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STATIC AND DYNAMIC STIFFNESS OF CNC MACHINE TOOL SERVO DRIVES

In this paper a model for the servo drive system with disturbance forces is given. Static and dynamic stiffness for the proposed model is analyzed. An equation for analytical calculation of the static stiffness is given. Correctness of the proposed equation is experimentally verified. Simulation of the influence of some parameters on the static and dynamic servo drive system stiffness is performed with simulation program MATLAB & SIMULINK.

1. INTRODUCTION

Servo drive systems are widely applied to CNC machine tools, robots, manipulators, assembly machines etc. The servo drive system stiffness may be defined as an influence of the disturbance force (torque) on the position (angular position) deviation.

In the theory of the automatic control stiffness can be defined as reciprocal value of the stationary error of the position (angular position) caused by the disturbance force (torque) [16, 19].

Investigations about servo drive systems stiffness are very seldom presented in the literature. The results shown in [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20] are of particular interest. The most of the previous articles don't take into the consideration influence of the mechanical transmission elements on the servo drive stiffness. The researches [7,8,13,17,18] are more complete, but still have one imperfection, absence of analytical equation for estimation the servo drive system static stiffness.

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2. A MODEL OF THE SERVO DRIVE SYSTEM WITH DISTURBANCE FORCES

Fig. 1 and Fig. 2 show an original model of the servo drive system with disturbance forces.

