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ON KNOWLEDGE DISCOVERY AND REPRESENTATIONS OF MOLECULAR STRUCTURES USING TOPOLOGICAL INDICES

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Abstract

The main purpose of a topological index is to encode a chemical structure by a number. A topological index is a graph invariant, which decribes the topology of the graph and remains constant under a graph automorphism. Topological indices play a wide role in the study of QSAR (quantitative structure-activity relationship) and QSPR (quantitative structure-property relationship). Topological indices are implemented to judge the bioactivity of chemical compounds. In this article, we compute the *ABC* (atom-bond connectivity); ABC_4 (fourth version of *ABC*), *GA*(geometric arithmetic) and GA_5(fifth version of *GA*) indices of some networks sheet. These networks include: octonano window sheet; equilateral triangular tetra sheet; rectangular sheet; and rectangular tetra sheet networks.

Keywords: graph network, sheet, topological index, cheminformatics, knowledge discovery

1 Introduction and preliminary results

In chemical graph theory, an interesting subfield called *Cheminformatics* deals with a chemical phenomenon known as quantitative structure-activity/ structure- property relationships of chemical compounds. The methods of cheminformatics are also used for prediction of properties relevant to the drug discovery and optimization process. For example,knowledge discoverycan be used for the identification of lead compounds in pharmaceutical data matching. An emerging tool, used in the study of these phenomena, is a topological index which remains constant for all chemical structures up to their symmetries. The study of the topological indices on chemical structure drugs can provide a theoretical basis for the manufacturing of drugs and chemical materials. As a consequence, lack of chemical experiments is made up. A number of topological indices are determined in view of edge dividing methods which provide remedy to the lack of medicine experiments. In other words, computation of topological indices provides a theoretical basis for pharmaceutical engineering. Correlation of many physico-chemical properties like boiling point; stability; and strain energy of these chemical compounds in a chemical structure is explained by their topological indices [2, 7, 17, 19, 20].

In this era of rapid technological development, chemical and pharmaceutical techniques in recent years have been rapidly evolved, and thus a large number of new nanomaterials, crystalline materials, and drugs emerge every year. To determine the chemical properties of such a large number of new compounds and new drugs requires a large amount of chemical experiments, thereby greatly increasing the workload of the chemical and pharmaceutical researchers. Fortunately, the chemical based experiments found that there was strong connection between topology molecular structures and their physical behaviors, chemical characteristics, and biological features, such as melting point, boiling point, and toxicity of drugs (see Wiener [24] as examples).

The description of a graph can be a number; a polynomial; a sequence of numbers; or a matrix. A numerical quantity related to a graph that represents the topology of the graph is a topological index. There are several main types of topological indices such as distance based topological indices and degree based topological indice. Among these indices, a degree-based topological index is very important and plays a vital role in chemical graph theory. More precisely, a topological index $Top(G)$ of a graph G is a number that remains the for every graph H such that$H \cong G$. In other words, we have $Top(H) - Top(G) = 0$. In [24], Wiener introduced the first topological index while working on the boiling point of paraffin. He named this index the *path number*. After a period of time, the path number was renamed *Wiener index* in [5] and the theory of topological indices got attention from many researchers.

In present work, by a graph G we always mean a *network* with vertex set $V(G)$ and edge set $E(G)$. We denote the degree of a vertex u of G by $deg(u)$ (degree of a vertex u of a graph is the number of edges that are incident to the vertex u) and S_u is the sum of degrees of vertex v which is the in neighbour of vertex u i.e., $S_u = \sum_{v\in N_G(u)} deg(v)$, where $N_G(u)$ is the of vretices which are in the neighbours of vertex u, i.e., $N_G(u) = v \in V(G)|uv \in E(G)$.

The *atom-bond connectivity* (ABC) is the well-known degree based topological index, which is introduced by Estrada *et al.* [7] and defined as

$$ABC(G) = \sum_{uv\in E(G)} \sqrt{\frac{deg(u)+deg(v)-2}{deg(u)deg(v)}}. \quad (1)$$

The *geometric-arithmetic* (GA) index is due to Vukičevic′ *et al.* [23] and defined as

$$GA(G) = \sum_{uv\in E(G)} \frac{2\sqrt{deg(u)deg(v)}}{(deg(u)+deg(v))}. \quad (2)$$

Ghorbani *et al.* [9] introduced the fourth version of ABC index denoted by $ABC_4(G)$ and defined as

$$ABC_4(G) = \sum_{uv\in E(G)} \sqrt{\frac{S_u+S_v-2}{S_uS_v}}. \quad (3)$$

Recently, the fifth version of GA index is proposed by Graovac *et al.* [10] and defined as

$$GA_5(G) = \sum_{uv\in E(G)} \frac{2\sqrt{S_uS_v}}{(S_u+S_v)}. \quad (4)$$

The aforementioned topological invariants are hugely studied in literature. Some of the work done in this direction can be found in [1-24].

In present work, we study the ABC, ABC_4, GA, and GA_5 indices of certain networks sheets including octo-nanowindows sheet; enhanced mesh or rectangular sheets; HDN like networks namely equilateral triangular tetra sheets; and rectangular tetra sheets networks.

2 Octo - nanowindows network sheet

In this Section, we are in the position to compute ABC, ABC_4, GA, and GA_5 indices of octo-nanowindow sheet networks.

2.1 Construction of octo - nanowindow sheet

First we draw an octagon where the vertices are the eight corners and an edge links these corners with a length of 2 units. We paste the p octagons row wise and q octagons column wise to get a graph G which is known as octo-sheet. Connect the degree 2 vertices of each octagon which are at distance 4 units. Finally, introduce 4 new corner vertices and connect them to the corner vertices of the corner octagon with a new edge of unit 1. The graph obtained in this way is called octo-nanowindow $ONW(4p,4q)$ having $2(2pq+p+q+2)$ vertices and $6pq+3p+3q+4$ edges. The graph of $ONW(4p,4q)$ is shown in Figure 1.

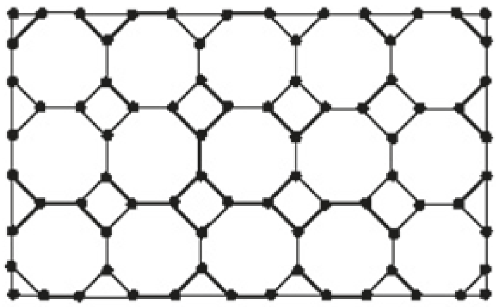

Figure 1. The octo-nanowindo $ONW(4p,4q)$ network, where $p=3$ and $q=5$.

Table 1. Edge partition of $ONW(4p,4q)$ which depends on the degrees of the final vertices located at unit distance from each edge.

(d_u,d_v) where $uv \in E(G)$	Number of Edges
$(2,3)$	8
$(3,3)$	$6pq+3p+3q-4$

The following theorem gives the ABC and GA indices of octo-nanowindow $ONW(4p,4q)$.

Theorem 2.1 *Suppose that G is the graph of octo-nanowindow $ONW(4p,4q)$. Then, $ABC(G) = 4pq+2p+2q+(4\sqrt{2}-\frac{8}{3})$, for $p,q>1$.* *Proof.* By following the data given in Table 1, we have $ABC(G)=\sum_{uv\in E(G)}\sqrt{\frac{d_u+d_v-2}{d_ud_v}}=8\sqrt{\frac{2+3-2}{2\times 3}}+(6pq+3p+3q-4)\sqrt{\frac{3+3-2}{3\times 3}}$. After an easy calculation, we get $ABC(G)=4pq+2p+2q+(4\sqrt{2}-\frac{8}{3})$.

Theorem 2.2 *With the same notations, we have $GA(G)=6pq+3p+3q+4(\frac{4\sqrt{6}}{5}-1)$, for $p,q>1$.* *Proof.* By using the information about the edge partition given in Table 1, we have $GA(G)=\sum_{uv\in E(G)}2\frac{\sqrt{d_ud_v}}{(d_u+d_v)}=2\frac{\sqrt{2\times 3}}{2+3}\times 8+2\frac{\sqrt{3\times 3}}{3+3}\times(6pq+3p+3q-4)$.

$$GA(G)=6pq+3p+3q+4(\frac{4\sqrt{6}}{5}-1).$$

Table 2. Edge partition of octo-nanowindow based on vertices degree sum which are located at unit distance from the final vertices of each edge.

(S_u,S_v) where $uv \in E(G)$	Number of edges
$(6,8)$	8
$(8,8)$	4
$(8,9)$	8
$(9,9)$	$6pq+3p+3q-16$

In next two results, we shall calculate the $ABC_4$and GA_5 index for the graph G of octo-nanowindow.

