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Operational model for vessel traffic using optimal control and calibration

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Abstract

Due to the ever-increasing economic globalization, the scale of transportation through ports and waterways has increased sharply. As the capacity of maritime infrastructure in ports and inland waterways is limited, it is important to simulate vessel behavior to balance safety and capacity in restricted waterways. Currently many existing vessel simulation models focus mainly on vessel dynamics and maritime traffic in the open ocean. These models are, however, inapplicable to simulating vessel behavior in ports and inland waterways, because behavior in such areas can be influenced by many factors, such as waterway geometry, external conditions and human factors.

To better simulate vessel behavior in ports and waterways, we developed a new maritime traffic model by adapting the theory of pedestrian models. This new model comprises two parts: the Route Choice Model and the Operational Model. The Route Choice Model has been demonstrated and calibrated in our recent study, in which the desired speed is generated. This paper presents the second part of the model, the Operational Model, which describes vessel behavior based on optimal control by using the output of the Route Choice Model. The calibration of the Operational Model is carried out as well.

In the Operational Model, the main behavioral assumption is that all actions of the bridge team, such as accelerating and turning, are executed to force the vessel to sail with the desired speed and course. In the proposed theory, deviating from the desired speed and course, accelerating, decelerating and turning will provide disutility (cost) to the vessel. By predicting and minimizing this disutility, longitudinal acceleration and angular acceleration can be optimized. This way, the Operational Model can be used to predict the vessel speed and course. Automatic Identification System (AIS) data of unhindered vessel behavior in the Port of Rotterdam, the Netherlands, were used to calibrate the Operational Model. The calibration results produced plausible parameter values that minimized the objective function. The paths generated with these optimal parameters corresponded reasonably well to the actual paths.

Introduction

One of the main concerns for maritime infrastructure managers is the balance between safety and capacity: when measures are taken to increase capacity, usually the safety decreases, and vice versa, especially in ports and inland waterways where the available area for vessels is restricted.

To improve maritime traffic management and optimize ports and waterway design, modeling tools are used mainly in three ways. Some models calculate the collision and grounding probability (Degre et al., 2003; Fowler et al., 2000; Pedersen, 1995). The second way is predicting the vessel maneuvering by including hydrodynamics of vessels (Sariöz et al., 2003; Sutulo et al., 2002; Yoon et al., 2003). Others are related to simulating routing in a shipping network (Hsu et al., 2007; Kosmas et al., 2012; Norstad et al., 2011).

These models focus mostly on vessel dynamics and maritime traffic in the open ocean and cannot be applied in constrained ports and waterways, where vessel behavior (speed, course and path) is influenced by such factors as waterway geometry, water depth and interaction between vessels. To predict the vessel behavior in ports and waterways, it is important to include the influence of these factors on the vessel behavior. However, these factors have received little research. Advances in maritime safety in port areas require additional effort in developing models that consider the additional complexity of port areas.

In recent studies (Hoogendoorn et al., 2013; Shu et al., 2015), significant progress has been made in developing a new maritime traffic model to predict vessel behavior and traffic in ports and waterways. To develop this model, vessel behavior is categorized into tactical and operational levels. The tactical level includes vessel route choice, which is reflected by the desired course at each location. This desired course represents the optimal course when the vessel is not influenced by other vessels or external conditions (e.g. current, waves, or wind). Like the desired course, the desired speed is the optimal vessel speed when the vessel is not influenced by other vessels or external conditions. Together with vessel route choice (desired course), the desired speed serves as the basis for vessel behavior at the operational level. The operational level includes the dynamics of vessel behavior, e.g. longitudinal acceleration and angular acceleration of the vessel.

According to this hierarchy, the new maritime traffic model comprises two parts: the Route Choice Model and the Operational Model. The Route Choice Model, including underlying theory and calibration, has already been presented before (Shu et al., 2014). In this Route Choice Model, the desired course is generated in continuous space by a dynamic programming approach and by a numerical solution approach. The concept of the Operational Model is based on optimal control, and was presented in our previous paper (Hoogendoorn et al., 2013), in which several basic scenarios showed the potential application of this method.