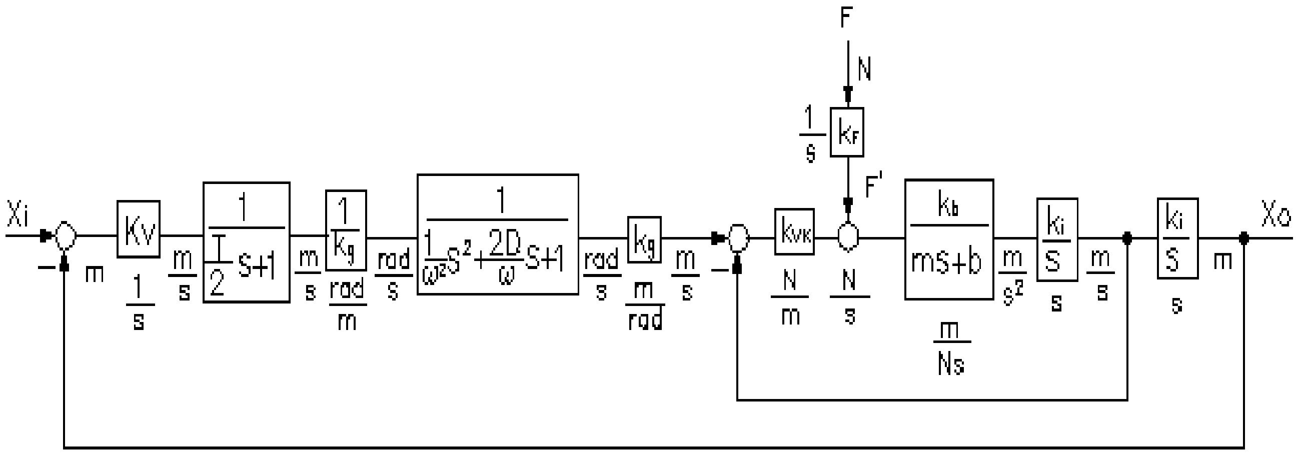

Fig. 1. Model of the servo drive system with disturbance forces

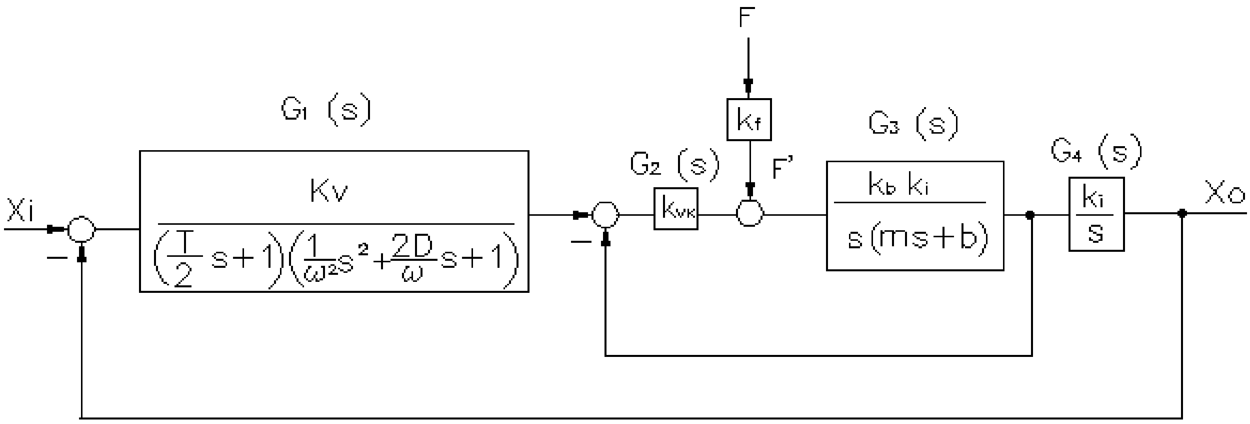

Fig. 2. The model from figure 1 after arranging

All the relevant parameters in Fig. 1 are given bellow: Kv-position loop gain 1/s, T-sampling period s, s-Laplace operator, kg-coefficient of transformation of rotation in translation m/rad, D-damping of the electrical parts, ω-nominal angular frequency of the electrical parts 1/s, kvk-total stiffness of the mechanical transmission elements N/m, F-disturbance force N, kf-disturbance force gain 1/s, kb-gain of the mechanical transmission damping m/Ns, m-mass of the mechanical transmission elements kg, b-damping quotient of the mechanical transmission elements Ns/m, ki-integrator gain s, Xi-input position, Xo-output position.

Factors kf, kb and ki are always 1. They are taken into the consideration, only to have dimensional correctness of the models, given on Fig. 1 and Fig. 2.

Transfer function between output position and disturbance force is given with equation (1).

$$\frac{Xo(s)}{F(s)}=\frac{k_f.G_3(s)\cdot G_4(s)}{1+G_2(s)\cdot G_3(s)+G_1(s)\cdot G_2(s)\cdot G_3(s)\cdot G_4(s)} \tag{1}$$

With substituting the transfer functions G1(s), G2(s), G3(s) and G4(s) from fig.2 in equation (1) we obtain

$$\frac{Xo(s)}{F(s)}=\frac{b_3s^3+b_2s^2+b_1s+b_0}{a_6s^6+a_5s^5+a_4s^4+a_3s^3+a_2s^2+a_1s+a_0} \tag{2}$$

Coefficients in the equation (2) are:

$$b_3=k_f k_b k_i^2\frac{T}{2\omega^2},\ b_2=k_f k_b k_i^2\left(\frac{1}{\omega^2}+\frac{DT}{\omega}\right),\ b_1=k_f k_b k_i^2\left(\frac{2D}{\omega}+\frac{T}{2}\right),\ a_6=m\frac{T}{2\omega^2},$$

$$a_5=\left[m\left(\frac{1}{\omega^2}+\frac{DT}{\omega}\right)+b\frac{T}{2\omega^2}\right],\ a_4=\left[m\left(\frac{2D}{\omega}+\frac{T}{2}\right)+b\left(\frac{1}{\omega^2}+\frac{DT}{\omega}\right)+k_{vk}k_b k_i\frac{T}{2\omega^2}\right],$$

$$a_3=\left[m+b\left(\frac{2D}{\omega}+\frac{T}{2}\right)+k_{vk}k_b k_i\left(\frac{1}{\omega^2}+\frac{DT}{\omega}\right)\right],\ a_2=\left[b+k_{vk}k_b k_i\left(\frac{2D}{\omega}+\frac{T}{2}\right)\right],$$

$a_1=k_{vk}k_b k_i$ and $a_0=Kv\cdot k_{vk}k_b k_i^2$.

3. STATIC AND DYNAMIC STIFFNESS OF THE SERVO DRIVE SYSTEM

One of the most important requirements with regard to servo drive system concerns its sensitivity to load disturbances.

The qualitative measure of this sensitivity is the servo drive system stiffness.