Theorem 2.3 *Let G be the graph of octo-nanowindow. Then,*

$$ABC_4(G)=[\tfrac{8}{3}]pq+\tfrac{4}{3}p+\tfrac{4}{3}q-\tfrac{28}{9}+\tfrac{\sqrt{14}}{2}+\tfrac{2\sqrt{30}}{3}$$

holds for $p,q>1$.

Proof. The formula for ABC_4 index is

$$ABC_4(G)=\sum_{uv\in E(G)}\sqrt{\frac{S_u+S_v-2}{S_uS_v}}.$$

By using the information given in Table 2, we get

$$ABC_4(G) = 8\sqrt{\frac{6+8-2}{6\times 8}}+4\sqrt{\frac{8+8-2}{8\times 8}}+8\sqrt{\frac{8+9-2}{8\times 9}}+(6pq+3p+3q-16)\sqrt{\frac{9+9-2}{9\times 9}},$$

$$ABC_4(G)=[\tfrac{8}{3}]pq+\tfrac{4}{3}p+\tfrac{4}{3}q-\tfrac{28}{9}+\tfrac{\sqrt{14}}{2}+\tfrac{2\sqrt{30}}{3}.$$

Theorem 2.4 *Suppose G is the graph of octo-nanowindow. Then, $GA_5(G)=6pq+3p+3q+\frac{32\sqrt{3}}{7}+\frac{96\sqrt{3}}{17}-12$, holds for $p,q>1$.*

Proof. By using the information in the second colmun of Table 2, we have $GA_5(G)=\sum_{uv\in E(G)}\frac{2\sqrt{S_uS_v}}{(S_u+S_v)}$ $=\frac{2\sqrt{6\times 8}}{6+8}\times 8+\frac{2\sqrt{8\times 8}}{8+8}\times 4+\frac{2\sqrt{8\times 9}}{8+9}\times 8+\frac{2\sqrt{9\times 9}}{9+9}\times(6pq+3p+3q-16)$

This implies

$$GA_5(G)=6pq+3p+3q+\frac{32\sqrt{3}}{7}+\frac{96\sqrt{3}}{17}-12.$$

3 Equilateral triangular tetra sheets network

This Section is devoted to the study of degree-based topological descriptors of the equilateral triangular tetra sheet.

We denote a graph of equilateral triangular tetra sheet network with dimension n by $ETTS(n)$. The dimension n of $ETTS(n)$ is represented by arranging n vertices on one side of the triangle structure.

3.1 Construction of the graph $ETTS(n)$

First we draw a triangle having n vertices on each side. We connect a vertex from one side of the triangle to the corresponding vertices of the other two sides of the triangle by an edge. Introduce new vertices in the intersecting edges. Replace all K_3 with K_4. (see Figure 3). The graph obtained in this way is called equilateral triangular tetra sheet with dimension n. The number of vertices and edges in $ETTS(n)$ are $\frac{(3n^2-3n+2)}{2}$ and $\frac{(9n^2-15n+6)}{2}$ respectively.We note that the corner vertices of the graph $ETTS(n)$ with degree $(3,3)$ are at equal distances to all other vertices for each n, where $n \geq 2$. In the following theorems, we compute the ABC and GA indices for the graph of equilateral triangular tetra sheet network.

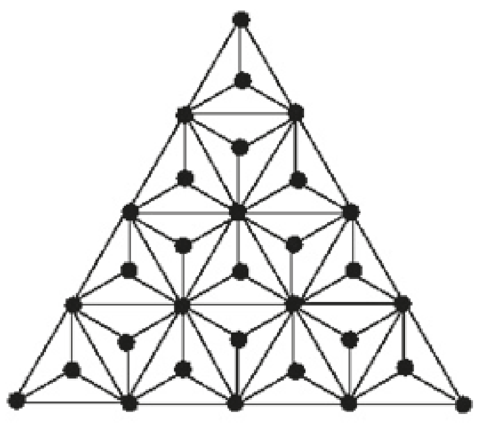

Figure 2. The equilateral tetra sheet network ETTS(5)

Table 3. Edge partition of $ETTS(5)$ based on degree of final vertices of each edge, where $uv \in E(G)$

(d_u,d_v)	Number of Edges
$(3,3)$	3
$(3,7)$	$9n-12$
$(7,7)$	$3n-6$
$(3,12)$	$3(n-3)(n-2)$
$(7,12)$	$6n-18$
$(12,12)$	$\frac{3}{2}(n-4)(n-3)$

Theorem 3.1 *Suppose that Gis a graph such that* $G \cong ETTS(n)$. *Then* $ABC(ETTS(n)) = (n^2)[\frac{\sqrt{22}}{8}+\frac{\sqrt{13}}{2}]+n[\frac{6}{7}(\sqrt{42})+\frac{6}{7}(\sqrt{3})-\frac{5}{2}(\sqrt{13})+\frac{\sqrt{357}}{7}-\frac{7}{8}\sqrt{22}]+[2+\frac{8}{7}(\sqrt{42})-\frac{12}{7}(\sqrt{3})+3\sqrt{13}-\frac{3}{7}(\sqrt{357})+\frac{3}{2}(\sqrt{22})]$, *holds for* $n \geq 3$. *Proof.* By using the information about the edge partition given in Table 3, we have $ABC(G)=\sum_{uv\in E(G)}\sqrt{\frac{d_u+d_v-2}{d_ud_v}} = 3\sqrt{\frac{3+3-2}{3\times 3}}+(9n-12)\sqrt{\frac{3+7-2}{3\times 7}}+(3n-6)\sqrt{\frac{7+7-2}{7\times 7}}+3(n-3)(n-2)\sqrt{\frac{3+12-2}{3\times 12}}+(6n-18)\sqrt{\frac{7+12-2}{7}\times 12}+\frac{3}{2}(n-4)(n-3)\sqrt{\frac{12+12-2}{12\times 12}}$.

$ABC(G) = (n^2)[\frac{\sqrt{22}}{8}+\frac{\sqrt{13}}{2}]+n[\frac{6}{7}(\sqrt{42})+\frac{6}{7}(\sqrt{3})-\frac{5}{2}(\sqrt{13})+\frac{\sqrt{357}}{7}-\frac{7}{8}\sqrt{22}]+[2+\frac{8}{7}(\sqrt{42})-\frac{12}{7}(\sqrt{3})+3\sqrt{13}-\frac{3}{7}(\sqrt{357})+\frac{3}{2}(\sqrt{22})]$.

In the following theorem, we calculate the GA index of $ETTS(n)$

Theorem 3.2 *Assume that G is a graph of* $ETTS(n)$. *Then, the following holds for* $n \geq 3$ $GA(G) = n^2[\frac{39}{10}]+n[\frac{9}{5}(\sqrt{21})+\frac{24}{19}(\sqrt{21})-\frac{39}{2}]+[\frac{147}{5}-12\frac{\sqrt{21}}{5}+72\frac{\sqrt{21}}{19}]$.

Proof. By following the information given in Table 3, we get $GA(G)=\sum_{uv\in E(G)}2\frac{\sqrt{d_ud_v}}{(d_u+d_v)} = 3\times 2\frac{\sqrt{3\times 3}}{3+3}+(9n-12)\times 2\frac{\sqrt{3\times 7}}{3+7}+(3n-6)\times 2\frac{\sqrt{7\times 7}}{7+7}+3(n-3)(n-2)\times 2\frac{\sqrt{3\times 12}}{3+12}+(6n-18)\times 2\frac{\sqrt{7\times 12}}{7+12}+\frac{3}{2}(n-4)(n-3)\times 2\frac{\sqrt{12\times 12}}{12+12}$.

$GA(G) = n^2[\frac{39}{10}]+n[\frac{9}{5}(\sqrt{21})+\frac{24}{19}(\sqrt{21})-\frac{39}{2}]+[\frac{147}{5}-12\frac{\sqrt{21}}{5}+72\frac{\sqrt{21}}{19}]$.

Table 4. Edge Partition of $ETTS(n)$ based on vertices degree sum which are located at unit distance from the final vertices of each edge.