The objective of this paper is twofold. Firstly, we would like to integrate the Route Choice Model (Shu et al., 2015) with the Operational Model (Hoogendoorn et al., 2013). The calibrated result of the Route Choice Model is used as the input for the Operational Model to test the new maritime traffic model as a whole. Secondly, the Operational Model is calibrated by using the Automatic Identification System (AIS) database, which contains vessel information transmitting between vessels and shore stations, such as vessel speed, course, position, etc. In this study, the interactions between vessels and the external conditions are not considered because the relationships between vessel behavior and these factors are unclear at this stage.

The paper is structured as follows. Firstly, the maritime traffic control frame is proposed, followed by the description of the Operational Model. Then, the calibration process and its results are described. Finally, discussion, conclusions and recommendations for future research are presented.

Maritime traffic control frame

In this section, the maritime traffic control framework for the new maritime traffic model is presented.

As mentioned, vessel behavior is described at tactical and operational levels, which are shown in Figure 1. The traffic state (sailing context) is observed by the bridge team and serves as input for the Operational Model. Using the desired course generated by the Route Choice Model at the tactical level as reference, the control in longitudinal and angular direction can be optimized, while taking into account relevant external conditions. The control includes longitudinal acceleration, u_1, and angular acceleration, u_2. With this optimized control, the bridge team will make a maneuver leading to the next traffic state, addressing vessel speed, course and position.

In this context, it should be noted that the vessel route choice denotes the optimal vessel course, which is related to the vessel's position in the waterway. Corresponding to these two levels, this new maritime traffic model will comprise two parts: the Route Choice Model resulting in preferred courses, and the Operational Model describing the sailing behavior.

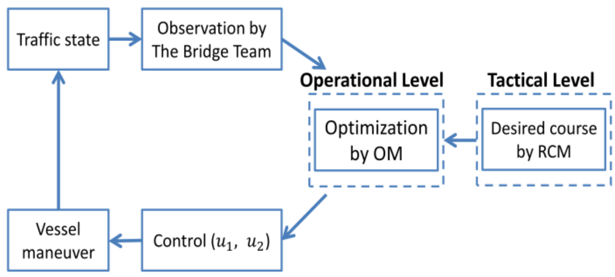

Figure 1. Maritime traffic control framework

Vessel operational model by optimal control

In this paper we do not consider drift angle (the difference between the heading and the course), water resistance to the vessel, or vessel geometry; rather, we use the simplified Operational Model proposed in our previous work (Hoogendoorn et al., 2013). Then we use Pontryagin's method to solve this optimal control problem. In this section, this model is briefly introduced.

Mathematical model

Figure 2 shows the vessel coordinate system and the controls, which are the longitudinal acceleration, u_1, and angular acceleration, u_2.

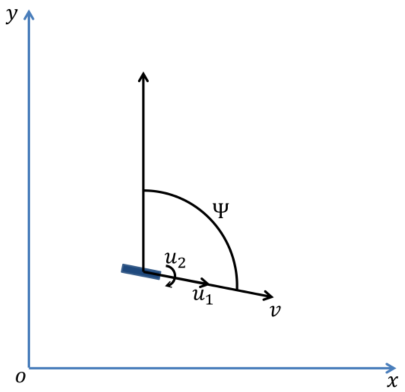

Figure 2. Vessel coordinate system and controls

Let $\vec{\xi} = (x, y, v, \psi)$ denote the state of the vessel, in which x and y determine the position, v is the vessel speed and ψ is the course angle. The following mathematical model describes vessel dynamics and controls:

$$\dot{x} = v\cos\left(\frac{\pi}{2} - \psi\right) \tag{1}$$

$$\dot{y} = v\sin\left(\frac{\pi}{2} - \psi\right) \tag{2}$$

$$\dot{v} = u_1 \tag{3}$$

$$\dot{\psi} = u_2 \tag{4}$$

Optimal control and solutions

By defining the control objective functions, we can turn the control of vessel dynamics into a cost minimization problem. The main behavioral assumption is that all actions of the bridge team, such as accelerating and turning, are executed to provide the disutility (cost) to the vessel. Minimizing the cost will force the vessel to sail with the desired speed and course. In this paper, we use the same objective function as in our previous work (Hoogendoorn et al., 2013). The objective function J is defined by:

$$J = \int_t^{t+H} L\left(s, \vec{\xi}, \vec{u}\right) \mathrm{d}s + \Phi\left(t + H, \vec{\xi}(t + H)\right) \tag{5}$$

where H is the prediction horizon used when making a decision at time instant t, L denotes the running cost (cost incurred in a small time interval $[\tau, \tau + \mathrm{d}\tau)$), $\vec{u} = (u_1, u_2)$ denotes the control, and Φ denotes the terminal costs at terminal conditions, which is the cost that is incurred when the vessel ends up at state $\vec{\xi}(t + H)$ at time instant $t + H$.

