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VALUE OF GEOLOGICAL INFORMATION IN EXPLOITATION MANAGEMENT: THE CASE OF EXPLOITATION UNITS OF THE POLKOWICE-SIEROSZOWICE MINE

WARTOŚĆ INFORMACJI GEOLOGICZNEJ W PROCESIE ZARZĄDZANIA EKSPLOATACJĄ NA PRZYKŁADZIE ODDZIAŁÓW WYDOBYWCZYCH KOPALNI POLKOWICE-SIEROSZOWICE

The application of mathematical techniques of management is particularly significant in managing mineral deposits as well as generally in the mining industry, in which the execution of geological-mining projects is usually time-consuming and expensive. Such projects are usually undertaken in conditions of uncertainty, and the incurred expenses do not always generate satisfactory revenues. Mineral deposit management requires close cooperation between the geologist providing necessary information about the deposit and the miner conducting exploitation work. A real decision-making problem was undertaken, in which three exploitation divisions of a certain area in the Polkowice-Sieroszowice mine, differing in ore quality, could be developed in an order which would guarantee maximisation of income. First, the ore price was calculated with the NSR formula; next, the decision-making problem was presented as a kind of game between the geologist (the mine) and states of Nature.

Keywords: ore deposit management, decision-making, geological information, game

Projekty geologiczno-górnicze (surowcowe) różnią się znacznie od innych form aktywności gospodarczej człowieka, ponieważ wiedza o przedmiocie zainteresowań opiera się głównie na ocenach, zaś samo złoże kopalin jest obiektem przyrodniczym i trudno jest jednoznacznie przewidzieć rzeczywiste efekty jego odkrycia. Geologiczna niepewność związana z modelem złoża i jego zasobami znajduje odzwierciedlenie w technicznych planach kopalni i przygotowaniu rozcinki złoża odpowiednim systemem i sposobem eksploatacji. Kwantyfikacja, ocena i zarządzanie niepewnością geologiczną jest kluczowe w strategicznym planowaniu działania kopalni.

Podstawowym celem, dla którego wykonuje się wyrobiska udostępniające jest przygotowanie złoża do eksploatacji górniczej. Wyrobiska udostępniające stanowią główne drogi transportu ludzi i urobku oraz spływu wód kopalnianych. Część z nich stanowi drogi jezdne i wentylacyjne, na innych zostaje ulokowany przenośnik taśmowy, a jeszcze innymi podprowadzone są niezbędne media. W celu zabezpieczenia wyrobisk w ich bezpośrednim sąsiedztwie wydzielane są ochronne filary oporowe (rys. 1) mające różną szerokość (100-260 m) w zależności od parametrów wytrzymałościowych otaczających je skał. W trakcie

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drążenia wyrobisk udostępniających i przygotowawczych duże znaczenie ma prawidłowe rozpoznanie geologiczne nowo rozcinanych obszarów złoża. Do głównych zadań służby geologicznej w kopalniach rud w tym zakresie należą:

- prawidłowe opróbowanie serii złożowej,
- kartowanie wyrobisk podziemnych i dokumentowanie wszelkiego typu zjawisk nietypowych,
- rozpoznanie złoża otworami wyprzedzającymi z wyrobisk podziemnych.

Podczas prac udostępniających gromadzona jest informacja o parametrach serii złożowej, która potem zostaje wykorzystana w prowadzeniu właściwej gospodarki złożem. O przydatności i użyteczności informacji decyduje wiele cech, niemniej za najbardziej istotne należy uznać jej aktualność, dokładność, dostępność, elastyczność i kompletność. Niebagatelne są także koszty jej pozyskania. Bardziej precyzyjna informacja wymaga często znacznych, dodatkowych wydatków. W górnictwie, w kwestii oceny warunków bezpieczeństwa pracy taka konieczność jest nieodzowna. Kompletna informacja umożliwia podejmowanie właściwych decyzji, przekłada się na sposób wykorzystania posiadanych środków i możliwości, zapewnia większą elastyczność w dopasowaniu do bieżących realiów rynkowych, technologicznych i in.

Zagadnienie dostępu do informacji jest problemem szerokim i dotyczy wielu aspektów rzeczywistości. Zdobywanie kolejnych informacji, służących zmniejszaniu ryzyka, jest zwykle kosztowne. Zdarza się, że pozyskanie kolejnych informacji jest od pewnego momentu nieopłacalne. Dodatkowa informacja jest potrzebna tylko do momentu, gdy oczekiwane korzyści z jej uzyskania przewyższają koszty z jej zdobywania. Wiąże się z tym problem wyceny informacji. Informacja, adekwatna do problemu, dla którego jest pozyskiwana, zwiększa zasób wiedzy podmiotu gospodarującego, jednak rosnąca jej ilość stwarza możliwość dezinformacji. Instrumentów analizy posiadanej przez decydenta informacji dostarczają m.in. narzędzia teoria gier. W teorii gier informacja modelowana jest zazwyczaj z wykorzystaniem koncepcji zbiorów informacyjnych (rys. 2). Zwykle uczestnicy gry nie dysponują pełną informacją, co implikuje fakt, ze sytuacja decyzyjna rozgrywa się w warunkach niepewności.