$(S_u,S_v) where uv \in E(G)$	Number of edges
$(17,17)$	3
$(17,38)$	12
$(38,38)$	3
$(38,47)$	6
$(38,70)$	6
$(47,70)$	6
$(70,80)$	6
$(38,26)$	12
$(26,70)$	9
$(31,70)$	6
$(36,70)$	3
$(47,80)$	$6(n-5)$
$(80,80)$	$3(n-5)$
$(47,47)$	$3(n-5)$
$(80,90)$	$6(n-6)$
$(90,90)$	$\frac{3(n^2-13n+42)}{2}$
$(47,26)$	$6(n-4)$
$(26,80)$	$3(n-5)$
$(47,31)$	$3(n-4)$
$(31,80)$	$6(n-5)$
$(36,80)$	$9(n-5)$
$(36,90)$	$3(n^2-11n+30)$

In Theorem 3.3 and Theorem 3.4, we are in the position to compute the ABC_4 and GA_5 indices of a graph G, where $G \cong ETTS(n)$. For this purpose, the edge partition of G based on the degree sum of final vertices of each edge is given in the following Table.

Theorem 3.3 *For $n \geq 6$, the ABC_4 index of a graph $G \cong ETTS(n)$ is* $ABC_4(G)) = (\frac{\sqrt{89}}{30\sqrt{2}} + \frac{\sqrt{31}}{3\sqrt{10}})n^2 + (\frac{15}{2\sqrt{47}} + \frac{3\sqrt{79}}{40\sqrt{2}} + 6\sqrt{\frac{23}{2209}} + \frac{3\sqrt{7}}{5\sqrt{3}} - \frac{13\sqrt{89}}{30\sqrt{2}} + 6\sqrt{\frac{71}{1222}} + \frac{3}{2\sqrt{5}} + 6\sqrt{\frac{19}{1457}} + \frac{3\sqrt{109}}{2\sqrt{155}} + \frac{9\sqrt{19}}{4\sqrt{30}} - \frac{11\sqrt{31}}{3\sqrt{10}})n + \frac{12\sqrt{2}}{17} + 12\sqrt{\frac{53}{646}} + \frac{3\sqrt{37}}{19\sqrt{2}} + 6\sqrt{\frac{83}{1786}} + 6\sqrt{\frac{53}{1330}} + 6\sqrt{\frac{23}{658}} + \frac{3\sqrt{37}}{5\sqrt{14}} + 12\sqrt{\frac{31}{494}} + 9\sqrt{\frac{47}{910}} + 18\sqrt{\frac{11}{2170}} + \sqrt{\frac{13}{35}} - \frac{75}{2\sqrt{47}} - \frac{3\sqrt{79}}{8\sqrt{2}} - 30\sqrt{\frac{23}{2209}} - \frac{18\sqrt{7}}{5\sqrt{3}} + \frac{7\sqrt{89}}{5\sqrt{2}} - 24\sqrt{\frac{71}{1222}} - \frac{15}{2\sqrt{5}} - 24\sqrt{\frac{19}{1457}} - \frac{15\sqrt{109}}{2\sqrt{155}} - \frac{45\sqrt{19}}{4\sqrt{30}} + 10\sqrt{\frac{31}{10}}$.

Proof. Since we know that, the formula for computing ABC_4 index of the given graph is $ABC_4(G) = \sum_{uv\in E(G)} \sqrt{\frac{S_u+S_v-2}{S_uS_v}}$. By inserting the information given in Table 4, whave $ABC_4(G) = 3\sqrt{\frac{17+17-2}{17\times17}} + 12\sqrt{\frac{17+38-2}{17\times38}} + 3\sqrt{\frac{38+38-2}{38\times38}} + 6\sqrt{\frac{38+47-2}{38\times47}} + 6\sqrt{\frac{38+70-2}{38\times70}} + 6\sqrt{\frac{47+70-2}{47\times70}} + 6\sqrt{\frac{70+80-2}{70\times80}} + 12\sqrt{\frac{38+26-2}{38\times26}} + 9\sqrt{\frac{26+70-2}{26\times70}} + 6\sqrt{\frac{31+70-2}{31\times70}} + 3\sqrt{\frac{36+70-2}{36\times70}} + 6(n-5)\sqrt{\frac{47+80-2}{47\times80}} + 3(n-5)\sqrt{\frac{80+80-2}{80\times80}} + 3(n-5)\sqrt{\frac{47+47-2}{47\times47}} + 6(n-6)\sqrt{\frac{80+90-2}{80\times90}} + (\frac{3(n^2-13n+42)}{2})\sqrt{\frac{90+90-2}{90\times90}} + 6(n-4)\sqrt{\frac{47+26-2}{47\times26}} + 3(n-5)\sqrt{\frac{26+80-2}{26\times80}} + 3(n-4)\sqrt{\frac{47+31-2}{47\times31}} + 6(n-5)\sqrt{\frac{31+80-2}{31\times80}} + 9(n-5)\sqrt{\frac{36+80-2}{36\times80}} + 3(n^2-11n+30)\sqrt{\frac{36+90-2}{36\times90}}$

$ABC_4(G) = 3\sqrt{\frac{17+17-2}{17\times17}} + 12\sqrt{\frac{17+38-2}{17\times38}} + 3\sqrt{\frac{38+38-2}{38\times38}} + 6\sqrt{\frac{38+47-2}{38\times47}} + 6\sqrt{\frac{38+70-2}{38\times70}} + 6\sqrt{\frac{47+70-2}{47\times70}} + 6\sqrt{\frac{70+80-2}{70\times80}} + 12\sqrt{\frac{38+26-2}{38\times26}} + 9\sqrt{\frac{26+70-2}{26\times70}} + 6\sqrt{\frac{31+70-2}{31\times70}} + 3\sqrt{\frac{36+70-2}{36\times70}} + 6(n-5)\sqrt{\frac{47+80-2}{47\times80}} + 3(n-5)\sqrt{\frac{80+80-2}{80\times80}} + 3(n-5)\sqrt{\frac{47+47-2}{47\times47}} + 6(n-6)\sqrt{\frac{80+90-2}{80\times90}} + (\frac{3(n^2-13n+42)}{2})\sqrt{\frac{90+90-2}{90\times90}} + 6(n-4)\sqrt{\frac{47+26-2}{47\times26}} + 3(n-5)\sqrt{\frac{26+80-2}{26\times80}} + 3(n-4)\sqrt{\frac{47+31-2}{47\times31}} + 6(n-5)\sqrt{\frac{31+80-2}{31\times80}} + 9(n-5)\sqrt{\frac{36+80-2}{36\times80}} + 3(n^2-11n+30)\sqrt{\frac{36+90-2}{36\times90}}$ $ABC_4(G) = 3\sqrt{\frac{17+17-2}{17\times17}} + 12\sqrt{\frac{17+38-2}{17\times38}} + 3\sqrt{\frac{38+38-2}{38\times38}} + 6\sqrt{\frac{38+47-2}{38\times47}} + 6\sqrt{\frac{38+70-2}{38\times70}} + 6\sqrt{\frac{47+70-2}{47\times70}} + 6\sqrt{\frac{70+80-2}{70\times80}} + 12\sqrt{\frac{38+26-2}{38\times26}} + 9\sqrt{\frac{26+70-2}{26\times70}} + 6\sqrt{\frac{31+70-2}{31\times70}} + 3\sqrt{\frac{36+70-2}{36\times70}} + 6(n-5)\sqrt{\frac{47+80-2}{47\times80}} + 3(n-5)\sqrt{\frac{80+80-2}{80\times80}} + 3(n-5)\sqrt{\frac{47+47-2}{47\times47}} + 6(n-6)\sqrt{\frac{80+90-2}{80\times90}} + (\frac{3(n^2-13n+42)}{2})\sqrt{\frac{90+90-2}{90\times90}} + 6(n-4)\sqrt{\frac{47+26-2}{47\times26}} + 3(n-5)\sqrt{\frac{26+80-2}{26\times80}} + 3(n-4)\sqrt{\frac{47+31-2}{47\times31}} + 6(n-5)\sqrt{\frac{31+80-2}{31\times80}} + 9(n-5)\sqrt{\frac{36+80-2}{36\times80}} + 3(n^2-11n+30)\sqrt{\frac{36+90-2}{36\times90}}$

Further simplification gives us the following form $ABC_4(G) = \frac{\sqrt{89}}{30\sqrt{2}} + \frac{\sqrt{31}}{3\sqrt{10}})n^2 + (\frac{15}{2\sqrt{47}} + \frac{3\sqrt{79}}{40\sqrt{2}} + 6\sqrt{\frac{23}{2209}} + \frac{3\sqrt{7}}{5\sqrt{3}} - \frac{13\sqrt{89}}{30\sqrt{2}} + 6\sqrt{\frac{71}{1222}} +$