Since we do not consider the interaction between vessels or external conditions, the running cost contains only two items:

1) Straying from the desired speed and course costs

$$L^{\text{stray}} = \frac{1}{2}\left(c_2^v\left(v^0(\vec{x}) - v\right)^2 + c_2^\psi\left(\psi^0(\vec{x}) - \psi\right)^2\right) \tag{6}$$

2) Maneuvering effort costs

$$L^{\text{effort}} = \frac{1}{2}\left(c_3^v u_1^2 + c_3^\psi u_2^2\right) \tag{7}$$

Here, $c_2{}^v$, $c_2{}^\psi$, $c_3{}^v$ and $c_3{}^\psi$ are weighting factors for these costs. The parameters $v^0(\vec{x})$ and $\psi^0(\vec{x})$ denote the desired speed and desired course in the position $\vec{x}$, respectively.

To minimize the objective function, we assume that the longitudinal acceleration and the angular acceleration the bridge team selects satisfies:

$$\vec{u}^*_{[t,t+H)} = \arg\min J\left(\vec{u}_{[t,t+H)}\right) \tag{8}$$

subject to (1–4).

For the vessel state $\vec{\xi} = (x, y, v, \psi)$, we define shadow costs (or co-state) as $\vec{\lambda} = (\lambda_x, \lambda_y, \lambda_v, \lambda_\psi)$ to formulate the so-called Hamiltonian function (Fleming et al., 2006):

$$\mathcal{H} = L + \vec{\lambda} \cdot \frac{\mathrm{d}\vec{\xi}}{\mathrm{d}t} \tag{9}$$

where the shadow costs describe the relative change in the optimal cost in case of a (small) change in the state.

The Hamiltonian function and the shadow costs satisfy:

$$-\frac{\mathrm{d}\vec{\lambda}}{\mathrm{d}t} = \frac{\partial \mathcal{H}}{\partial \vec{\xi}} \tag{10}$$

In this optimal control, the initial condition is the vessel state in $\vec{\xi}$ at t, which is the vessel's current position, speed and course. For the terminal condition, we assume that the vessel will reach its optimal speed and course at the end of the prediction horizon at instant $t + H$, which means the shadow costs are zero.

According to the so-called optimality conditions for the optimal control:

$$\mathcal{H}\left(t, \vec{\xi}, \vec{u}^*, \vec{\lambda}\right) \leq \mathcal{H}\left(t, \vec{\xi}, \vec{u}, \vec{\lambda}\right) \forall \vec{u}$$

And the optimal control u_1^* and u_2^* can be determined as follows:

$$u_1^* = -\lambda_v / c_3^v \quad (11)$$

$$u_2^* = -\lambda_\psi / c_3^\psi \quad (12)$$

By substituting (6), (7) and (9) in (10), we obtain the following equations, which express the shadow costs dynamics:

$$-\dot{\lambda}_v = c_2^v\left(v - v^0\right) + \lambda_x \cos\left(\frac{\pi}{2} - \psi\right) + \lambda_y \sin\left(\frac{\pi}{2} - \psi\right) \quad (13)$$

$$-\dot{\lambda}_x = c_2^v\left(v^0 - v\right)\frac{\partial v^0}{\partial x} + c_2^\psi\left(\psi^0 - \psi\right)\frac{\partial \psi^0}{\partial x} \quad (14)$$

$$-\dot{\lambda}_y = c_2^v\left(v^0 - v\right)\frac{\partial v^0}{\partial y} + c_2^\psi\left(\psi^0 - \psi\right)\frac{\partial \psi^0}{\partial y} \quad (15)$$

$$-\dot{\lambda}_\psi = c_2^\psi\left(\psi - \psi^0\right) + \lambda_x v \sin\left(\frac{\pi}{2} - \psi\right) + \lambda_y \cos\left(\frac{\pi}{2} - \psi\right) \quad (16)$$

We then use numerical solution procedures to solve the system of these equations (Hoogendoorn et al., 2012).