Działalność operacyjna kopalni zmierzająca do udostępnienia i eksploatacji poszczególnych części złoża realizowana jest etapami. Nieodzownym staje się zatem podejmowanie decyzji i wybór optymalnej strategii na każdym z nich. Dążeniem kopalni może być uzyskanie najkorzystniejszych wypłat, niezależnie od zaistniałych warunków otoczenia. Działanie takowe zaprezentowano na przykładzie trzech parcel eksploatacyjnych A, B i C uruchamianych w obrębie ZG Polkowice-Sieroszowice. Charakterystykę parcel zestawiono w tabelach 1, 2 i 3, a schematyczne przekroje zmienności zobrazowano na figurach 3, 4 oraz 5. Na podstawie formuły *NSR* (3) obliczono ceny rudy w parcelach. Bazując na analogii określono prawdopodobieństwa napotkania konkretnych układów wskaźników jakościowych rudy w parcelach. Zestawiając oczekiwane przychody w odniesieniu do możliwych dwóch stanów natury zdefiniowano macierz gry (4). Wygenerowany problem decyzyjny sprowadzony został do modelu gry koordynacji, a rozstrzygano w nim, które z parcel należy poddać eksploatacji. Opierając się na zakresie informacji o parametrach złoża we wszystkich parcelach będącej w posiadaniu geologa nadzorującego postęp frontu eksploatacyjnego, przeanalizowano dostępne struktury informacyjne, wskazując te optymalne. Oczekiwane średnie wielkości przychodów dla poddawanych ocenie struktur zestawiono w tabelach 4, 5 oraz 6.

Słowa kluczowe: zarządzanie złożem rud, podejmowanie decyzji, informacja geologiczna, gra

1. Introduction

Geological-mining (raw material) projects differ significantly from other forms of economic activity. This is because knowledge about the raw material in question is mostly based on estimates, while the potential fluctuation of income and the volume of the resource result from the global market situation and prices of raw materials, as well as the rate of change of these factors (Snowden et al., 1986; Abdel Sabour & Dimitrakopoulos, 2011). A mineral deposit is a natural object and hence it is difficult to unequivocally predict the actual results of its discovery. It is also difficult to unequivocally estimate the actual financial effects of these results or the probability of their occurrence. Undertaking exploration work, or even exploitation of a mineral deposit, does not guarantee financial success. Investigations conducted by the U.S. Bureau of Mines

(Jolly, 1991) proved that among 100 researched cases of copper deposits, only one turned out to be recoverable ore; and that only one deposit in 1000 assures profitable production to follow. In the years 1956-1986, only 69 such deposits were discovered. Burmeister (vide Li et al., 2008) informs us that among 35 gold mines active in Australia in the years 1983-1987, 68% did not achieve the projected head grade intended for processing. Similar studies conducted in North America by Harquail (Li et al., 2008) proved that among 50 gold ore mining projects, only 10% achieved their intended targets, while 38% went bankrupt during their first year.

Geological uncertainty connected with the model of the deposit and its resources is reflected in the mine's technical plans and preparation of the opening of the deposit using the proper system and method of exploitation. Quantification, assessment and management of geological uncertainty is critical to strategic mine planning (Elkington & Durham, 2011; Godoy & Dimitrakopoulos, 2011; Groeneveld & Topal, 2011, Azimi et al., 2011). An inaccurate evaluation of geological-mining conditions (inaccurate information) results in the likelihood of unforeseen (uncertain) occurrences, leading in turn to financial losses, decreased profitability or even reduced lifetime of the project (Dowd, 1997). These occurrences cover, among others: areas of difficult geological-mining conditions (of tectonic, depth, hydrogeological, geological engineering, gas or geothermal character) influencing the method of exploitation, its safety and results; inability to achieve assumed mining capacities; inadequate volume of material transferred for processing; overestimating the quality and/or quantity of useful mineral; etc. The extreme consequence of geological uncertainty is premature closing or liquidation of the mine. Underestimating the quality or quantity of useful material is just as disadvantageous, although it leads to different consequences. The significance of geological information is proved by cases where mines had to change mine plans, reduce production, or even have been closed down (Berry & McCarthy, 2006).

2. Copper ore deposits in Poland, the Polkowice-Sieroszowice mine – access and development

Economically significant copper ores in Poland occur in Permian (Zechstein) rocks of the North-Sudetic Trough and Fore-Sudetic Monocline. The deposit in the Monocline, currently being mined, represents one of the largest copper deposits in the world. In typological classification it is ranked as a model stratoidal deposit in sedimentary rocks (Cox & Singer, 1986; Paulo & Strzelska-Smakowska, 2000). It shows substantial continuity and is associated with a single rock lithology. Faults divide the deposit into irregular blocks of different shapes and dimensions, causing its mosaic structure. The majority of faults have throws of several meters and heaves of thirty to forty meters, creating numerous horsts and grabens.

Copper mineralization in the Fore-Sudetic Monocline is connected with sediments of a shallow sea of increasing salinity. The most characteristic rocks, which usually occur at the bottom of the Zechstein formation, are black dolomite shales. In some places mineralization also includes underlying littoral sandstones of white Rotliegend as well as overlying dolomites and limestones. The intensity of the mineralization is diversified: the richest are shales (5-10% Cu), in which, despite their low thickness (0.3-1 m), about one-fourth of the copper is concentrated. The thickness of the remaining rock formations (sandstones and carbonate rocks) ranges from a few to several meters, respectively. Thus, depending on location, the thickness of the ore deposit varies between 0.2 and 19 m, with an average of about 4.8 m. Copper mineralization disappears in the so-called “Rote Fäule” (red spots) zone, a transitional area between the reducing and oxidized facies. The most important minerals, economically speaking, include sulfides of copper (chalcocite, bornite, and chalcopyrite, accessory covellite, digenite and tennantite); also common are galena and pyrite.

