{z=a \\ x=t}} . \qquad (12)$$

Similarly, we find the main curvature in the perpendicular direction. To do this, we calculate the derivatives $\frac{dz}{dy}$ and $\frac{d^2 z}{dy^2}$ using equation (2):

$$\frac{dz}{dy} = -\frac{y}{(z\cos\alpha - x\sin\alpha)\cos\alpha}$$

$$\frac{d^2 z}{dy^2} = -\frac{1 + \left(\dfrac{dz}{dy}\right)^2 \cos^2\alpha}{(z\cos\alpha - x\sin\alpha)\cos\alpha} . \qquad (13)$$

Then the main curvature K_2 equals:

$$K_2 = \left. \frac{\left| \dfrac{d^2 z}{dy^2} \right|}{\left(1 + \left(\dfrac{dz}{dy}\right)^2\right)^{3/2} \cdot \cos\alpha} \right|_{\substack{y=0 \\ x=0 \\ z=t}} . \qquad (14)$$

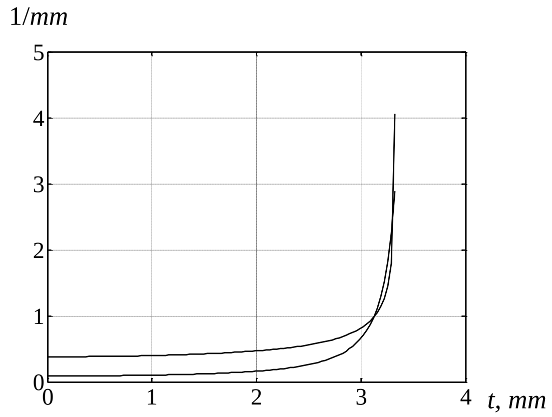

Fig. 3. Dependence of curvature 1 – K_1, 2 – K_2 of contact ellipse on the step between grooves *t*

Formulas (12, 14) and a graph of the curvature of contact ellipse between steps of grooves *t* (Fig. 3) indicate that with an increase of step between the grooves to 2 mm the contact ellipse curvature is in constant range.

Force P is acting on the edge and presses it into the seed. We can investigate the defomational state of seeds using the solution of the problem from Hertz's monograph.

If you choose to start a new coordinate system at the point of the edge and the seed contact on the *x* and *y*-axis and they are situated in a common area, then in the circle of points of contact surfaces equation can be represented as:

$$z_3 = a_{23}x^2 + a_{32}xy + a_{22}y^2,$$
$$z_2 = -\left(d_{22}x^2 + b_{32}xy + b_{22}y^2\right). \qquad (15)$$

Then the distance between corresponding points before deformation:

$$z_3 - z_2 = (a_{33}+b_{22})x^2 + (a_{32}+b_{32})xy + (a_{22}+b_{22})y^2. \qquad (16)$$

The coordinate system can be selected so that $(a_{32} + b_{32}) = 0$ and then:

$$z_3 - z_2 = Ax^2 + By^2. \qquad (17)$$

Axis are marked so that $A \geq B$.

Further, we believe that the edge has a shape of a parabolic cylinder:

$$z_3 = \frac{k}{x} x^2. \qquad (18)$$

Then A and B can be expressed due to previously defined curvatures:

$$A = k_3 + k; \quad b = k_2 . \qquad (19)$$

According to Hertz theory after applying to the edge force *N* the area of contact occurs, which has the shape of an ellipse:

$$\frac{x^2}{a_3^2} + \frac{y^2}{b_3^2} = 1 , \qquad (20)$$

where: $(a_1 \leq b_1)$.

Pressure distribution in the contact area can be calculated by the formula:

$$p = (x, y) = \frac{3P}{2\pi a_3 b_3}\sqrt{1 - \frac{x^2}{a_3^2} - \frac{y^2}{b_3^2}}. \qquad (21)$$

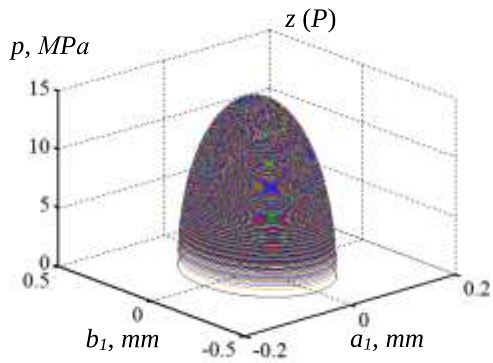

Fig. 4. Graph of pressure distribution on the contact area

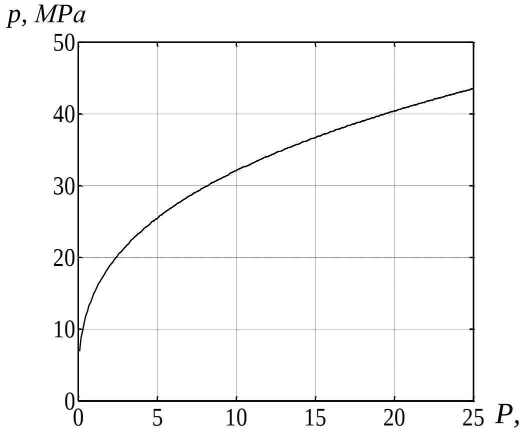

Fig. 5. Graph of pressure dependence on the area of contact *p* according to normal force of compression *P*

It is significant that seed and groove form an ellipse of contact with semiaxes a_1, b_1, reduction of mechanical stress and pressure *p* decreases with more distance from the contact area (Figure 4). With increasing normal force *P* (Figure 5), groove's pressure on the seed will also increase, and contact stresses occurring in fruit shell will be transmitted to the core and this can lead to its destruction.

CONCLUSIONS

We revealed geometric relationship between an ellipsoidshaped seed and grooved working surfaces parameters that make it impossible to turn the seed on its axis. Mathematical formulas of curvatures K_1, K_2 of ellipse of contact between grooves with step *t* are obtained , as well as the dependences of the contact pressure *p* with respect to compressive normal force *P*.

A mathematical model of the interaction of milk thistle seed with grooved working surfaces allows us to substantiate the basic parameters of means for peeling the seeds of milk thistle - the height of the working surfaces grooves, the gap between the working surfaces.

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