Desired speed and course

As we can see in (5), the desired speed and course are taken as reference in determining the running cost. The difference between the actual speed and the desired speed, and between the actual course and the desired course, determine the costs of straying from the optimal path. Since the aim of this control is to minimize the total cost, this running cost item forces vessels to follow the desired speed and course. In this section, we introduce the manner in which desired speed and course are determined in this version of the Operational Model.

Based on the variation of vessel speed over time, vessel behavior can be categorized as either being in an equilibrium or a non-equilibrium state. In the equilibrium state, the vessel is sailing at its desired speed and will try to maintain this speed, with no need to accelerate or decelerate. Non-equilibrium states occur when, for whatever reason, the vessel is not moving at the desired speed, and needs to accelerate or decelerate to reach this desired speed. This typically occurs due to external circumstances, such as strong cross current or the influence of waterway geometry.

Let us consider the contribution in running costs for the equilibrium and the non-equilibrium situation. Since the speed does not change in the equilibrium situation, the contribution to the running costs of speed difference and longitudinal acceleration is equal to zero according to (6). So using the equilibrium situation, we cannot get calibration results for parameters c_2^v and c_3^v. However, the non-equilibrium situation does not have this problem. In this situation, the vessel speed after the acceleration or deceleration is considered to be the desired speed.

In this paper, we will use the path segment of non-equilibrium state (blue paths in Figure 6) to calibrate the Operational Model.

In the Route Choice Model (Shu et al., 2014), we already showed the calibrated desired course by optimal control. To calculate the desired course, the research area (see Figure 4) is discretized into a grid. The desired course is calculated in each cell based on the AIS data set of vessel category (small General Dry Cargo (GDC) vessels less than 3600 gross tonnage) in the direction Sea – Nieuwe Maas in the Botlek area of the Port of Rotterdam.

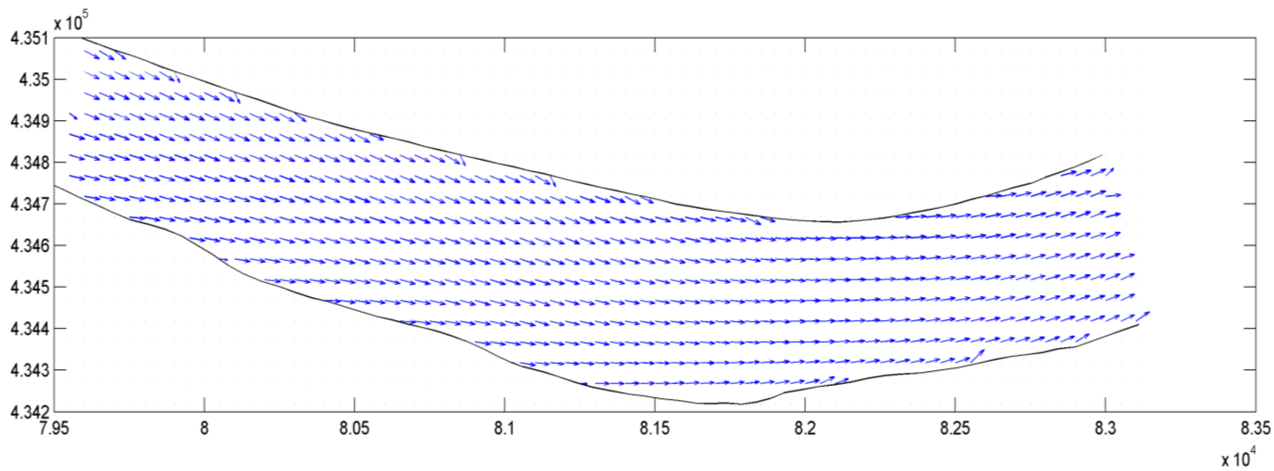

Figure 3. The desired course in sea – Nieuwe Maas direction in the Botlek area

Figure 3 shows the desired course generated by the Route Choice Model in the grid. The arrows in each cell show the desired course angle and form the desired course field. In this course field, it can be seen that when the vessel is close to shore, it will be repelled away from the shoreline. It should be noted that these arrows are equal in length, except in the boundary, where the uncertainty exists.

Based on this field, the desired course for any location in this area can be calculated by interpolation. We then use this desired course field as the reference in the Operational Model.

