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ENERGY ANALYSIS OF AN IDEAL SUCTION-PRESSURE UNIT

The article presents an energy analysis of an ideal suction-pressure unit of a radial piston pump of high efficiency. The pump is an important element of a multiplying hydrostatic transmission, which may be used in small or micro hydros. Because the suction-pressing unit plays a crucial role as far as pump parameters are concerned, the design and working principles have been described in the article. Not only has the ideal model been described, but also all the processes it triggers. The mass balance, volume balance and energy balance of the unit have been studied. Also, mathematical models of energy conversion of working liquid have been described. Indicator diagrams of each chamber of the suction-pressure unit have been presented. The diagrams served to determine technical work of processes present there. This article is an introduction to a detailed analysis of a real suction-pressing set, which includes all factors influencing its work as well as generated losses.

Key words: hydraulics, pump, hydrostatic gear

1. INTRODUCTION

There has been a recent growth of attention focused on application of multiplying hydrostatic gears into small or micro hydros [3, 4, 5]. This conception requires designing novel low speed hydraulic pumps of relatively high volume of geometric workspace per one revolution. The idea of the design was presented in figure 1. The structure of the pump consists of radially distributed suction-pressure units driven by eccentric shaft. Efficiency of the device is an important factor to be evaluated prior to manufacturing a prototype unit. It can be performed by calculating energy balance which allows in-depth analysis of energy losses and possibilities to increase the overall efficiency.

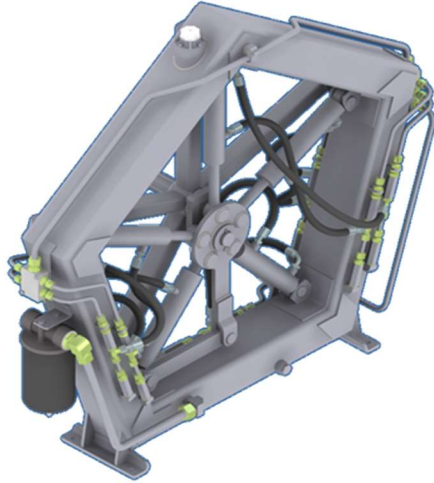


Fig. 1. Low speed hydraulic pump

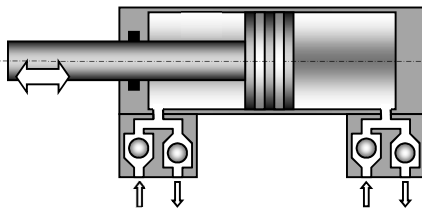


Fig. 2. Suction-pressure unit

The number of work cycles per one revolution of the pump is equal to the number of suction-pressure units. Therefore, in the considered case, a set of suction-pressing units (fig. 2) plays a major role as it determines the efficiency of an entire device. The pump efficiency usually exceeds 90%. Thus, it is important to identify the source of energy losses, estimate their scale and develop a method of their minimization. This article constitutes an attempt to determine the sources of energy losses and their mathematical formulation. In the literature [6, 7, 8, 9] simplified methods of calculation efficiency coefficients were presented. Balawender presented a detailed systematic energy analysis of volumetric hydraulic devices, determining the energy balance, taking into account all the important phenomena [1]. The described approach allowed to identify the source of the losses and their mathematical formulation.

2. ENERGY AND SUBSTANTIAL BALANCE OF AN IDEAL SUCTION-PRESSURE UNIT

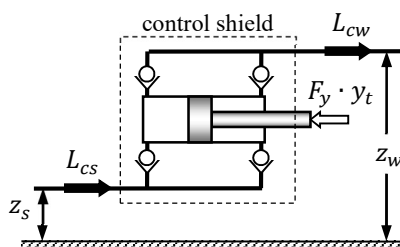


Fig. 3. Functional diagram

An ideal suction-pressure unit can be described as a one where its working chambers does not possess any dead volume and input mechanical energy is entirely converted into technical work during adiabatic process. In the unit there are no leakages, pressure drops related to flow friction, and mechanical friction.

In addition, a hydraulic medium does not exchange any heat with an environment. While calculating an energy balance of suction-pressure set, it has to be treated as periodically working machine which constitutes an open system, fixed to the reference level and surrounded by control balance shield (fig. 3). When interaction between the system and the environment is equal and balanced, the system is in a stable state. In the described case, the energy balance has to be formulated for parameters of the hydraulic fluid at the inlet and outlet channel cross-section. In addition, it has to be assumed that the flow system is geometrically homogenous in every cross-section of the channel which is normal to its main axis [11]. The inlet and outlet are determined by the control balance shield. This approach enables assigning average parameters to every point of the particular cross-sections.