$\frac{3}{2\sqrt{5}} + 6\sqrt{\frac{19}{1457}} + \frac{3\sqrt{109}}{2\sqrt{155}} + \frac{9\sqrt{19}}{4\sqrt{30}} - \frac{11\sqrt{31}}{3\sqrt{10}})n + (\frac{12\sqrt{2}}{17} + 12\sqrt{\frac{53}{646}} + \frac{3\sqrt{37}}{19\sqrt{2}} + 6\sqrt{\frac{83}{1786}} + 6\sqrt{\frac{53}{1330}} + 6\sqrt{\frac{23}{658}} + \frac{3\sqrt{37}}{5\sqrt{14}} + 12\sqrt{\frac{31}{494}} + 9\sqrt{\frac{47}{910}} + 18\sqrt{\frac{11}{2170}} + \sqrt{\frac{13}{35}} - \frac{75}{2\sqrt{47}} - \frac{3\sqrt{79}}{8\sqrt{2}} - 30\sqrt{\frac{23}{2209}} - \frac{18\sqrt{7}}{5\sqrt{3}} + \frac{7\sqrt{89}}{5\sqrt{2}} - 24\sqrt{\frac{71}{1222}} - \frac{15}{2\sqrt{5}} - 24\sqrt{\frac{19}{1457}} - \frac{15\sqrt{109}}{2\sqrt{155}} - \frac{45\sqrt{19}}{4\sqrt{30}} + 10\sqrt{\frac{31}{10}}$.

Theorem 3.4 *For $n \geq 6$, the GA_5 index of a graph G of the equilateral triangular tetra sheet $ETTS(n)$ is* $GA_5(G) = (\frac{6\sqrt{10}}{7} + \frac{3}{2})n^2 + (\frac{48\sqrt{235}}{127} - \frac{27}{2} + \frac{72\sqrt{2}}{17} + \frac{12\sqrt{1222}}{73} + \frac{12\sqrt{130}}{53} + \frac{\sqrt{1457}}{13} + \frac{48\sqrt{155}}{111} + \frac{108\sqrt{5}}{29} - \frac{66\sqrt{10}}{7})n + (\frac{24\sqrt{646}}{55} + \frac{12\sqrt{1786}}{85} + \frac{\sqrt{2660}}{9} + \frac{4\sqrt{3290}}{39} + \frac{8\sqrt{14}}{5} + \frac{3\sqrt{247}}{4} + \frac{3\sqrt{455}}{8} + \frac{12\sqrt{2170}}{101} + \frac{18\sqrt{70}}{53} - \frac{240\sqrt{235}}{127} - \frac{432\sqrt{2}}{17} - \frac{48\sqrt{1222}}{73} - \frac{60\sqrt{130}}{53} - \frac{4\sqrt{1457}}{13} - \frac{240\sqrt{155}}{111} - \frac{540\sqrt{5}}{29} + \frac{180\sqrt{10}}{7} + 39)$.

Proof. Inserting the information about the edge partitition from Table 4, in the following,$A_5(G) = \sum_{uv\in E(G)} 2\frac{\sqrt{S_uS_v}}{S_u+S_v} = 3 \times \frac{2\sqrt{17\times17}}{17+17} + 12 \times \frac{2\sqrt{17\times38}}{17+38} + 3 \times \frac{2\sqrt{38\times38}}{38+38} + 6 \times \frac{2\sqrt{38\times47}}{38+47} + 6 \times \frac{2\sqrt{38\times70}}{38+70} + 6 \times \frac{2\sqrt{47\times70}}{47+70} + 6 \times \frac{2\sqrt{70\times80}}{70+80} + 12 \times \frac{2\sqrt{38\times26}}{38+26} + 9 \times \frac{2\sqrt{26\times70}}{26+70} + 6 \times \frac{2\sqrt{31\times70}}{31+70} + 3 \times \frac{2\sqrt{36\times70}}{36+70} + 6(n-5) \times \frac{2\sqrt{47\times80}}{47+80} + 3(n-5) \times \frac{2\sqrt{80\times80}}{80+80} + 3(n-5) \times \frac{2\sqrt{47\times47}}{47+47} + 6(n-6) \times \frac{2\sqrt{80\times90}}{80+90} + \frac{3(n^2-13n+42)}{2} \times \frac{2\sqrt{90\times90}}{90+90} + 6(n-4) \times \frac{2\sqrt{47\times26}}{47+26} + 3(n-5) \times \frac{2\sqrt{26\times80}}{26+80} + 3(n-4) \times \frac{2\sqrt{47\times31}}{47+31} + 6(n-5) \times \frac{2\sqrt{31\times80}}{31+80} + 9(n-5) \times \frac{2\sqrt{36\times80}}{36+80} + 3(n^2-11n+30) \times \frac{2\sqrt{36\times90}}{36+90}$.

After simplification, we get $GA_5(G) = (\frac{6\sqrt{10}}{7} + \frac{3}{2})n^2 + (\frac{48\sqrt{235}}{127} - \frac{27}{2} + \frac{72\sqrt{2}}{17} + \frac{12\sqrt{1222}}{73} + \frac{12\sqrt{130}}{53} + \frac{\sqrt{1457}}{13} + \frac{48\sqrt{155}}{111} + \frac{108\sqrt{5}}{29} - \frac{66\sqrt{10}}{7})n + (\frac{24\sqrt{646}}{55} + \frac{12\sqrt{1786}}{85} + \frac{\sqrt{2660}}{9} + \frac{4\sqrt{3290}}{39} + \frac{8\sqrt{14}}{5} + \frac{3\sqrt{247}}{4} + \frac{3\sqrt{455}}{8} + \frac{12\sqrt{2170}}{101} + \frac{18\sqrt{70}}{53} - \frac{240\sqrt{235}}{127} - \frac{432\sqrt{2}}{17} - \frac{48\sqrt{1222}}{73} - \frac{60\sqrt{130}}{53} - \frac{4\sqrt{1457}}{13} - \frac{240\sqrt{155}}{111} - \frac{540\sqrt{5}}{29} + \frac{180\sqrt{10}}{7} + 39)$.

4 Extended mesh or rectangular network sheet

This Section is about the study of degree-based topological descriptors for the extended mesh.

4.1 Construction of extended mesh network

Assume that P_n is a path with n vertices. Th $P_m \times P_n$, $form, n \geq 2$, represents the two dimensional mesh, where m is the number of rows and n is the number of columns. We denote this by $M(m,n)$. Note that $M(m,n)$ is a graph with vertex set $V(M(m,n)) = v_{ij}, 1 \leq i \leq m, 1 \leq j \leq n$ and edge set $E(M(m,n)) = (v_{ij}, v_{i(j+1)}) : 1 \leq i \leq m, 1 \leq j \leq n-1 \cup v_{ij}, v_{(i+1)j} : 1 \leq i \leq m-1, 1 \leq j \leq n$.

By constructing all 4-cycle in a $m \times n$ mesh into a complete subgraph, we get an architecture called the extended mesh that is denoted by $EX(m,n)$. The number of vertices in $EX(m,n)$ is mn and the number of edges in $EX(m,n)$ is $4mn - 3m - 3n + 2$.

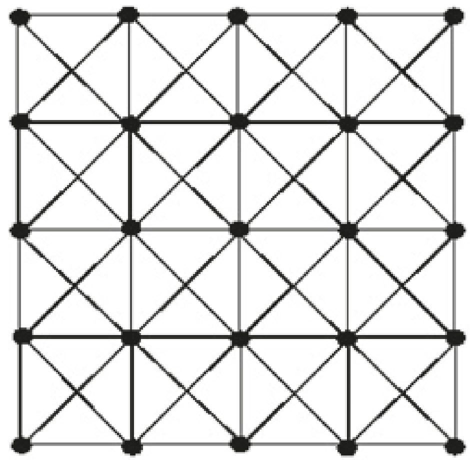

Figure 3. The rectangular sheet network (extended mesh) $EX(5,5)$

Table 5. Edge partition of $EX(m,n)$, based on degree of end vertices of each edge, where $uv \in E(G)$.