Calibration of the Operational Model

In this section, the Operational Model is calibrated with AIS data. AIS data have proven to provide powerful means to investigate maritime traffic (Aarsæther et al., 2009; Mou et al., 2010). We first introduce the AIS data used for calibration. We then describe the calibration set-up and conclude by illustrating the objective function and the parameter constraints.

AIS data used for calibration

As shown in Figure 4, 69 cross sections with intervals of about 50 meters are defined (Shu et al., 2013). The AIS data are recorded on these cross sections. These cross sections are approximately perpendicular to the waterway axis. End points of these cross sections are located at the five meter water depth line. For the areas without five meters water depth line, such as entrances to basins or the Oude Maas junction, end points are created such that the boundary remains smooth. In the model, these end points of cross sections will form the effective waterway bank for vessel sailing.

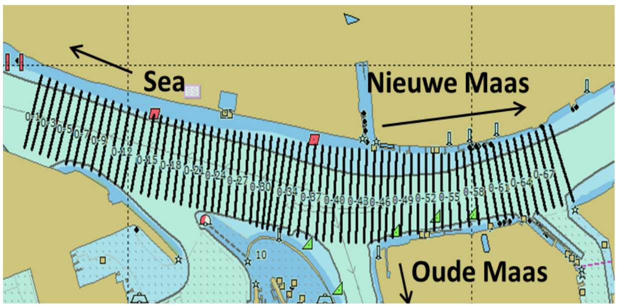

Figure 4. A series of 69 cross sections in sea – Nieuwe Maas

To calibrate the Operational Model without interaction between vessels, we used unhindered vessel behavior, which is the behavior unaffected by other vessels. Here, an unhindered path is defined if the distance to other vessels is at least 2 km, during the whole trip of the vessel.

Since we generated the desired course for small GDC vessels, we investigate the Operational Model for the same vessel category. AIS data for these vessels in the Botlek area from January 2009 to April 2011 were selected.

As mentioned in the section *Desired speed and course*, we need to select the AIS data from a non-equilibrium state. Speed changes larger than 1.5 m/s during three minutes are considered as the threshold to select the vessel path segment in non-equilibrium states. In this way, 67 partial trajectories (blue lines in Figure 6) were selected and used in the calibration.

Calibration set-up and objective function

In the Operational Model, the weight factors c_2^v, c_2^ψ, c_3^v and c_3^ψ must be calibrated. It should be noted that all weights cannot be uniquely determined from the data, since only the relative importance of the weights can be determined. Without loss of generality, we can set $c_2^v = 1$. Then, the parameters to be calibrated are found in the vector $\beta^T = (c_2^\psi, c_3^v, c_3^\psi)$.

We formulate the objective function based on the selected vessel path and AIS data. For each vessel partial trajectory, we considered the vessel movement between the start of the cross section and its end.

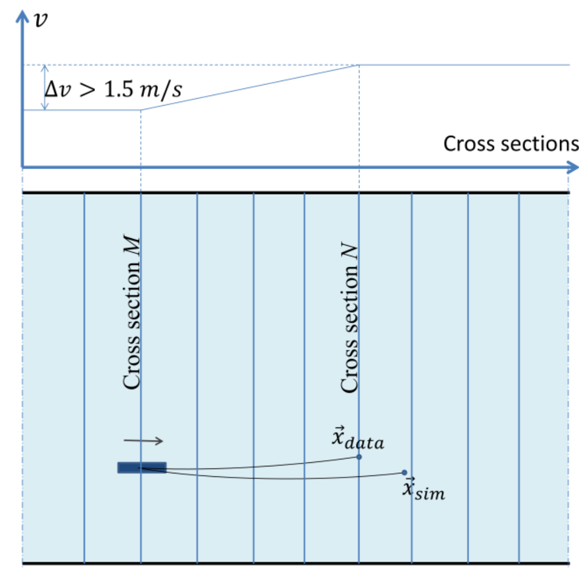

Figure 5. Vessel partial trajectory and speed change by AIS data and simulation trajectory

As shown in Figure 5, a non-equilibrium state is expressed when a vessel sails from cross section *M* to the right and passes cross section *N*. As mentioned in the section *AIS data used for calibration,* the speed change is larger than 1.5 m/s in this period, which is less than three minutes. In the AIS data, this vessel passes the cross section N at the location $\vec{x}_{\text{data}}$ at the time moment t. Using the Operational Model, we start the vessel from the same position on the cross section M and predict the vessel position $\vec{x}_{\text{sim}}$ at the same time instant t. We determine the difference between AIS data and simulated results by comparing the vessel position $\vec{x}_{\text{data}}$ and $\vec{x}_{\text{sim}}$, since the position is the consequence of the speed and course during the sailing. In other words, the position difference reflects the difference in speed and course.