The deposit on the Fore-Sudetic Monocline has diffused borders delineated by weighted-average equivalent copper (Cu_e) content combined with silver content in the ore profile. The conventional border of technical availability is 1250 m, accepted by analogy with active mines of non-ferrous metal ores. The user of the deposit, the Copper Mining and Smelting Industrial Complex (KGHM Polska Miedź S.A.), was established in 1961 as a state enterprise. Since 1991, KGHM Polska Miedź S.A. has been a joint stock company. At present the total annual production of copper in three active mines: Lubin, Rudna and Polkowice-Sierszowice amounts to 500,000 tons of copper, along with about 1,000 tons of silver.

Construction of the Polkowice mine began in 1962 and initial mineral extraction began in 1968. The decision to build the Sieroszowice mine was made in 1972 and it was commissioned at the beginning of 1980. On 1 January 1996, the two mines were combined and the Polkowice-Sieroszowice mine division was created. Geological-mining conditions of exploitation in the Polkowice-Sieroszowice mine are difficult, and there are variations in the development of the deposit series. Variable morphology of ore bodies results in ore dilution. The considerable depth of the deposit causes an increase in underground temperatures and pressure of the rock mass. The strength of the roof rocks is variable; the strength parameters of sandstones and dolomites are the most important in roof-control operations. Rock bursts represent a serious hazard in areas where the roof is strong and fairly thick. During the current exploitation, the variable deposit parameters observed include:

- thickness,
- form and structure,
- ore lithology,
- intensity and type of mineralization.

The ore horizon in the Polkowice-Sieroszowice is relatively thin, which is a characteristic feature of this mine. Around 35.4% of the reserve has a thickness of 2.01-3.0 m; 21.3%, of 0.1-1.5 m. Shale ore, though constituting only about 17% of the reserve, contains the highest percentage of copper. Average deposit thickness is as high as 1.7 m with 3.19% of copper content. About 40% of the silver content is also obtained from shale. The extraction of ore under such conditions requires the use of suitably low-silhouette machinery. The Polkowice-Sieroszowice mine yields about 190,000 tons of copper annually.

The basic aim of development work is preparation of the deposit for mining exploitation. From a mining industry point of view, a prepared part of a deposit can be defined as a deposit parcel cut through by a number of works, drilled in a complex of between two and five drifts (Butra et al., 1996; Piechota, 2007). Development works are the main paths for transport of people and output, as well as ducts for the flow of mine water. Some of them will act as transport roads and ventilation ducts; others will be fitted with belt conveyors; while still others will be used for providing necessary utilities such as electric power or water. In order for the works to fulfill these functions for a long time, they need to be protected from the negative effects of nearby mining works or old guags. For that purpose, barrier pillars of various widths (100-260 m), depending on the resistance parameters of the surrounding rocks (Piechota, 2007), are placed in the immediate area (Fig. 1).