(d_u, d_v)	Number of Edges
$(3,5)$	8
$(3,8)$	4
$(5,8)$	$(6m+6n-32)$
$(5,5)$	$2(m+n-4)$
$(8,8)$	$(4mn-11m-11n+30)$

Theorem 4.1 *The ABC index of extended mesh $EX(m,n)$ is*

$ABC(EX(m,n)) = (\sqrt{\frac{7}{2}})mn + (3\sqrt{\frac{11}{10}} + \frac{4\sqrt{2}}{5} - \frac{11\sqrt{7}}{4\sqrt{2}})m + (3\sqrt{\frac{11}{10}} + \frac{4\sqrt{2}}{5} - \frac{11\sqrt{7}}{4\sqrt{2}})n + (8\sqrt{\frac{2}{5}} + 2\sqrt{\frac{3}{2}} - 16\sqrt{\frac{11}{10}} - \frac{16\sqrt{2}}{5} + \frac{15\sqrt{7}}{2\sqrt{2}})$, for $m,n \geq 5$.

Proof. Let G be a graph of extended mesh $EX(m,n)$. The information given in Table 5 leads us to the following calculation

$ABC(G) = \sum_{uv\in E(G)} \sqrt{\frac{d_u+d_v-2}{d_u d_v}} = 8\sqrt{\frac{3+5-2}{3\times5}} + 4\sqrt{\frac{3+8-2}{3\times8}} + (6m+6n-32)\sqrt{\frac{5+8-2}{5\times8}} + 2(m+n-4)\sqrt{\frac{5+5-2}{5\times5}} + (4mn-11m-11n+30)\sqrt{\frac{8+8-2}{8\times8}}$.

More simplification reduce the above exprssion in to the following form $ABC(EX(m,n)) = (\sqrt{\frac{7}{2}})mn + (3\sqrt{\frac{11}{10}} + \frac{4\sqrt{2}}{5} - \frac{11\sqrt{7}}{4\sqrt{2}})m + (3\sqrt{\frac{11}{10}} + \frac{4\sqrt{2}}{5} - \frac{11\sqrt{7}}{4\sqrt{2}})n + (8\sqrt{\frac{2}{5}} + 2\sqrt{\frac{3}{2}} - 16\sqrt{\frac{11}{10}} - \frac{16\sqrt{2}}{5} + \frac{15\sqrt{7}}{2\sqrt{2}})$.

Table 6. Edge Partition of $EX(m,n)$ based on vertices degree sum which are located at unit distance from the final vertices of each edge.

(S_u,S_v) *where* $uv \in E(G)$	Number of edges
$(18,29)$	8
$(29,34)$	8
$(18,47)$	4
$(29,55)$	8
$(29,29)$	4
$(47,64)$	4
$(47,55)$	8
$(34,47)$	8
$(29,47)$	8
$(34,34)$	$2(m+n-10)$
$(55,55)$	$2(m+n-8)$
$(55,64)$	$(6m-6n-56)$
$(34,55)$	$(6m+6n-56)$
$(64,64)$	$(4mn-19m-19n+90)$

Theorem 4.2 *For $m,n \geq 5$, $GA(EX(m,n)) = 4mn + (\frac{24\sqrt{10}}{13} - 9)m + (\frac{24\sqrt{10}}{13} - 9)n + (2\sqrt{15} + \frac{16\sqrt{6}}{11} - \frac{128\sqrt{10}}{13} + 22)$.*

Proof. To compute the GA index of the graph $G = EX(m,n)$, use the edge partition given in Table 5 to get $GA(G) = \sum_{uv\in E(G)} 2\frac{\sqrt{d_u d_v}}{(d_u+d_v)} = 8\times\frac{2\sqrt{3\times5}}{3+5} + 4\times\frac{2\sqrt{3\times8}}{3+8} + (6m+6n-32)\times\frac{2\sqrt{5\times8}}{5+8} + 2(m+n-4)\times\frac{2\sqrt{5\times5}}{5+5} + (4mn-11m-11n+30)\frac{2\sqrt{8\times8}}{8+8}$

simple calculation implies $GA(EX(m,n)) = 4mn + (\frac{24\sqrt{10}}{13} - 9)m + (\frac{24\sqrt{10}}{13} - 9)n + (2\sqrt{15} + \frac{16\sqrt{6}}{11} - \frac{128\sqrt{10}}{13} + 22)$ The next two results are about the computation of the ABC_4 and GA_5 indices of the graph of $EX(m,n)$. We give the edge partition of a graph G, where $G \cong EX(m,n)$, based on the degree sum of the final vertices of each edge in the following table.

Theorem 4.3 *For $m,n \geq 5$, the ABC_4 index of $EX(m,n)$ is computed as*

$ABC_4(EX(m,n)) = (\frac{3\sqrt{7}}{8\sqrt{2}})mn + (\frac{2\sqrt{33}}{17\sqrt{2}} + \frac{12\sqrt{3}}{55} + \frac{9\sqrt{13}}{4\sqrt{55}} + 6\sqrt{\frac{87}{1870}} - \frac{57\sqrt{7}}{32\sqrt{2}})m + (\frac{2\sqrt{33}}{17\sqrt{2}} + \frac{12\sqrt{3}}{55} + \frac{9\sqrt{13}}{4\sqrt{55}} + 6\sqrt{\frac{87}{1870}} - \frac{57\sqrt{7}}{32\sqrt{2}})n + (8\sqrt{\frac{5}{58}} + 8\sqrt{\frac{61}{986}} + 4\sqrt{\frac{7}{94}} + 8\sqrt{\frac{82}{1595}} + 8\sqrt{\frac{14}{841}} + \frac{\sqrt{109}}{2\sqrt{47}} + 16\sqrt{\frac{5}{517}} + 8\sqrt{\frac{79}{1598}} + 8\sqrt{\frac{74}{1363}} - \frac{20\sqrt{33}}{17\sqrt{2}} - \frac{96\sqrt{3}}{55} - 21\sqrt{\frac{13}{55}} - 56\sqrt{\frac{87}{1870}} + \frac{135\sqrt{7}}{16\sqrt{2}})$.

Proof. By following the data in Table 6, we have

$ABC_4(G) = \sum_{uv\in E(G)} \sqrt{\frac{S_u+S_v-2}{S_u S_v}} = 8\sqrt{\frac{18+29-2}{18\times29}} + 8\sqrt{\frac{29+34-2}{29\times34}} + 4\sqrt{\frac{18+47-2}{18\times47}} + 8\sqrt{\frac{29+55-2}{29\times55}} + 4\sqrt{\frac{29+29-2}{29\times29}} + 4\sqrt{\frac{47+64-2}{47\times64}} + 8\sqrt{\frac{47+55-2}{47\times55}} + 8\sqrt{\frac{34+47-2}{34\times47}} + 8\sqrt{\frac{29+47-2}{29\times47}} + 2(m+n-10)\sqrt{\frac{34+34-2}{34\times34}} + 2(m+n-8)\sqrt{\frac{55+55-2}{55\times55}} + (6m+6n-56)\sqrt{\frac{55+64-2}{55\times64}} + (6m+6n-56)\sqrt{\frac{34+55-2}{34\times55}} + (4mn-19m-19n+90)\sqrt{\frac{64+64-2}{64\times64}}$.

$ABC_4(EX(m,n)) = (\frac{3\sqrt{7}}{8\sqrt{2}})mn + (\frac{2\sqrt{33}}{17\sqrt{2}} + \frac{12\sqrt{3}}{55} + \frac{9\sqrt{13}}{4\sqrt{55}} + 6\sqrt{\frac{87}{1870}} - \frac{57\sqrt{7}}{32\sqrt{2}})m + (\frac{2\sqrt{33}}{17\sqrt{2}} + \frac{12\sqrt{3}}{55} + \frac{9\sqrt{13}}{4\sqrt{55}} + 6\sqrt{\frac{87}{1870}} - \frac{57\sqrt{7}}{32\sqrt{2}})n + (8\sqrt{\frac{5}{58}} + 8\sqrt{\frac{61}{986}} + 4\sqrt{\frac{7}{94}} + 8\sqrt{\frac{82}{1595}} + 8\sqrt{\frac{14}{841}} + \frac{\sqrt{109}}{2\sqrt{47}} + 16\sqrt{\frac{5}{517}} + 8\sqrt{\frac{79}{1598}} + 8\sqrt{\frac{74}{1363}} - \frac{20\sqrt{33}}{17\sqrt{2}} - \frac{96\sqrt{3}}{55} - 21\sqrt{\frac{13}{55}} - 56\sqrt{\frac{87}{1870}} + \frac{135\sqrt{7}}{16\sqrt{2}})$.