Dynamic servo drive system stiffness can be defined as a measure of influence of disturbance force F (torque T) on the output position Xo (angular position θo) deviation in the transient period.

$$Sd(s)=\frac{F(s)}{Xo(s)}=\frac{T(s)}{\theta o(s)} \tag{3}$$

For the model from Fig. 1 and Fig. 2 the equation for dynamic stiffness for servo drive systems become:

$$Sd(s)=\frac{a_6s^6+a_5s^5+a_4s^4+a_3s^3+a_2s^2+a_1s+a_0}{b_3s^3+b_2s^2+b_1s+b_0} \quad (4)$$

where the coefficients $a_6,a_5,a_4,a_3,a_2,a_1,a_0,b_3,b_2,b_1$ and b_0 are equal to the coefficients in the equation (2).

The static stiffness Sst of the servo drive system in [11,12,15,16,19] is defined as:

$$Sst=\lim_{s\to 0} Sd(s) \quad (5)$$

The static stiffness of the servo drive system model on Fig. 1 and Fig. 2 can be defined as:

$$Sst=\lim_{s\to 0}\frac{a_6s^6+a_5s^5+a_4s^4+a_3s^3+a_2s^2+a_1s+a_0}{b_3s^3+b_2s^2+b_1s+b_0}=\frac{a_0}{b_0} \quad (6)$$

With substitution $a_0=Kv\cdot k_{vk}k_bk_i^2$ and $b_0=k_fk_bk_i^2$ in equation (6) we obtain

$$Sst=\frac{Kv\cdot k_{vk}k_bk_i^2}{k_fk_bk_i^2}=\frac{Kv\cdot k_{vk}}{k_f}\ \text{N/m} \quad (7)$$

As we mentioned always k_f=1 1/s. In that case equation (7) becomes

$$Sst=\frac{Kv[1/s]\cdot k_{vk}[N/m]}{1\cdot[1/s]}=Kv\cdot k_{vk}\quad \text{N/m} \quad (8)$$

From equation (8) it is obvious that the static servo drive system stiffness is defined as a product between position loop gain Kv and total stiffness of the mechanical transmission elements k_{vk}.

4. SIMULATION OF THE SERVO DRIVE WITH DISTURBANCE FORCE

Influence of the sampling period T, nominal angular frequency of the electrical parts ω, damping of the electrical parts D, damping gradient of the mechanical transmission elements b, mass of the mechanical transmission elements m, total stiffness of the mechanical transmission elements kvk and position loop gain Kv, on the dynamic and static stiffness has been investigated with simulation program MATLAB & SIMULINK.

In the simulations one parameter has been changed, and the other parameters were held constant.

In fact with the simulation, a position deviation in time domain Xo(t) caused by step disturbance force F=1 N, is shown.

Simulation objects are models shown on Fig. 1 and Fig. 2.

$$Xo(t)=L^{-1}\left[\frac{Xo(s)}{F(s)}\cdot\frac{1}{s}\right]m \qquad (9)$$

To estimate dynamic stiffness Sd we can use following equation,

$$Sd=\frac{F}{\max Xo(t)}\ N/m \qquad (10)$$

where F is disturbance force and maxXo(t) is maximal position deviation.

In all simulation examples F was 1N.

We can calculate static stiffness Sst when the transient process will disappear and Xo(t) become constant. Theoretically