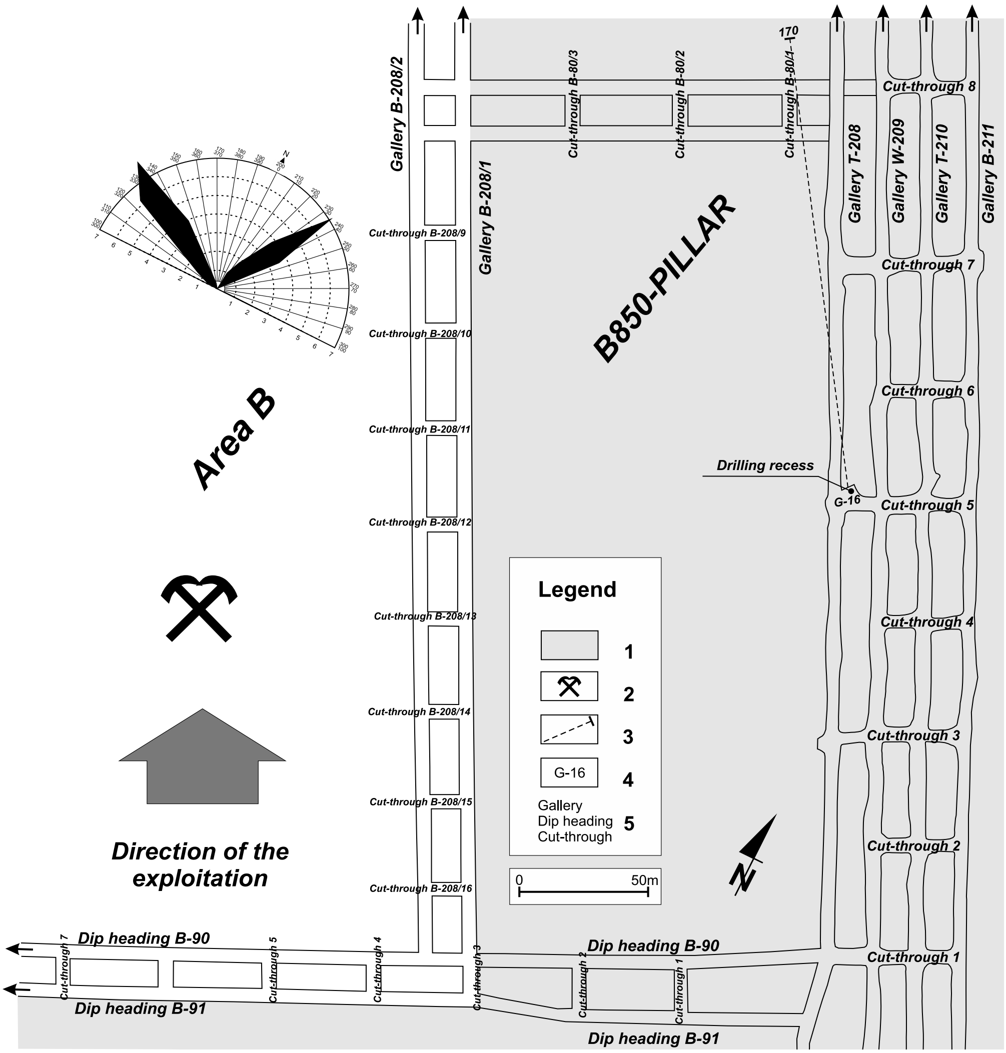

Fig. 1. Simplified configuration of development works applied in the Polkowice-Sieroszowice mine: 1 – fragment of barrier pillar, 2 – fragment of exploitation field, 3 – hydrogeological hole, 4 – hole number, 5 – gallery, dip heading, cut-through

At the stage of drilling access and development works, an important role is played by the proper geological identification of newly-drilled parts of the deposit. The key tasks of geological service in ore mines in this area are (Krzak & Panajew, 2008):

- proper sampling of deposit series. It sometimes happens that the adit through which the works pass does not correspond to the range of the deposit. Based on obtained information, it is possible to estimate the resources in the delineated deposit parcel, determine the average deposit parameters (content, yield and thickness of the deposit series), choose the best method of exploitation, and optimally conduct deposit management,

- charting underground works and documenting all kinds of atypical phenomena. Charting enables a forecast of the geological-mining conditions which may be encountered in the path of the planned exploitation works,
- evaluation of the deposit by advance drilling from within underground works. Although preparatory works themselves are aimed at such an evaluation, it is sometimes necessary to conduct auxiliary drilling works. Drilling works enable, to a great extent, the rapid estimation of geological and hydrogeological conditions in front of and to the sides of preliminary works. Such information allows for flexible reactions and changes in cutting methods in a particular exploitation field.

3. Information, game theory

It is commonly understood that good, reliable and timely information enables proper decision-making. In most situations, including those modelled by game-theory applications, the key issue is the scope of information possessed by the players. It would be superfluous to convince anyone of its significance in practically every sort of economic activity and decision-making. Unfortunately, as accurately cited by Robbins (2003): "…You will never possess all the information you need to make the decision. If you had it, it would be an obvious conclusion, not a decision…" In recent years, information has been considered a resource (production factor) of strategic importance, enabling the execution of the management function. Suitability and usability of information are determined by many features, although the most important of them are the following: timeliness, accuracy, availability, flexibility and completeness.

Costs of obtaining the information are important as well. More accurate information is usually connected with higher and additional expenses. In the mining industry such a link cannot be avoided, particularly in relation to health and safety issues. Complete information enables proper decision-making so as to optimally use the means and opportunities at hand; it also provides flexibility in adapting to current market and technological circumstances.

The theory of games is a field of mathematics, analyzing situations of conflict or cooperation between many subjects in a formal approach. Hence, it researches the capabilities for optimal behaviour, where the consequences of actions (succeeds or fails) of participants of the game (players) depend on mutual decisions (actions, moves). Game theory is often used for analyzing situations differing from one another from the point of view of availability of information. Information is usually modelled by applying the concept of information sets. In general, an information set is a point where the players make decisions. It happens very often that the players possess different ranges of information (asymmetry of information). Hiding or disclosing information is usually crucial to the result of the game. A graphic visualization of interaction between the players is represented by the so-called decision tree, showing all possible components of the decision-making process. The elements of the tree are vertices (points where decisions are made), tracks (edges, branches) showing possible directions within the game, and information sets (Fig. 2). Game tracks are the lines running from the initial to the terminal vertex via intermediate vertices, corresponding to certain actions taken by the players (decision-makers). Each terminal vertex corresponds to exactly one track running from the initial point. An information set is a point where players make decisions. In other words, an information set is a set of vertices which cannot be distinguished from each other by the player at the moment of making the decision.

In the game shown in Fig. 2, player 2, who makes a sequential decision after player 1's move, does not know at which vertex he is located; his information is incomplete.

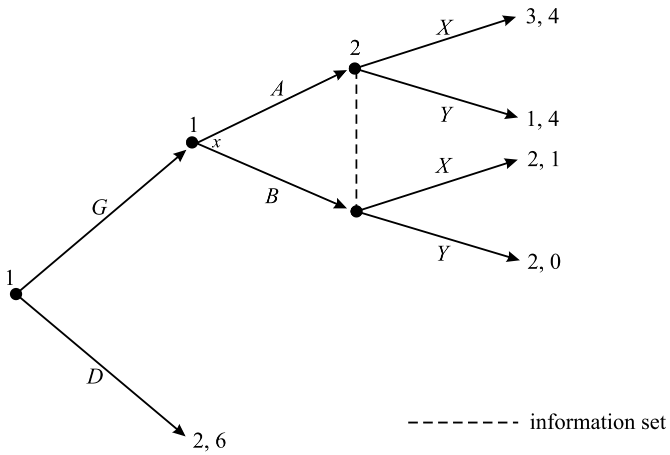

Fig. 2. A sample decision tree of a game and an information set

In a game with perfect (complete) information, the information set contains one element. Otherwise, it is a game of imperfect (incomplete) information, which means that the players do not know the sets of strategies and payoff functions of other players and Nature can make an initial move which is not known to all players (Aumann & Maschler, 1995; Watson, 2002; Rasmusen, 2007). Perfect information implies that the game is played in certainty conditions; otherwise, it takes place in uncertainty conditions.