Theorem 4.4 *For $m,n \geq 5$, we have*

$GA_5(EX(m,n)) = 4mn + (\frac{12\sqrt{1870}}{89} - \frac{1689}{119})m + (\frac{12\sqrt{1870}}{89} - \frac{1689}{119})n + (\frac{48\sqrt{58}}{47} + \frac{16\sqrt{986}}{63} + \frac{24\sqrt{94}}{65} + \frac{4\sqrt{1595}}{21} + \frac{4\sqrt{841}}{29} + \frac{64\sqrt{47}}{111} + \frac{8\sqrt{2585}}{51} + \frac{16\sqrt{1598}}{81} -$

$\frac{4\sqrt{1363}}{19} - \frac{112\sqrt{1870}}{89} + \frac{5530}{119})$.

Proof. The information in Table 6 leads us to the required result.

$GA_5(G) = \sum_{uv\in E(G)} 2\frac{\sqrt{S_uS_v}}{S_u+S_v} = 8\times\frac{2\sqrt{18\times29}}{18+29} + 8\times\frac{2\sqrt{29\times34}}{29+34} + 4\times\frac{2\sqrt{18\times47}}{18+47} + 8\times\frac{2\sqrt{29\times55}}{29+55} + 4\times\frac{2\sqrt{29\times29}}{29+29} + 4\times\frac{2\sqrt{47\times64}}{47+64} + 8\times\frac{2\sqrt{47\times55}}{47+55} + 8\times\frac{2\sqrt{34\times47}}{34+47} + 8\times\frac{2\sqrt{29\times47}}{29+47} + 2(m+n-10)\times\frac{2\sqrt{34\times34}}{34+34} + 2(m+n-8)\times\frac{2\sqrt{55\times55}}{55+55} + (6m+6n-56)\times\frac{2\sqrt{55\times64}}{55+64} + (6m+6n-56)\times\frac{2\sqrt{34\times55}}{34+55} + (4mn-19m-19n+90)\times\frac{2\sqrt{64\times64}}{64+64}$.

After simplification,we get

$GA_5(EX(m,n)) = 4mn + (\frac{12\sqrt{1870}}{89} - \frac{1689}{119})m + (\frac{12\sqrt{1870}}{89} - \frac{1689}{119})n + (\frac{48\sqrt{58}}{47} + \frac{16\sqrt{986}}{63} + \frac{24\sqrt{94}}{65} + \frac{4\sqrt{1595}}{21} + \frac{4\sqrt{841}}{29} + \frac{64\sqrt{47}}{111} + \frac{8\sqrt{2585}}{51} + \frac{16\sqrt{1598}}{81} - \frac{4\sqrt{1363}}{19} - \frac{112\sqrt{1870}}{89} + \frac{5530}{119})$.

5 Rectangular tetra sheets network

In this Section, we compute the ABC, ABC_4, GA, and GA_5 indices of a graph of rectangular tetra sheet network.

5.1 Construction of Rectangular Tetra Sheet

Draw a grid graph P_mP_n of dimension (m,n). Join the diagonal vertices of the paraellelogram so that each rectangle is divided into two triangles. Replace each triangle K_3 by K_4 , the resulting graph is known as the graph of rectangular tetra sheet network and is denoted by $RTS(m,n)$, (see Figure 4). In this graph, the number of vertices and edges are $3mn-2(m+n)+2$ and $9mn-8n-8m+7$ respectively.

In the next two theorems, we compute the ABC and GA indices of the graph of rectangular tetra sheet network by using the edge partition given in the following table.

Table 7. Edge partition of $RTS(m,n)$. It depends on the degrees of the final vertices located at unit distance of each edge.

(d_u,d_v) where $uv\in E(G)$	Number of Edges
$(3,3)$	2
$(5,7)$	4
$(5,12)$	2
$(3,5)$	4
$(3,7)$	$6m+6n-20$
$(3,12)$	$6mn-12m-12n+24$
$(7,7)$	$2m+2n-10$
$(7,12)$	$4m+4n-20$
$(12,12)$	$3mn-8m-8n+21$

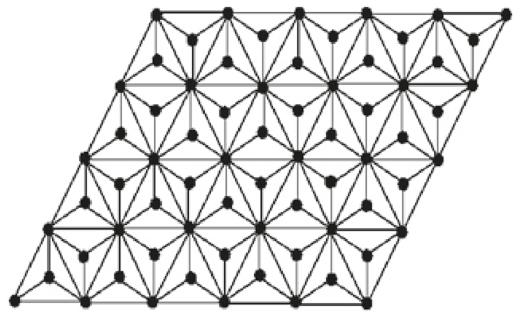

Figure 4. The graph of rectangular tetra sheet RTS(m,n) where m=5 and n=6

Theorem 5.1 *Let $G \cong RTS(m,n)$. Then, for $m,n \geq 3$, the $ABC(G)$ is*

$ABC(G) = (\sqrt{13} + \frac{\sqrt{22}}{4})mn + [\frac{4}{7}\sqrt{3} + \frac{4}{7}\sqrt{42} - \frac{2}{3}\sqrt{22} - 2\sqrt{13} + \frac{2}{21}\sqrt{357}](m+n) \quad \frac{4}{3} + \frac{4}{7}\sqrt{14} + \frac{4}{5}\sqrt{10} + 4\sqrt{13} - \frac{10}{21}\sqrt{357} \quad -\frac{20}{7}\sqrt{3} - \frac{40}{21}\sqrt{42} + \frac{7}{4}\sqrt{22} + 1.$

Proof. Put the information given in Table 7 in $ABC(G) = \sum_{uv\in E(G)}\sqrt{\frac{d_u+d_v-2}{d_ud_v}}$, we have $ABC(G) = 2\sqrt{\frac{3+3-2}{3\times3}} + 4\sqrt{\frac{5+7-2}{5\times7}} + 2\sqrt{\frac{5+12-2}{5\times12}} + 4\sqrt{\frac{3+5-2}{3\times5}} + (6(m-2)(n-2))\sqrt{\frac{3+12-2}{3\times12}} + (4m+4n-20)\sqrt{\frac{7+12-2}{7\times12}} + (2m+2n-10)\sqrt{\frac{7+17-2}{7\times7}} + (6m+6n-20)\sqrt{\frac{3+7-2}{3\times17}} + (3mn-8m-8n+21)\sqrt{\frac{12+12-2}{12\times12}}$

Further simplification reduces the above expression to the following form

$ABC(G) = (\sqrt{13} + \frac{\sqrt{22}}{4})mn + [\frac{4}{7}\sqrt{3} + \frac{4}{7}\sqrt{42} - \frac{2}{3}\sqrt{22} - 2\sqrt{13} + \frac{2}{21}\sqrt{357}](m+n) \quad \frac{4}{3} + \frac{4}{7}\sqrt{14} +$

$\frac{4}{5}\sqrt{10} + 4\sqrt{13} - \frac{10}{21}\sqrt{357} - \frac{20}{7}\sqrt{3} - \frac{40}{21}\sqrt{42} + \frac{7}{4}\sqrt{22} + 1$.

Theorem 5.2 *Let $G \cong RTS(m,s)$. Then, for $m,n \geq 3$, the $GA(A)$ is*

$$GA(G) = [\tfrac{39}{5}]mn + [\tfrac{6}{5}\sqrt{21} + \tfrac{16}{19}\sqrt{21} - \tfrac{48}{5} - 6]m + [\tfrac{6}{5}\sqrt{21} + \tfrac{16}{19}\sqrt{21} - \tfrac{48}{5} - 6]n + \tfrac{2}{3}\sqrt{35} + \tfrac{25}{17}\sqrt{15} - \tfrac{80}{19}\sqrt{21} - 4\sqrt{21} + \tfrac{161}{5}.$$

Proof. Put the information given in Table 7 in $GA(G) = \sum_{\mu\nu E(G)} 2\frac{\sqrt{d_u d_v}}{(d_u+d_v)}$, we have

$$GA(G) = 2 \times 2\sqrt{\tfrac{3\times 3}{3+3}} + 4 \times 2\sqrt{\tfrac{5\times 7}{5+7}} + 2 \times 2\sqrt{\tfrac{5\times 12}{5+12}} + 4 \times 2\sqrt{\tfrac{3\times 5}{3+5}} + (6m+6n-20) \times 2\sqrt{\tfrac{3\times 7}{3+7}} + (6mn-12m-12n+24) \times 2\sqrt{\tfrac{3\times 12}{3+12}} + (2m+2n-10) \times 2\sqrt{\tfrac{7\times 7}{7+7}} + (4m+4n-20) \times 2\sqrt{\tfrac{7\times 12}{7+12}} + (3mn-8m-8n+21) \times 2\sqrt{\tfrac{12\times 12}{12+12}}.$$

After simplification, we get

$$GA(G) = [\tfrac{39}{5}]mn + [\tfrac{6}{5}\sqrt{21} + \tfrac{16}{19}\sqrt{21} - \tfrac{48}{5} - 6]m + [\tfrac{6}{5}\sqrt{21} + \tfrac{16}{19}\sqrt{21} - \tfrac{48}{5} - 6]n + \tfrac{2}{3}\sqrt{35} + \tfrac{25}{17}\sqrt{15} - \tfrac{80}{19}\sqrt{21} - 4\sqrt{21} + \tfrac{161}{5}.$$

In the table below, we give the edge partition of a graph $G \cong RTS(m,s)$ based on vertices degree sum which are located at unit distance from the final vertices of each edge.