The calibration process is intended to minimize the difference between vessel position measured from AIS data and vessel behavior predicted by the Operational Model. If m denotes the total number of the AIS records, then we can express the objective function as:

$$E(\beta) = \frac{1}{m}\sum_{i=1}^{m}\left|\vec{x}^{\,i}_{\text{data}} - \vec{x}^{\,i}_{\text{sim}}\right| \tag{17}$$

In this way, the calibration problem becomes a multi-variable nonlinear optimization problem as follows:

$$\beta^{*} = \arg\min E(\beta) \tag{18}$$

Parameter constraints

In the Operational Model, the parameters to be calibrated are the weight factors of the running cost, and should be positive values. By comparing the vessel tracks generated by the Operational Model, we apply the following constraints (minimum and maximum values) for these parameters to restrict them to reasonable and positive values without excluding possible solutions. The parameter c_2^{ψ} and c_2^{v} are restricted to the interval [0, 100], and the parameters c_3^{ψ} is restricted to [0, 1000].

Calibration results

Optimal parameters and simulated paths

By applying the optimization method described in the previous section, we have found the best fit of the Operational Model prediction to the selected AIS data. The calibration results are shown in Table 1. It should be noted that the running cost is dimensionless. The weight factors have been given the units shown in Table 1.

Table 1. Calibration Results

	c_2^{ψ}	c_3^{v}	c_3^{ψ}
Unit	1/rad^2	s^4/m^2	s^4/rad^2
Optimized parameters	7.99	33.6	393.41
Error	25.45 m		

It can be seen that all of these optimal values are positive as we expected, as they are the weight factors of the cost function terms, which need to be minimized in optimal control.

We then compare the contribution to the running cost of the difference between current speed and desired speed (speed difference), and the difference between current course and desired course (course difference) using these optimal parameters. Comparing the contribution of one unit of the speed difference (m/s) to one unit of course difference (rad) in the running cost, we find that the latter is 8 times (c_2^{ψ}/c_2^{v}) the former. That means one unit of the course difference is more significant than one unit of speed difference. Similarly, one unit of the angular acceleration (rad/s^2) is more significant than the longitudinal acceleration (m/s^2).

Based on the optimal parameters in Table 1, simulated paths are generated by the Operational Model and used to compare the real path against AIS data. As shown in Figure 6, the real paths

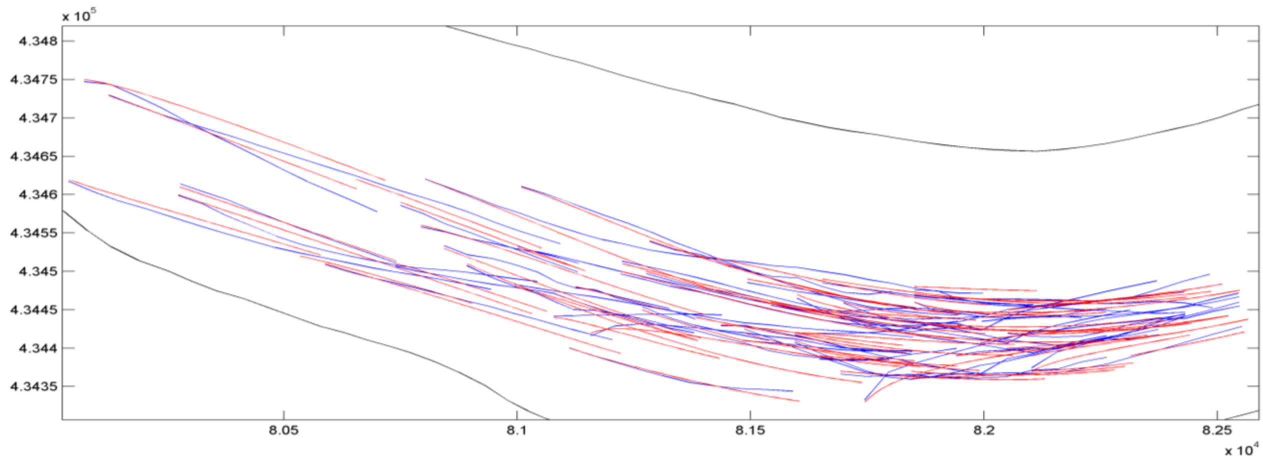

Figure 6. Real paths (blue) and simulated paths (red) by the Operational Model

(blue) and simulated paths (red) correspond reasonably well.