Games against Nature are a specific kind of games. Nature is a pseudo-player who takes random actions at specified points in the game with specified probabilities. Such situations are usually modeled with the conventional decision tree approach. In the following part of a paper a payoff matrix to geological information evaluation will be applied. The idea of using payoff matrix is a rudimentary form of the work by Kaufmann (1963). Planning over extraction that have uncertain payoffs was developed in several works, inter alia, by Brennan & Schwartz (1985), Torries (1998), Walls & Eggert (1996), Newendorp & Schuyler (2000), Davis & Samis (2006), Topal (2008), Dehghani & Ataee-Pour (2011). Most of them were related to the oil and gas exploration. Among Polish authors by Butra et. al. (2002), Wanielista et. al. (2002), Saługa & Sobczyk (2005), Magda (2011).

4. Geological information modeling for the purposes of deposit management

The operational activity of the mine, aimed at opening and exploiting separate parts of the deposit, is conducted in stages. Hence, decision-making and choosing an optimal strategy becomes necessary at each stage. Basing on available information, the mine's geological service is particularly responsible for:

- geological deposit evaluation,
- supervision of front-line work areas,
- documentation of reserves.

The activities listed above, along with others, constitute deposit management. In performing these tasks, the geological service of the Polkowice-Sieroszowice mine is obliged to solve the following decision-making problem.

In a certain mining area, development and exploitation of three divisions, A, B and C, is planned. The deposit series in division A includes shales and dolomites only. Oxidized zones appear in the underlying sandstone series and partially in the shales and dolomites up to 2.5 m. Barren zones extend from several meters up to thirty or forty. On the border between barren rocks and ore, a sudden increase in the content of copper sulfides is observed. Balanced copper mineralization is often shifted from shale to overlying dolomites, or disappears altogether (Fig. 3). In the "Rote Fäule" zones, increased concentrations of gold, platinum and palladium are observed. Within the borders of the division in question, no discontinuous tectonics were reported. The average deposit parameters of division A are shown in Table 1.

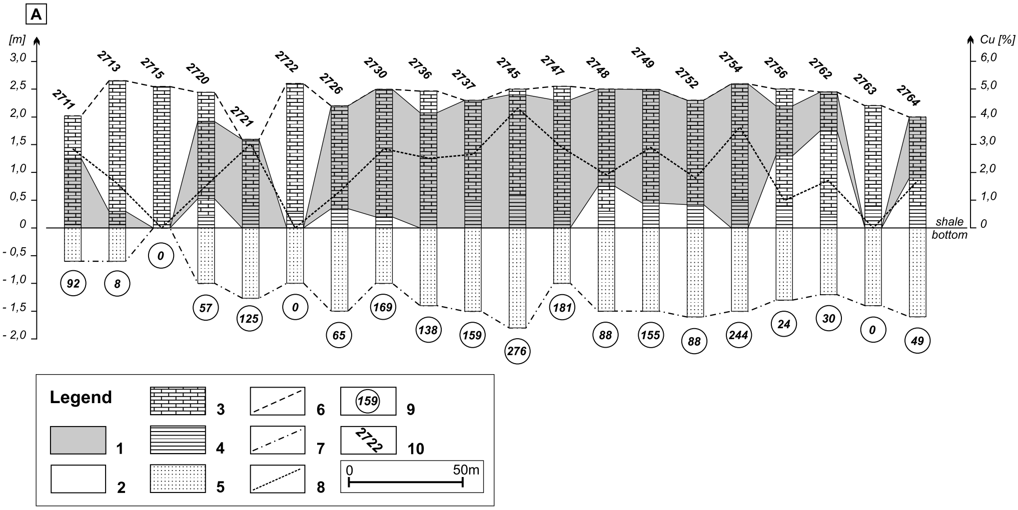

Fig. 3. Simplified cross-section of deposit series variability in division A: 1 – deposit series, 2 – barren rocks, 3 – dolomite, 4 – shale, 5 – sandstone, 6 – gallery roof, 7 – gallery floor, 8 – Cu content in deposit series [%], 9 – Cu yield [kg/m^2], 10 – sample number

TABLE 1

Average deposit parameters in division A

Dolomite		Shale		Total		
Thickness [m]	Cu content [%]	Thickness [m]	Cu content [%]	Thickness [m]	Cu content [%]	Cu yield [kg/m^2]
1.46	2.16	0.14	6.33	1.60	2.53	108.9

Balanced mineralization in division B applies to shale and dolomite only. Locally, oxidized zones occur in sandstone series indicated by red spots. These zones extend up to 1 m in overlying dolomites and shales, partially creating barren zones extending up to 50 m. On the border between barren rocks and ore, a sudden increase of copper sulfides content is also observed (Fig. 4). Within the borders of division B, individual small faults (0.5-1.0 m. throw) have been observed. The average deposit parameters in division B are shown in Table 2.