In the next two results, we calculate the ABC_4 and GA_5 indices for a graph G of rectangular tetra sheet $RTS(m,n)$.

Theorem 5.3 *For $m,n \geq 6$, the formula for ABC_4 index of the graph $G \cong RTS(m,n)$ is*

$$ABC_4(G) = [\tfrac{\sqrt{310}}{15} + \tfrac{\sqrt{178}}{30}]mn + [\tfrac{2}{611}\sqrt{876762} + \tfrac{2}{65}\sqrt{845} + \tfrac{4}{1457}\sqrt{27683} + \tfrac{\sqrt{16895}}{155} + \tfrac{\sqrt{570}}{20} - \tfrac{4}{15}\sqrt{310} + \tfrac{2}{47}\sqrt{92} + \tfrac{5}{47}\sqrt{47} + \tfrac{2}{15}\sqrt{21} - \tfrac{7}{30}\sqrt{178}](m+n) + \tfrac{8}{17}\sqrt{2} + \tfrac{4}{323}\sqrt{34328} + \tfrac{\sqrt{74}}{19} + \tfrac{2}{893}\sqrt{148238} + \tfrac{2}{665}\sqrt{70490} + \tfrac{3}{4}\sqrt{2} + \tfrac{\sqrt{2010}}{45} + \tfrac{\sqrt{41610}}{219} + \tfrac{\sqrt{30}}{6} + \tfrac{\sqrt{15038}}{292} + \tfrac{4}{247}\sqrt{15314} + \tfrac{2}{295}\sqrt{26910} - \tfrac{16}{611}\sqrt{86762} + \tfrac{6}{455}\sqrt{42770} - \tfrac{4}{65}\sqrt{845} + \tfrac{4}{465}\sqrt{11470} - \tfrac{32}{1457}\sqrt{27683} + \tfrac{6}{1085}\sqrt{23870} + \tfrac{4}{2263}\sqrt{230826} - \tfrac{8}{155}\sqrt{16895} + \tfrac{2}{105}\sqrt{455} + \tfrac{2}{219}\sqrt{7811} - \tfrac{2}{5}\sqrt{570} + \tfrac{16}{15}\sqrt{310} + \tfrac{4}{47}\sqrt{94} + \tfrac{8}{1095}\sqrt{10585} + \tfrac{\sqrt{123}}{15} - \tfrac{20}{47}\sqrt{92} + \tfrac{2}{329}\sqrt{15134} - \tfrac{45}{47}\sqrt{47} + \tfrac{\sqrt{518}}{35} + \tfrac{\sqrt{55115}}{365} + \tfrac{\sqrt{117530}}{730} - \tfrac{9}{40}\sqrt{158} - \tfrac{6}{5}\sqrt{21} + \tfrac{13}{18}\sqrt{178}.$$

Table 8. Table 8: Edge Partition of a graph G $\cong$ RTS(m,n) based on vertices degree sum which are located at unit distance from the final vertices of each edge.

(S_u, S_v) where $uv \in E(G)$	Number of edges
$(17,17)$	2
$(17,38)$	8
$(38,38)$	2
$(38,47)$	4
$(38,70)$	4
$(24,32)$	4
$(24,45)$	4
$(24,73)$	4
$(32,45)$	4
$(32,73)$	2
$(26,38)$	8
$(26,45)$	4
$(26,47)$	$4m+4n-32$
$(26,70)$	6
$(26,80)$	$2m+2n-16$
$(31,45)$	4
$(31,47)$	$2m+2n-16$
$(31,70)$	4
$(31,73)$	4
$(31,80)$	$4m+4n-32$
$(36,70)$	2
$(36,73)$	4
$(36,80)$	$6m+6n-48$
$(36,90)$	$6mn-24m-24n+96$
$(45,47)$	4
$(45,73)$	4
$(45,80)$	4
$(47,47)$	$2m+2n-20$
$(47,70)$	4
$(47,80)$	$4m+4n-36$
$(70,80)$	4
$(73,80)$	4
$(73,90)$	2
$(80,80)$	$2m+2n-18$
$(80,90)$	$4m+4n-36$
$(90,90)$	$3mn-14m-14n+65$

Proof. Based on the information given in Table 8, we compute the ABC_4 index of G as follows

$ABC_4(G) = \sum_{uv\in E(G)} \sqrt{\frac{S_u+S_v-2}{S_uS_v}} = 2\sqrt{\frac{17+17-2}{17\times17}} + 8\sqrt{\frac{17+38-2}{17\times38}} + 2\sqrt{\frac{38+38-2}{38\times38}} + 4\sqrt{\frac{24+32-2}{24\times32}} + 4\sqrt{\frac{24+45-2}{24\times45}} + 4\sqrt{\frac{32+45-2}{32\times45}} + 4\sqrt{\frac{45+47-2}{45\times47}} + 4\sqrt{\frac{47+38-2}{47\times38}} + 8\sqrt{\frac{26+38-2}{26\times38}} + 4\sqrt{\frac{26+45-2}{26\times45}} + 4\sqrt{\frac{38+70-2}{38\times70}} + 4\sqrt{\frac{24+73-2}{24\times73}} + 2\sqrt{\frac{32+73-2}{32\times73}} + (4m+4n-32)\sqrt{\frac{26+47-2}{26\times47}} + 6\sqrt{\frac{26+70-2}{26\times70}} + (2m+2n-16)\sqrt{\frac{26+80-2}{26\times80}} + 4\sqrt{\frac{31+45-2}{31\times45}} + (2m+2n-16)\sqrt{\frac{31+47-2}{31\times47}} + 4\sqrt{\frac{31+70-2}{31\times70}} + 4\sqrt{\frac{31+73-2}{31\times73}} + (4m+4n-32)\sqrt{\frac{31+80-2}{31\times80}} + 2\sqrt{\frac{36+70-2}{36\times70}} + 4\sqrt{\frac{36+73-2}{36\times73}} + (6m+6n-48)\sqrt{\frac{36+80-2}{36\times80}} + (6mn-24m-24n+96)\sqrt{\frac{36+90-2}{36\times90}} + 4\sqrt{\frac{45+73-2}{45\times73}} + 4\sqrt{\frac{45+80-2}{45\times80}} + +(2m+2n-20)\sqrt{\frac{47+47-2}{47\times47}} + 4\sqrt{\frac{47+70-2}{47\times70}} + (4m+4n-36)\sqrt{\frac{47+80-2}{47\times80}} + 4\sqrt{\frac{70+80-2}{70\times80}} + 4\sqrt{\frac{73+80-2}{73\times80}} + 2\sqrt{\frac{73+90-2}{73\times90}} + (2m+2n-18)\sqrt{\frac{80+80-2}{80\times80}} + (4m+4n-36)\sqrt{\frac{80+90-2}{80\times90}} + (3mn-14m-14n+65)\sqrt{\frac{90+90-2}{90\times90}}.$