2.1. Mass balance of suction-pressure unit

In suction and pressure channel, the changes of mass in the system (m_u) occurred during a time dt can be denoted as

$$dm_u = \sum_1^n dm_u = \sum_1^n \dot{m}_u dt. \quad (1)$$

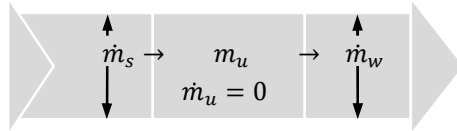


Fig. 4. Band chart of mass flux

The flux of mass changes in the system (\dot{m}_u) can be described as the velocity of mass changes of hydraulic fluid in the system [10, 11]. Assuming periodical work of the ideal suction-pressure unit, the mass fluxes: discharged \dot{m}_w and sucked \dot{m}_s are balanced in the stable state (fig. 4). Therefore, there are no changes in the mass of the working medium in the system, what can be formulated as:

$$\dot{m}_s = \dot{m}_w = \dot{m}_c \Rightarrow m_u = \text{const} \Rightarrow \dot{m}_u = 0. \quad (2)$$

2.2. Volumetric balance of suction-pressure unit

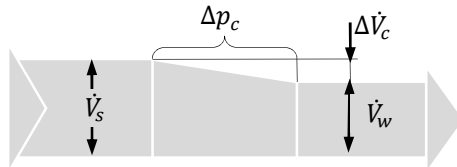


Fig. 5. Band chart of volume flux

In figure 5, the band chart of volumetric flux for the presented suction-pressure unit was presented. During its work, the periodically changing volumetric flux of the medium is simultaneously sucked and discharged. Balancing the flows, they have to be considered for the steady

stable states. Pressure p_s acts on the discharged medium. The fluid flowing through the system is isentropically compressed reaching the pressure of p_w .

As a result of the compression, the volume decreases. It leads to the situation that the inlet stream \dot{V}_s is greater than the outlet stream \dot{V}_w . The difference between these streams $\Delta\dot{V}_c$ can be called the stream losses and described mathematically as:

$$\Delta\dot{V}_c = \dot{V}_s - \dot{V}_w = \dot{m}_c \cdot (v_s - v_w) , \quad (3)$$

where: v_s – specific volume of the working fluid in the inlet channel,
 v_w – specific volume of the working fluid in the outlet channel,
 $\dot{m}_c = \dot{m}_s = \dot{m}_w$ – mass flux of the working fluid flowing through the system.

2.2. Energy balance of suction-pressure unit

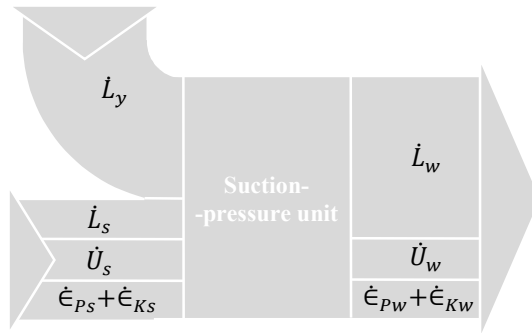


Fig. 6. Band chart of energy

From the motionless observer's point of view the energy of the system in the stable state does not change [1, 11]. Thus, the amount of input energy is equal to the amount of output energy (fig. 6). During the moment of formulating the energy balance, for the considered unit, the energy flux related to the moved piston and piston rod \dot{L}_y is put into the system:

$$\dot{L}_y = F_t \cdot \dot{y} , \quad (4)$$

where: F_t – force put to the piston, \dot{y} – velocity of piston rod.

In addition, the working fluid of mass m_s flows in the system. Its energy consists of:

– internal energy:

$$\dot{U}_s = \dot{m}_s \cdot u_s , \quad (5)$$

– kinetic energy:

$$\dot{e}_{Ks} = \frac{\dot{m}_w \cdot w_s^2}{2} , \quad (6)$$

– potential energy:

$$\dot{e}_{Ps} = \dot{m}_w \cdot g \cdot z_s , \quad (7)$$

– work related to supply mass flux of working fluid \dot{m}_s of specific volume v_s and pressure p_s :

$$\dot{L}_s = \dot{m}_s \cdot p_s \cdot v_s , \quad (8)$$

where: u_s – specific internal energy of working fluid at the suction channel, w_s – flow velocity at the suction channel, z_s – height of the mass centre at the control cross-section (axis of the inlet channel), g – local gravitational acceleration.