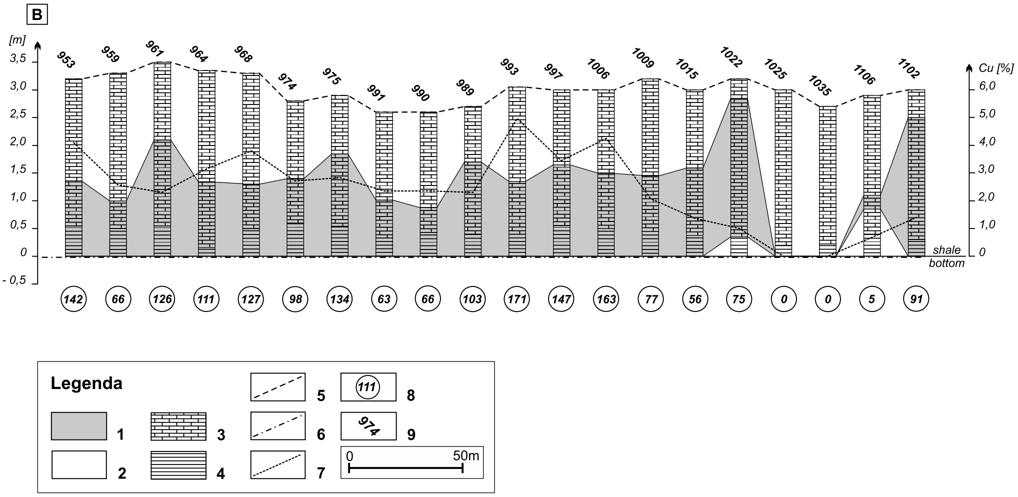

Fig. 4. Simplified cross-section of deposit series variability in division B: 1 – deposit series, 2 – barren rocks, 3 – dolomite, 4 – shale, 5 – gallery roof, 6 – gallery floor, 7 – Cu content in deposit series [%], 8 – Cu yield [kg/m^2], 9 – sample number

TABLE 2

Average deposit parameters in division B

Dolomite		Shale		Total		
Thickness [m]	Cu content [%]	Thickness [m]	Cu content [%]	Thickness [m]	Cu content [%]	Cu yield [kg/m^2]
1.13	1.88	0.34	5.44	1.47	2.71	104.2

Balanced mineralization in division C covers shale and dolomite only (Fig. 5). Barren sandstone is also extracted in order to rock mass distressing. Within the borders of division C, individual small en echelon faults with summarized 6 m throw have been observed. Ore series is continuous over the division, partially is reduced to the shale only, where the highest content of copper is reported. The average deposit parameters in division C are shown in Table 3.

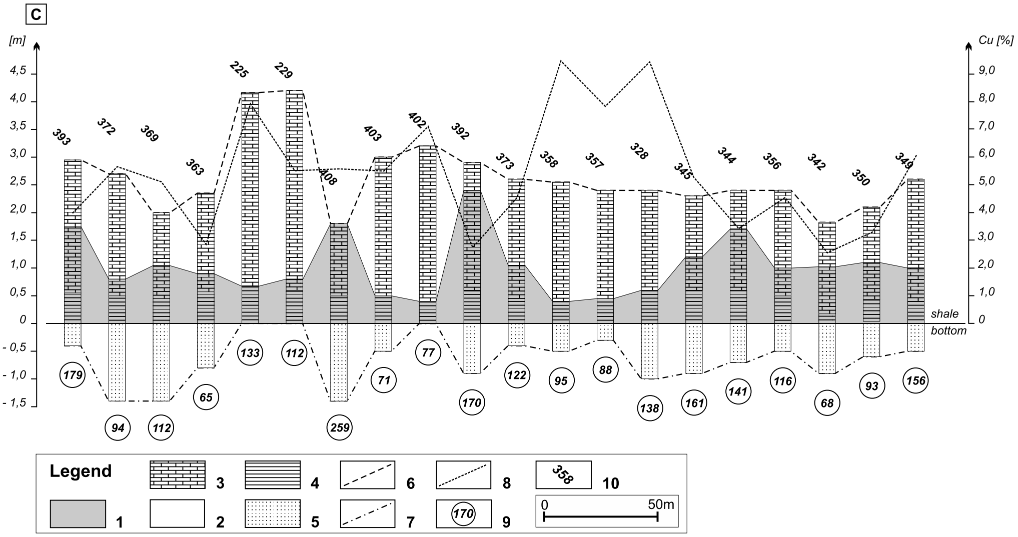

Fig. 5. Simplified cross-section of deposit series variability in division C: 1 – deposit series, 2 – barren rocks, 3 – dolomite, 4 – shale, 5 – sandstone, 6 – gallery roof, 7 – gallery floor, 8 – Cu content in deposit series [%], 9 – Cu yield [kg/m^2], 10 – sample number

TABLE 3

Average deposit parameters in division C

Dolomite		Shale		Total		
Thickness [m]	Cu content [%]	Thickness [m]	Cu content [%]	Thickness [m]	Cu content [%]	Cu yield [kg/m^2]
0.86	1.45	0.37	5.94	1.23	2.80	123.5

The accuracy of the deposit parameters reported in Tables 1, 2 and 3, based on identification to date, were considered sufficient and are regarded as the maximum expected in ore exploitation. As well, geological-mining conditions, given the environment of the divisions, are similar. The resulting operating costs are being estimated at identical levels for all divisions. However, the potential income to be generated from the ore exploitation is determined differently, as it depends mainly on the ore quality in each division.