Further simplification give us the required result $ABC_4(G) = [\frac{\sqrt{310}}{15} + \frac{\sqrt{178}}{30}]mn + [\frac{2}{611}\sqrt{876762} + \frac{2}{65}\sqrt{845} + \frac{4}{1457}\sqrt{27683} + \frac{\sqrt{16895}}{155} + \frac{\sqrt{570}}{20} - \frac{4}{15}\sqrt{310} + \frac{2}{47}\sqrt{92} + \frac{5}{47}\sqrt{47} + \frac{2}{15}\sqrt{21} - \frac{7}{30}\sqrt{178}](m+n) + \frac{8}{17}\sqrt{2} + \frac{4}{323}\sqrt{34328} + \frac{\sqrt{74}}{19} + \frac{2}{893}\sqrt{148238} + \frac{2}{665}\sqrt{70490} + \frac{3}{4}\sqrt{2} + \frac{\sqrt{2010}}{45} + \frac{\sqrt{41610}}{219} + \frac{\sqrt{30}}{6} + \frac{\sqrt{15038}}{292} + \frac{4}{247}\sqrt{15314} + \frac{2}{295}\sqrt{26910} - \frac{16}{611}\sqrt{86762} + \frac{6}{455}\sqrt{42770} - \frac{4}{65}\sqrt{845} + \frac{4}{465}\sqrt{11470} - \frac{32}{1457}\sqrt{27683} + \frac{6}{1085}\sqrt{23870} + \frac{4}{2263}\sqrt{230826} - \frac{8}{155}\sqrt{16895} + \frac{2}{105}\sqrt{455} + \frac{2}{219}\sqrt{7811} - \frac{2}{5}\sqrt{570} + \frac{16}{15}\sqrt{310} + \frac{4}{47}\sqrt{94} + \frac{8}{1095}\sqrt{10585} + \frac{\sqrt{123}}{15} - \frac{20}{47}\sqrt{92} + \frac{2}{329}\sqrt{15134} - \frac{45}{47}\sqrt{47} + \frac{\sqrt{518}}{35} + \frac{\sqrt{55115}}{365} + \frac{\sqrt{117530}}{730} - \frac{9}{40}\sqrt{158} - \frac{6}{5}\sqrt{21} + \frac{13}{18}\sqrt{178}.$

Theorem 5.4 *For $m,n \geq 6$, the GA_5 index of a graph $G \cong RTS(m,n)$ is*

$GA_5(G) = [\frac{12}{7}\sqrt{10} + 3]mn + [\frac{8}{73}\sqrt{1222} + \frac{8}{53}\sqrt{65} + \frac{72}{29}\sqrt{5} - \frac{48}{7}\sqrt{10} + \frac{32}{127}\sqrt{235} + \frac{48}{17}\sqrt{2} + \frac{2}{39}\sqrt{1457} + \frac{32}{111}\sqrt{155} - 11](m+n) + \frac{16}{55}\sqrt{646} + \frac{8}{85}\sqrt{1786} + \frac{96}{77}\sqrt{10} + \frac{16}{105}\sqrt{146} + \frac{16}{7}\sqrt{3} + \frac{16}{23}\sqrt{30} + \frac{\sqrt{247}}{2} + \frac{8}{71}\sqrt{1170} - \frac{64}{73}\sqrt{1222} + \frac{\sqrt{455}}{4} - \frac{64}{53}\sqrt{65} + \frac{6}{19}\sqrt{155} + \frac{12}{53}\sqrt{70} + \frac{48}{109}\sqrt{73} - \frac{576}{29}\sqrt{5} + \frac{192}{7}\sqrt{10} + \frac{6}{23}\sqrt{235} + \frac{12}{59}\sqrt{365} + \frac{8}{117}\sqrt{3290} - \frac{288}{127}\sqrt{235} + \frac{16}{17}\sqrt{14} + \frac{32}{153}\sqrt{365} + \frac{12}{163}\sqrt{730} - \frac{432}{17}\sqrt{2} + \frac{96}{77}\sqrt{10} + \frac{32}{97}\sqrt{438} - \frac{16}{39}\sqrt{1457} + \frac{8}{101}\sqrt{2170} + \frac{\sqrt{2263}}{13} - \frac{256}{111}\sqrt{155} + \frac{96}{25} + 61.$

Proof. By following the instructions about the edge partitioning in Table 8, we compute the GA_5 index of the graph G as follows

$GA_5(G) = \frac{2\sqrt{S_uS_v}}{(S_u+S_v)} = \frac{2\sqrt{17\times17}}{17+17} \times 2 + \frac{2\sqrt{17\times38}}{17+38} \times (8) + \frac{2\sqrt{38\times38}}{38+38} \times 2 + \frac{2\sqrt{38\times47}}{38+47} \times 4 + \frac{2\sqrt{38\times70}}{38+70} \times 4 + \frac{2\sqrt{24\times32}}{24+32} \times 4 + \frac{2\sqrt{24\times45}}{24+45} \times 4 + \frac{2\sqrt{24\times73}}{24+73} \times 4 + \frac{2\sqrt{32\times45}}{32+45} \times 4 + \frac{2\sqrt{32\times73}}{32+73} \times 2 + \frac{2\sqrt{26\times38}}{26+38} \times 8 + \frac{2\sqrt{26\times45}}{26+45} \times 4 + \frac{2\sqrt{26\times47}}{26+47} \times (4m+4n-32) + \frac{2\sqrt{26\times70}}{26+70} \times 6 + \frac{2\sqrt{26\times80}}{26+80} \times (2m+2n-16) + \frac{2\sqrt{31\times45}}{31+45} \times 4 + \frac{2\sqrt{31\times47}}{31+47} \times (2m+2n-16) + \frac{2\sqrt{31\times70}}{31+70} \times 4 + \frac{2\sqrt{31\times73}}{31+73} \times 4 + \frac{2\sqrt{31\times80}}{31+80} \times (4m+4n-32) + \frac{2\sqrt{36\times70}}{36+70} \times 2 + \frac{2\sqrt{36\times73}}{36+73} \times 4 + \frac{2\sqrt{36\times80}}{36+80} \times (6m+6n-48) + \frac{2\sqrt{36\times90}}{36+90} \times (6mn-24m-24n+96) + \frac{2\sqrt{45\times47}}{45+47} \times 4 + \frac{2\sqrt{45\times73}}{45+73\times4} + \frac{2\sqrt{45\times80}}{45+80} \times 4 + \frac{2\sqrt{47\times47}}{47+47} \times)(2m+2n-20) + \frac{2\sqrt{47\times70}}{47+70} \times 4 + \frac{2\sqrt{47\times80}}{47+80} \times (4m+4n-36) + \frac{2\sqrt{70\times80}}{70+80} \times 4 + \frac{2\sqrt{73\times80}}{73+80} \times 4 + \frac{2\sqrt{73\times90}}{73+90} \times 2 + \frac{2\sqrt{80\times80}}{80+80} \times (2m+2n-18) + \frac{2\sqrt{80\times90}}{80+90} \times (4m+4n-36) + \frac{2\sqrt{90\times90}}{90+9} \times (3mn-14m-14n+65).$

Further simplification give us the required result

$GA_5(G) = [\frac{12}{7}\sqrt{10} + 3]mn + [\frac{8}{73}\sqrt{1222} + \frac{8}{53}\sqrt{65} + \frac{72}{29}\sqrt{5} - \frac{48}{7}\sqrt{10} + \frac{32}{127}\sqrt{235} + \frac{48}{17}\sqrt{2} + \frac{2}{39}\sqrt{1457} + \frac{32}{111}\sqrt{155} - 11](m+n) + \frac{16}{55}\sqrt{646} + \frac{8}{85}\sqrt{1786} + \frac{96}{77}\sqrt{10} + \frac{16}{105}\sqrt{146} + \frac{16}{7}\sqrt{3} + \frac{16}{23}\sqrt{30} + \frac{\sqrt{247}}{2} + \frac{8}{71}\sqrt{1170} - \frac{64}{73}\sqrt{1222} + \frac{\sqrt{455}}{4} - \frac{64}{53}\sqrt{65} + \frac{6}{19}\sqrt{155} + \frac{12}{53}\sqrt{70} + \frac{48}{109}\sqrt{73} - \frac{576}{29}\sqrt{5} + \frac{192}{7}\sqrt{10} + \frac{6}{23}\sqrt{235} + \frac{12}{59}\sqrt{365} + \frac{8}{117}\sqrt{3290} - \frac{288}{127}\sqrt{235} + \frac{16}{17}\sqrt{14} + \frac{32}{153}\sqrt{365} + \frac{12}{163}\sqrt{730} - \frac{432}{17}\sqrt{2} + \frac{96}{77}\sqrt{10} + \frac{32}{97}\sqrt{438} - \frac{16}{39}\sqrt{1457} + \frac{8}{101}\sqrt{2170} + \frac{\sqrt{2263}}{13} - \frac{256}{111}\sqrt{155} + \frac{96}{25} + 61.$

6 Conclusion

In this article, we have done computation of some degree based topological indices for certain networks sheets. As a consequence, we got formulas for these networks. We hope this will help people in the field of network science understand and explore the basic topology of these chemical networks.

For future work, we plan or study the design of some new architectures / networks and their topological properties which play an important role to understand their underlying topologies.

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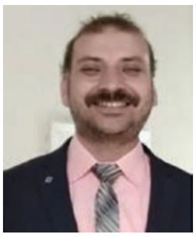

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