$$Sst=\frac{F}{\lim\limits_{t\to\infty} Xo(t)}\quad N/m \qquad (11)$$

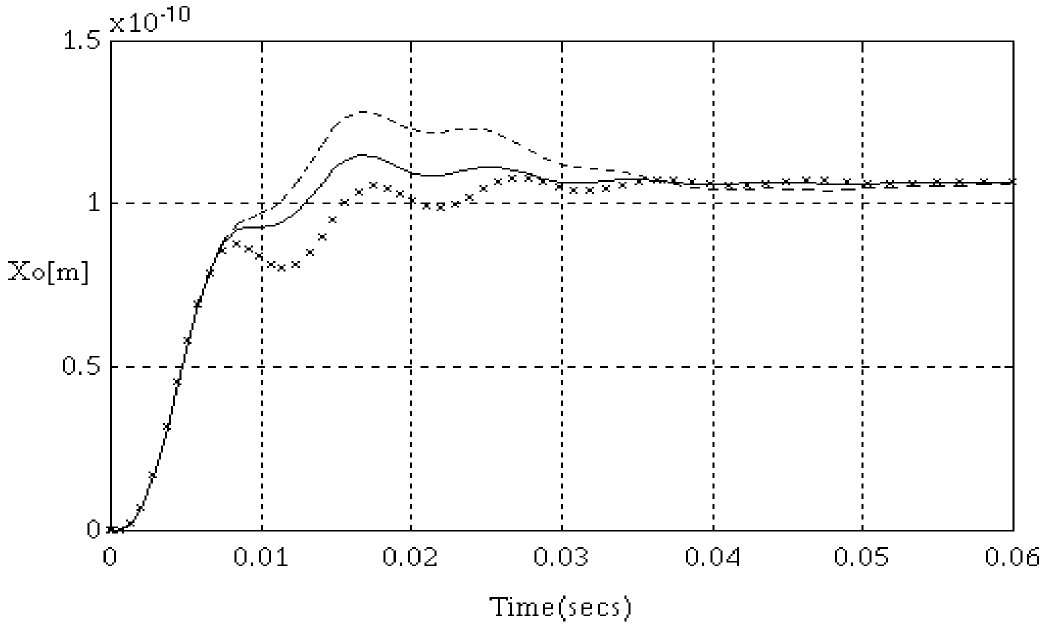

Fig. 3. Influence of the sampling period T on the position deviation caused by disturbance force F=1N (D=0.7, ω=1000 s^{-1}, m=180kg, b=25220 Ns/m, kvk=88.34·10^6 N/m, Kv=106.34 s^{-1})

(——) T=0.006 s (- - -) T=0.010 s (x x x) T=0.002 s

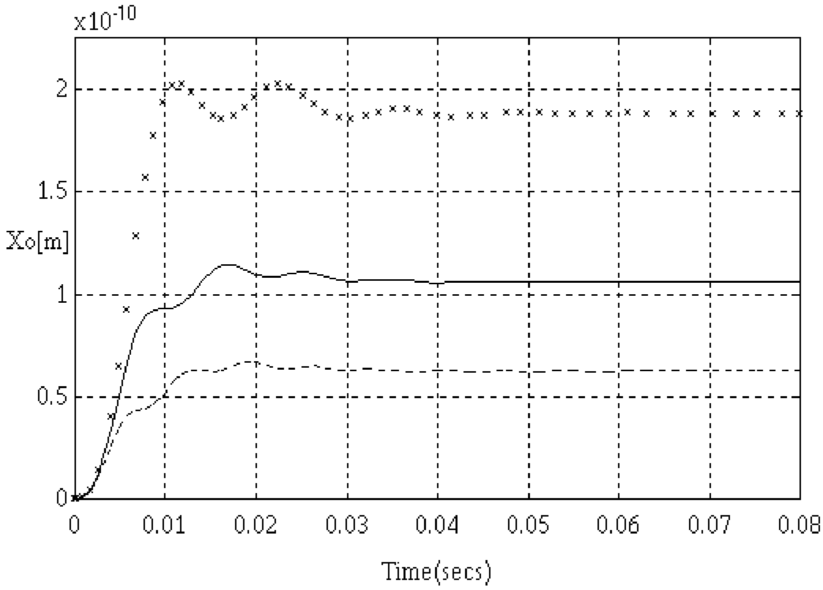

Fig. 4. Influence of the total stiffness of the mechanical transmission elements kvk on the position deviation caused by disturbance force F=1N

(D=0.7, ω=1000 s^{-1}, T=0.006s, m=180 kg, b=25220 Ns/m, Kv=106.34 s^{-1}).
(——)kvk=88.34·10^6 N/m, (- - -)kvk=150·10^6 N/m, (x x x)kvk=50·10^6 N/m

Simulations have shown that increasing sampling period T, damping of the electrical parts D and mass of the mechanical transmission elements m, decrease dynamic stiffness. On the other hand increasing nominal angular frequency of the electrical parts ω, damping gradient of the mechanical transmission elements b, total stiffness of the mechanical transmission elements kvk and position loop gain Kv, increase dynamic stiffness. With increasing total stiffness of the mechanical transmission elements kvk and position loop gain Kv, increase static stiffness too.

Some examples of the simulation are shown on Fig. 3 and Fig. 4.

5. EXPERIMENTAL VERIFICATION OF THE EQUATION FOR CALCULATING THE STATIC STIFFNESS FOR THE SERVO DRIVE SYSTEM

In order to verify the validity of the equation (8), an experiment on real servo drive system of CNC milling machine was performed.

The experimental installation is given on Fig. 5.