Ore price (value) has been calculated with the *NSR* (Net Smelter Return) formula (Wellmer, 1989; Wills, 2006), which is commonly applied for estimating the value of ores and concentrates of non-ferrous metals. In accordance to this rule, the price of 1 tone of concentrate makes:

$$NSR_k = a \cdot P - \left(TC + RC \cdot a\right) - K + B \tag{1}$$

where:

a — quantity of metal to be paid,
P — stock exchange price of metal,

TC — treatment charges for smelting 1t of concentrate,
RC — refining charges for 1t of metal,
K — penalty for presence of noxious tramp elements above the agreed level,
B — bonus for presence of useful tramp elements above the agreed level.

The quantity of material to be paid is a result of the applied processing technology. In metallurgical process, it is not the entire quantity of metal included in the concentrate that becomes returned, but only a part of it; thus, respective unit deductions are agreed between the mine and the metallurgical plant. For copper concentrates, the unit deduction makes usually 1% (Wellmer, 1989). In order to optimize the enrichment process, recovery (ε) is introduced into formula (1) as the function of concentrates' weight:

$$Q = \frac{\varepsilon \cdot \alpha}{\beta} \qquad (2)$$

where:
Q — weight of concentrate obtained from 1000 kg of feed,
α — metal content in the ore,
β — metal content in the concentrate,
ε — processing recovery.

After introducing notion (2) into formula (1), we get an equation allowing calculating the value of 1 ton of ore:

$$NSR_r = \frac{a \cdot P - TC - K + B}{\dfrac{\beta}{\varepsilon \cdot \alpha}} \qquad (3)$$

After assuming average copper price (cash LME) at 9,710 USD/t, and respective treatment/refining charges at $77/t and 7.7 US cents a pound prices of ore in the divisions were calculated. Bonuses and penalties for tramp elements of other metals were ignored. Ore price calculated with the NSR formula equals $200.44/t in division A, $214.70/t in division B and $221.83/t in division C.

The fluctuation of Cu content between blocks, formulated with variability factors (50.9% in A, 66.1% in B, 79.2% in C), indicates a higher variation of that parameter in division C. Thus the Cu content in the ore can drop to levels of 2.40% (A), 2.33% (B) and 2.31% (C). All represent poor ore quality. The appropriate NSR price will fall to $190.14 (A), $184.59 (B), and $183.01 (C).

Without conducting a more accurate assessment of the deposit series border or of Cu content, it can be assumed, by analogy to neighboring divisions, that meeting a certain level of ore quality can be assigned the following levels of probability:

good in division A, good in division B, good in division C – 0.125,
good in division A, good in division B, poor in division C – 0.175,
good in division A, poor in division B, good in division C – 0.175,
good in division A, poor in division B, poor in division C – 0.100,
poor in division A, good in division B, good in division C – 0.175,
poor in division A, good in division B, poor in division C – 0.100,
poor in division A, poor in division B, good in division C – 0.100,
poor in division A, poor in division B, poor in division C – 0.050.

Returning to the decision-making problem posed at the beginning of that chapter, it is possible to combine the expected incomes in the matrix (4).

		Cu content	
		good	poor
Incomes	from division A	200.44	190.14
	from division B	214.70	184.59
	from division C	221.83	183.01

(4)

The presented matrix of incomes represents game in which two players are involved: the mine's geological service which has three moves at its disposal: the choice of one of division for the exploitation and the state of Nature which can assume one of two possible positions: good or poor. Of course, the number of states of Nature may be continuous within a certain range; in the example, two discrete states of Nature were assumed. The strategy used in further solutions will guarantee the most beneficial payoffs even under the worst conditions.

The decision-making problem boils down here to the decision: which block should be developed and then exploited? Perhaps both? Perhaps all? An additional factor in the problem is that simultaneous opening of the divisions is connected with additional costs for improving transport roads unprepared for output coming from divisions, construction of conveyors, additional reinforcement of exploitation divisions and higher consumption of the means of production. The cost of such an investment is estimated at \$100/t of ore in case of two divisions developed, and \$300/t of ore in case of two divisions developed.

Such a defined decision-making problem can be treated as a game of coordination in which the geologist is obliged to make an optimal decision in relation to mining works development. The geologist is making a decision in conditions of uncertainty and lack of information. From payoff matrix (4) we find that even a poor quality of ore does not exclude exploitation. Important here is the range of information related to ore quality in all divisions the geologist has at his disposal. There are eight combinations of information structures in the game describing his information. These combinations are (Z Z Z), (Z Z N), (Z N Z), (N Z Z), (Z N N) (N N Z) (N Z N), and (N N N), where the positions of each combination stand for:

- pos. 1 – ore characteristic of division A,
- pos. 2 – ore characteristic of division B,
- pos. 3 – ore characteristic of division C.

The letter "Z" stands for knowledge of deposit characteristics and ore quality, while "N" indicates the lack of such knowledge. For instance, combination (Z Z Z) means complete knowledge of deposit parameters in all divisions, while (Z N N) means that geologists possess knowledge about the division A deposit only. The best point to start the analysis is set (N N N), representing the geologist's total ignorance concerning the deposit characteristics (fortunately, this is a rare and rather theoretical situation only). This case is reflected by eight behaviour schemes:

- the geologist decides to start exploitation of all blocks,
- the geologist decides to start exploitation of both divisions A and B only,
- the geologist decides to start exploitation of both divisions A and C only,
- the geologist decides to start exploitation of both divisions B and C only,
- the geologist decides to start exploitation of divisions A only,

- the geologist decides to start exploitation of divisions B only,
- the geologist decides to start exploitation of divisions C only,
- the geologist decides to start exploitation of neither block.