At the same time, the working fluid of mass m_w flows out through the pressure channel. Its energy consists of:

- internal energy:

$$\dot{U}_w = \dot{m}_w \cdot u_w , \quad (9)$$

- kinetic energy:

$$\dot{\epsilon}_{Kw} = \frac{\dot{m}_w \cdot w_w^2}{2} , \quad (10)$$

- potential energy:

$$\dot{\epsilon}_{Pw} = \dot{m}_w \cdot g \cdot z_w , \quad (11)$$

- work related to discharge the mass flux of working fluid \dot{m}_w of specific volume v_w and pressure p_w :

$$\dot{L}_w = \dot{m}_w \cdot p_w \cdot v_w , \quad (12)$$

where: u_w – specific internal energy of working fluid at the discharge channel, w_w – flow velocity at the discharge channel, z_w – height of the mass centre at the control cross-section (axis of the outlet channel).

Using a balance form of the first law of thermodynamics [10, 11], energy fluxes characterized by relations (4)÷(12) can be transformed into the following form:

$$\dot{\epsilon}_{Ps} + \dot{\epsilon}_{Ks} + \dot{L}_s + \dot{U}_s + \dot{L}_y = \dot{\epsilon}_{Pw} + \dot{\epsilon}_{Kw} + \dot{L}_w + \dot{U}_w , \quad (13)$$

what can be rewritten in the form:

$$\dot{L}_y = \Delta \dot{\epsilon}_P + \Delta \dot{\epsilon}_K + \Delta \dot{L}_c + \Delta \dot{U}_c . \quad (14)$$

Assuming that, in the suction-pressure unit, both suction and pressure channels are characterized by the same area of cross-section S_k , through which the same mass flux flows (2), the changes of kinetic energy of the fluid can be formulated as it follows

$$\Delta \dot{\epsilon}_K = \dot{\epsilon}_{Kw} - \dot{\epsilon}_{Ks} = \frac{\dot{m}_c \cdot (w_w^2 - w_s^2)}{2} = \frac{\dot{m}_c \cdot \left(\left(\frac{\dot{V}_w}{S_k} \right)^2 - \left(\frac{\dot{V}_s}{S_k} \right)^2 \right)}{2} , \quad (15)$$

assuming: $\dot{V}_s = \dot{m}_c \cdot v_s$, $\dot{V}_w = \dot{m}_c \cdot v_w$, the equation (15) can be rewritten in the form:

$$\Delta \dot{\epsilon}_K = \frac{\dot{m}_c^3 \cdot (v_w^2 - v_s^2)}{2 \cdot S_k^2} . \quad (16)$$

Potential energy of the working fluid depends on the location of pressure connector in relation to the suction one. The changes of the potential energy, based on relation (7) and (10) can be expressed by:

$$\Delta \dot{E}_p = \dot{E}_{p_w} - \dot{E}_{p_s} = \dot{m}_c \cdot g \cdot (z_w - z_s). \quad (17)$$

The increase of work stream related to the discharge of working fluid in relation to the work stream related to the suction of the working fluid is a function dependent on sucked and pressured volume and the pressure difference between inlet and outlet connectors:

$$\Delta \dot{L}_c = \dot{L}_w - \dot{L}_s = \dot{m}_c \cdot (p_w \cdot v_w - p_s \cdot v_s). \quad (18)$$

During the compression of working fluid, its internal energy increases. This gain can be described by the formula:

$$\Delta \dot{U}_c = \dot{U}_w - \dot{U}_s = \dot{m}_c \cdot (u_w - u_s). \quad (19)$$

Taking into account relations: (16)÷(19), the equation (14) takes a new form:

$$\dot{L}_y = \dot{m}_c \cdot \left(g \cdot (z_s - z_w) + \frac{\dot{m}_c^2 \cdot (v_s^2 - v_w^2)}{2 \cdot S_k^2} + p_w \cdot v_w - p_s \cdot v_s + u_w - u_s \right) \quad (20)$$