Sensitivity analysis

A sensitivity analysis was performed to verify the influence of each parameter on the error.

Starting from the optimized model parameters in Table 1 and varying a single parameter while keeping the other parameters constant, we estimate the effect of single parameter on the objective function. By analyzing the parameter's change, we can get an insight into the parameter's properties and how they affect the error, as well as the model.

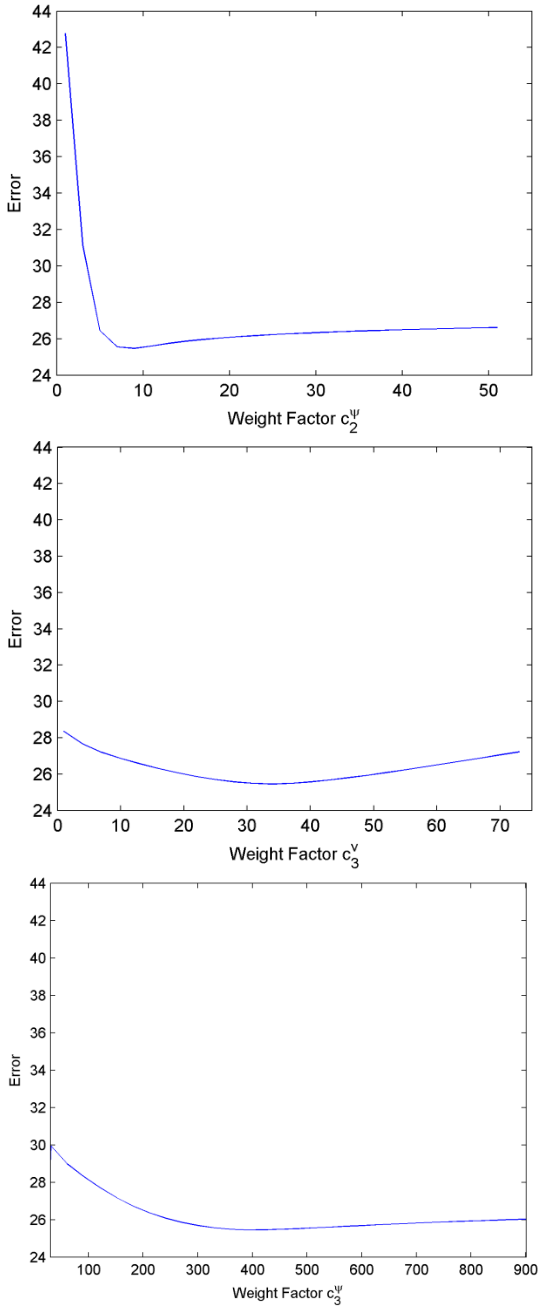

Figure 7. Variation of each parameter while keeping the other parameters at the optimal values listed in Table 1

Figure 7 shows the variation of each parameter while keeping the other parameters at the optimal values listed in Table 1. It can be seen that all three curves are smooth and have only one minimum, which means that the optimal parameters are easy to determine and generally reliable.

Conclusions

In this paper, we have integrated the Route Choice Model and the Operational Model to predict vessel sailing behavior in ports and waterways without considering the interactions between vessels and external conditions. The integration of these two models shows the potential of this method to predict vessel behavior and simulate maritime traffic. The Operational Mode was calibrated by using the AIS data in a non-equilibrium situation, the desired course generated by the Route Choice Model, as well as the desired speed, i.e. the final speed of the path segment used for calibration.

Based on the calibration results, we found that one unit of course difference is more significant than one unit of speed difference, and one unit of the angular acceleration is more significant than one unit of the acceleration. The calibrated optimal parameter values are positive as expected. Based on these optimal parameters, simulated paths are generated which correspond reasonably well with the real paths. The sensitivity analysis shows smooth curves and a unique minimum for each curve. That means the optimal parameters are generally reliable.