The results of the experiment are shown on Fig. 6.

It is obvious that with the appropriate changing of the position loop gain Kv, the elasticity of the system can be controlled.

From Table 1 it is evident that the difference between experimentally obtained end analytically calculated value of the static stiffness of the servo drive system is ±10% which is acceptable and sufficient for practice.

Table 1. Survey between experimentally obtained and analytically calculated (with equation 8) static stiffness of the real servo drive system.

	$Kv=28.33s^{-1}$	$Kv=100s^{-1}$
Sst (exper.) N/µm	2650	8025
Sst (analytic..) N/µm	2503	8834
difference %	-5.55	+10.08

difference%=[Sst(analytic.)-Sst(exper.)]/Sst(exper.)·100%

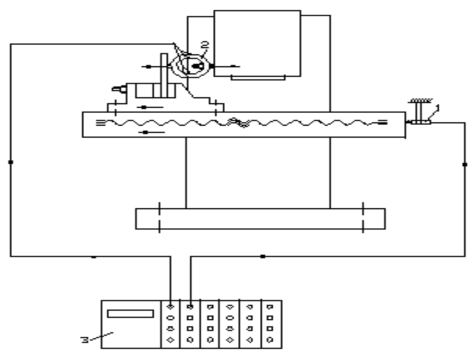

Fig. 5. Experimental installation for determining static stiffness of the position servo system for X-axes for CNC milling machine: 1.inductive transducer for displacement (HBM W1T/2), 2.device for generating disturbance force F, 3.device for registration the values of the displacement (position deviation) Xo and disturbance force F (HBM DA24 KWS3073)

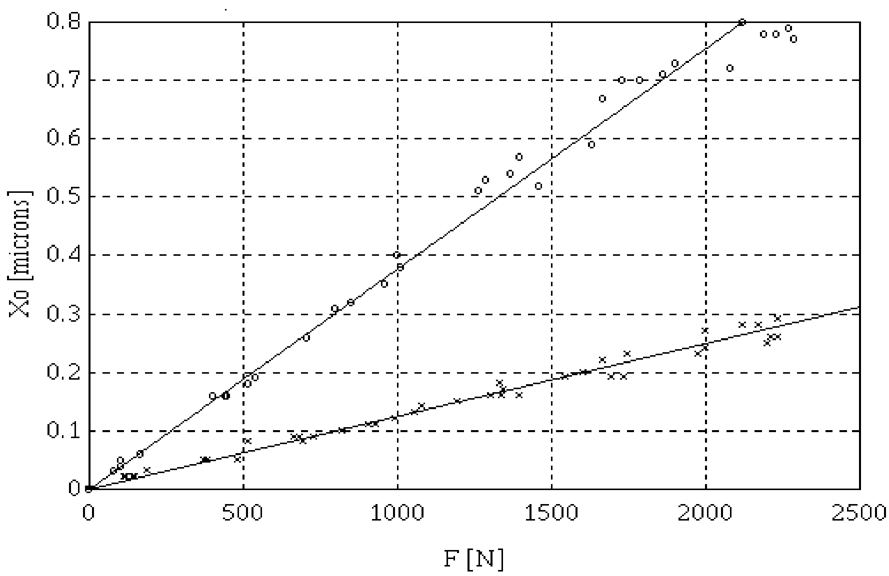

Fig. 6. Experimentally obtained dependency of the position deviation Xo [microns] and disturbance force F [N] for position servo system of CNC milling machine
(o - $Kv=28.33\ s^{-1}$), (x - $Kv=100\ s^{-1}$)

6. CONCLUSIONS

A model of servo drive system with disturbance force was proposed. It has been shown by simulation that bigger values for sampling period T, damping of the electrical parts D and mass of the mechanical transmission elements m, decrease dynamic stiffness. On the other hand bigger values for nominal angular frequency of the electrical parts ω, damping gradient of the mechanical transmission elements b, total stiffness of the mechanical transmission elements kvk and position loop gain Kv, increase dynamic stiffness. Increasing total stiffness of the mechanical transmission elements kvk and position loop gain Kv, increase static stiffness too.

An equation for analytically calculation of the servo drive systems static stiffness as a product of the total stiffness of mechanical transmission elements kvk and position loop gain Kv, was proposed.

Correctness of the equation was experimentally verified. The difference was in ±10% limits, which is acceptable for practical use.

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