By introducing Gibbs formula [10] for suction inlet:

$$i_s = u_s + p_s \cdot v_s \quad (21)$$

and pressure inlet:

$$i_w = u_w + p_w \cdot v_w, \quad (22)$$

the equation (20) takes the form:

$$\dot{L}_y = \dot{m}_c \cdot (g \cdot (z_w - z_s) + k_{EK} \cdot (v_w^2 - v_s^2) + i_w - i_s), \quad (23)$$

where a coefficient of kinetic energy conversion is described by formula:

$$k_{EK} = \frac{\dot{m}_c^2}{2 \cdot S_k^2}. \quad (24)$$

Based on relation (23), specific work of the unit can be formulated as it follows:

$$l_y = g \cdot (z_w - z_s) + k_{EK} \cdot (v_w^2 - v_s^2) + i_w - i_s, \quad (25)$$

Characterized by equation (24), the kinetic energy conversion coefficient transforms the formula (25) into implicit function, which can be easily solved by computational methods. Literature [1, 10, 11] describes that, the differences of potential and kinematic energy can be neglected. This approach allows to simplify the equation (25) and present it as a difference between the enthalpies of sucked and discharged working fluid:

$$l_y = i_w - i_s. \quad (26)$$

3. TECHNICAL WORK OF AN IDEAL SUCTION-PRESSURE UNIT

While testing flow devices, the empirical functions of working medium pressure, depending on an angle of shaft rotation or a linear displacement of a piston, is determined. Transforming the functions into the relation between the pressure of working medium and its volume, a closed indicator diagram is obtained. Its area characterizes the indicated work performed by the particular machine. Determination of an indicator diagram is therefore essential for conducting energy analysis of the considered system.

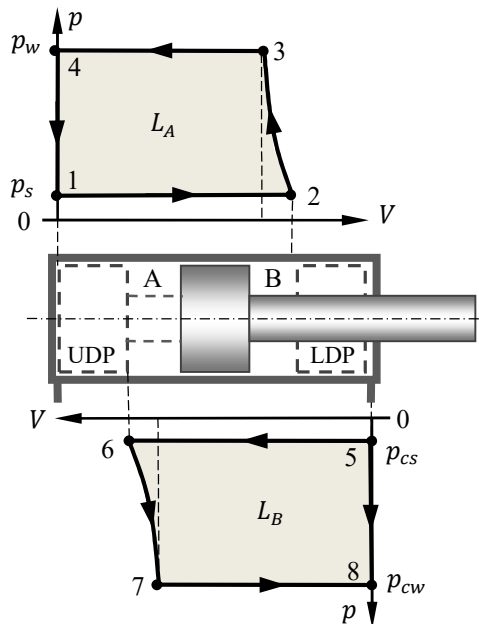


Fig. 7. Indicator diagrams of ideal suction-pressure unit

3.1. Indicator diagrams of an ideal suction-pressure unit

Working cycle of the ideal suction-pressure unit (one revolution of piston hydraulic pump) was presented in p - V diagram (fig. 7), which characterized two thermodynamic processes occurring at the same time but phase-shifted. The beginning of each cycle is a point at which the piston is positioned at upper dead position (UDP). Therefore, the volume of piston chamber A is equal to zero ($V_A = 0$) and the volume of piston rod B chamber is equal to cylinder capacity ($V_B = S_B \cdot y$), where y is a linear displacement of the piston and S_B is the area of piston calculated from A chamber side. The thermodynamic cycle of suction-pressure unit

consists of isobaric suction of the fluid (1-2, 5-6), isentropic compression (2-3, 6-7), isobaric discharge of the fluid (3-4, 7-8) and a virtual isochoric process. During a single cycle the working medium of mass m_c characterizes a specific mass efficiency of the ideal suction-pressure unit. Theoretical specific mass efficiency corresponds to theoretical specific volumetric efficiency:

$$V_c = m_c \cdot v_c. \quad (27)$$

Since two different work chambers have to be considered, theoretical specific volumetric efficiency of the described unit can be presented as a sum of volumetric efficiency of both chambers:

$$V_c = V_A + V_B = (m_A + m_B) \cdot v_c . \quad (28)$$

where: V_A – volume of A chamber, V_B – volume of B chamber.