In the calibration process, we use the path segment of the AIS data in the non-equilibrium situation. However, we are not certain which factor forces the vessel to change between the equilibrium and the non-equilibrium situation. This will require additional analysis, especially for the external factors and the impact of waterway geometry.

Future research will focus on adding more factors to the model, such as interaction between vessels, human factors and external conditions.

Acknowledgments

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References

1. AARSÆTHER, K.G. & MOAN, T. (2009) Estimating navigation patterns from AIS. *Journal of Navigation*. 62. pp. 587–607.
2. DEGRE, T., GLANSDORP, C. & VAN DER TAK, C. (2003) The importance of a risk based index for vessels to enhance maritime safety. *Proceedings of the the 10th IFAC Symposium on Control in Transportation Systems*. Tokyo, Japan.
3. FLEMING, W.H., SONER, H.M. & SONER, H.M. (2006) *Controlled Markov processes and viscosity solutions*. Springer.
4. FOWLER, T.G. & SØRGÅRD, E. (2000) Modeling ship transportation risk. *Risk Analysis*. 20. pp. 225–244.
5. HOOGENDOORN, S., DAAMEN, W., SHU, Y. & LIGTERINGEN, H. (2013) Modeling human behavior in vessel manoeuver simulation by optimal control and game theory. *Transportation Research Record: Journal of the Transportation Research Board*. 2326. pp. 45–53.
6. HOOGENDOORN, S., HOOGENDOORN, R., WANG, M. & DAAMEN, W. (2012) Driver, driver support, and cooperative systems modeling by dynamic optimal control. *Transportation Research Record: Journal of the Transportation Research Board*. 2316. pp. 20–30.
7. HSU, C.-I. & HSIEH, Y.-P. (2007) Routing, ship size, and sailing frequency decision-making for a maritime hub-and-spoke container network. *Mathematical and Computer Modelling*. 45. pp. 899–916.
8. KOSMAS, O. & VLACHOS, D. (2012) Simulated annealing for optimal ship routing. *Computers & Operations Research*. 39. pp. 576–581.
9. MOU, J.M., TAK, C. V. D. & LIGTERINGEN, H. (2010) Study on collision avoidance in busy waterways by using AIS data. *Ocean Engineering*. 37. pp. 483–490.
10. NORSTAD, I., FAGERHOLT, K. & LAPORTE, G. (2011) Tramp ship routing and scheduling with speed optimization. *Transportation Research Part C: Emerging Technologies*. 19. pp. 853–865.
11. PEDERSEN, P.T. (1995) Collision and grounding mechanics. *Proceedings of the WEMT*. Copenhagen.
12. SARIÖZ, K. & NARLI, E. (2003) Assessment of manoeuvring performance of large tankers in restricted waterways: a real-time simulation approach. *Ocean engineering*. 30. pp. 1535–1551.
13. SHU, Y., DAAMEN, W., LIGTERINGEN, H. & HOOGENDOORN, S. (2013) *Vessel Speed, Course, and Path Analysis in the Botlek Area of the Port of Rotterdam, Netherlands*. Transportation Research Board of the National Academies.
14. SHU, Y., DAAMEN, W., LIGTERINGEN, H. & HOOGENDOORN, S. (2014) Vessel route choice model by optimal control and calibration. *Proceedings of the IWNTM 2014: International Workshop on Nautical Traffic Models*. Wuhan, China, 15–17 October 2014.
15. SHU, Y., DAAMEN, W., LIGTERINGEN, H. & HOOGENDOORN, S. (2015) Vessel route choice theory and modeling. *Proceedings of the 94th Annual Meeting Transportation Research Board*. Washington, USA, 11–15 January 2015; Authors version.
16. SUTULO, S., MOREIRA, L. & GUEDES SOARES, C. (2002) Mathematical models for ship path prediction in manoeuvring simulation systems. *Ocean engineering*. 29. pp. 1–19.
17. YOON, H.K. & RHEE, K.P. (2003) Identification of hydrodynamic coefficients in ship manoeuvering equations of motion by Estimation-Before-Modeling technique. *Ocean Engineering*. 30. pp. 2379–2404.