During the filling of A chamber with the fluid (1-2), the work is performed in the isobaric process:

$$L_{1-2} = - m_A \cdot p_s \cdot v_s . \quad (29)$$

Afterwards, inlet valve is closed and isentropic compression process takes place (2-3). The work supplied to the system, which is needed for the process, can be formulated as it follows:

$$L_{2-3} = m_A \cdot (u_w - u_w) = \frac{\kappa}{\kappa - 1} \cdot p_s \cdot V_A \cdot \left[\left(\frac{p_w}{p_s} \right)^{\frac{\kappa-1}{\kappa}} - 1 \right] . \quad (30)$$

Assuming $k_\kappa = \frac{\kappa-1}{\kappa}$, it can be written that:

$$L_{2-3} = m_A \cdot p_s \cdot v_s \cdot \left[\frac{\left(\frac{p_w}{p_s} \right)^{k_\kappa} - 1}{k_\kappa} \right] , \quad (31)$$

where: $\kappa = c_p/c_v$, and: c_p – specific heat at a constant pressure, c_v – specific heat at a constant volume.

Once reaching the pressure p_w , outlet valve opens and the fluid is discharged through the channel into hydraulic system (3-4). The process can be mathematically described by relation:

$$L_{3-4} = m_A \cdot p_{cw} \cdot v_{cw} . \quad (32)$$

As an ideal piston pump is concerned, the dead volume is equal to zero. Thus, it can be assumed that there is no need to take isochoric process (4-1) into account. Technical work of the cycle for A chamber is expressed by:

$$L_A = \int_1^4 V dp = L_{1-2} + L_{2-3} + L_{3-4} . \quad (33)$$

Analogically, for B chamber it can be written that:

$$L_{5-6} = - m_B \cdot p_s \cdot v_s , \quad (34)$$

$$L_{6-7} = m_B \cdot p_{cs} \cdot v_s \cdot \left[\frac{\left(\frac{p_w}{p_s} \right)^{k_\kappa} - 1}{k_\kappa} \right] , \quad (35)$$

$$L_{7-8} = m_B \cdot p_{cw} \cdot v_w , \quad (36)$$

$$L_B = \int_5^8 V dp = L_{5-6} + L_{6-7} + L_{7-8} . \quad (37)$$

Based on the aforementioned relations, the total technical work of the ideal suction-pressure unit takes the form:

$$L = m_c \cdot \left(p_s \cdot v_s \cdot \left(\left[\frac{\left(\frac{p_w}{p_s} \right)^{k_\kappa} - k_\kappa - 1}{k_\kappa} \right] \right) + p_w \cdot v_w \right) \quad (38)$$

4. SUMMARY

In the article, the presented ideal suction-pressure unit differs significantly from the real one. However, it is vital to mathematically describe the processes occurring during its work cycle. Devised mathematical models describing the phenomena of interest will be used for conducting further research into suction-pressure units and, subsequently, the entire low speed pump. Based on the derived formulas, considering the additional internal and external factors such as friction, pressure drops, flow friction, the deformations of chambers and mechanical parts influencing the performance of the unit, a real model will be created. Energy analysis of suction-pressure unit plays a vital role in theoretical research into efficiency of different variants and designs of low speed pumps. This publication constitutes an introduction into detailed analysis of low speed radial piston pumps and multiplying hydrostatic gears.

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ANALIZA ENERGETYCZNA IDEALNEGO UKŁADU SSĄCO-TŁOCZĄCEGO

S t r e s z c z e n i e

W artykule zaprezentowano analizę energetyczną idealnego zespołu ssąco-tłoczącego promieniowej pompy tłokowej o dużej wydajności właściwej. Pompa ta ma stanowić ważną część multiplikującej przekładni hydrostatycznej, mogącej znaleźć zastosowanie w Małych Elektrowniach Wodnych. Ze względu na kluczową rolę zespołu ssąco-tłoczącego dla parametrów pompy, opisano jego budowę i zasadę działania. Scharakteryzowano model idealny oraz przedstawiono zjawiska w nim zachodzące. Wykonano bilans masowy, objętościowy oraz energetyczny prezentowanego zespołu. Opisano modele matematyczne zachodzących przemian energetycznych cieczy roboczej. Przedstawiono wykresy indykatorowe każdej z komór zespołu ssąco-tłoczącego, a na ich podstawie wyznaczono pracę techniczną przemian w nich zachodzących. Opracowanie to stanowi wstęp do szczegółowej analizy rzeczywistego zespołu ssąco-tłoczącego, uwzględniającej wszystkie czynniki wpływające na jego pracę, oraz powstające w nim straty.

Key words: hydraulika, pompa tłokowa