Each scheme can be assigned average expected payoffs (income):

- in scheme 1:

$$\begin{aligned}&0.125\cdot\left(200.44+214.70+221.83-300\right)+0.175\cdot\left(200.44+214.70+183.01-300\right)+\\&+0.175\cdot\left(200.44+184.59+221.83-300\right)+0.1\cdot\left(200.44+184.59+183.01-300\right)+\\&+0.175\cdot\left(190.14+214.70+221.83-300\right)+0.1\cdot\left(190.14+214.70+183.01-300\right)+\\&+0.1\cdot\left(190.14+184.59+221.83-300\right)+0.05\cdot\left(190.14+184.59+183.01-300\right)=\$303{,}30\,/\,\mathrm{t}\end{aligned}$$

- in scheme 2:

$$\begin{aligned}&0.125\cdot\left(200.44+214.70-100\right)+0.175\cdot\left(200.44+214.70-100\right)+\\&+0.175\cdot\left(200.44+184.59-100\right)+0.1\cdot\left(200.44+184.59-100\right)+\\&+0.175\cdot\left(190.14+214.70-100\right)+0.1\cdot\left(190.14+214.70-100\right)+\\&+0.1\cdot\left(190.14+184.59-100\right)+0.05\cdot\left(190.14+184.59-100\right)=\$297.97\,/\,\mathrm{t}\end{aligned}$$

- in scheme 3:

$$\begin{aligned}&0.125\cdot\left(200.44+221.83-100\right)+0.175\cdot\left(200.44+183.01-100\right)+\\&+0.175\cdot\left(200.44+221.83-100\right)+0.1\cdot\left(200.44+1183.01-100\right)+\\&+0.175\cdot\left(190.14+1221.83-100\right)+0.1\cdot\left(190.14+183.01-100\right)+\\&+0.1\cdot\left(190.14+1221.83-100\right)+0.05\cdot\left(190.141+183.01-100\right)=\$301.39\,/\,\mathrm{t}\end{aligned}$$

- in scheme 4:

$$\begin{aligned}&0.125\cdot\left(214.70+221.83-100\right)+0.175\cdot\left(214.70+183.01-100\right)+\\&+0.175\cdot\left(184.59+221.83-100\right)+0.1\cdot\left(184.59+183.01-100\right)+\\&+0.175\cdot\left(214.70+221.83-100\right)+0.1\cdot\left(214.70+183.01-100\right)+\\&+0.1\cdot\left(184.59+221.83-100\right)+0.05\cdot\left(184.59+183.01-100\right)=\$307.23\,/\,\mathrm{t}\end{aligned}$$

- in scheme 5:

$$\begin{aligned}&0.125\cdot200.44+0.175\cdot200.44+0.175\cdot200.44+0.1\cdot200.44+\\&+0.175\cdot190.14+0.1\cdot190.14+0.1\cdot190.14+0.05\cdot190.14=\$196.06\,/\,\mathrm{t}\end{aligned}$$

- in scheme 6:

$$\begin{aligned}&0.125\cdot214.70+0.175\cdot214.70+0.175\cdot184.59+0.1\cdot184.59+\\&+0.175\cdot214.70+0.1\cdot214.70+0.1\cdot184.59+0.05\cdot184.59=\$201.90\,/\,\mathrm{t}\end{aligned}$$

- in scheme 7:

$$\begin{aligned}&0.125\cdot221.83+0.175\cdot183.01+0.175\cdot221.83+0.1\cdot183.01+\\&+0.175\cdot221.83+0.1\cdot183.01+0.1\cdot221{,}83+0.05\cdot183.01=\$205.33\,/\,\mathrm{t}\end{aligned}$$

- in scheme 8: $=\$0\,/\,\mathrm{t}$

The best decision is to start exploitation in division B and C only. Changing the information structure can result in broadening the set of behaviour schemes. For information structure (Z N N), corresponding to the knowledge of deposit parameters in division A only, there are sixteen possible behaviour schemes (Table 4). Notation "E" stands for taking up exploitation, B for giving it up. The position of the letter indicates the geologists' decisions given the knowledge of the following facts:

- pos. 1 – decisions of geologist in relation to division A under a good state of Nature,
- pos. 2 – decisions of geologist in relation to division A under a poor state of Nature,
- pos. 3 – decisions of geologist in relation to division B under a good state of Nature,
- pos. 4 – decisions of geologist in relation to division B under a poor state of Nature,
- pos. 5 – decisions of geologist in relation to division C under a good state of Nature,
- pos. 6 – decisions of geologist in relation to division C under a poor state of Nature.

A sample calculation of expected income for EB EE EE is as follows:

$$\begin{aligned}&0.125\cdot(200.44+214.70+221.83-300)+0.175\cdot(200.44+214.70+183.01-300)+\\&+0.175\cdot(200.44+184.59+221.83-300)+0.1\cdot(200.44+184.59+183.01-300)+\\&+0.175\cdot(214.70+221.83-100)+0.1\cdot(214.70+183.01-100)+\\&+0.1\cdot(184.59+221.83-100)+0.05\cdot(184.59+183.01-100)=\$307.49\,/\,\mathrm{t}\end{aligned}$$

Details of calculation for other schemes were omitted here. The results are presented in Table 4. Some of the schemes are repetitions of conditions of ignorance of deposit characteristics. Eight further schemes, unavailable for the previous information structure, were obtained in the current one. One, slightly better scheme EB EE EE, unavailable in the previous information structure, has now been obtained. It refers to exploitation of division A only under good ore quality, with exploitation both remaining irrespective of ore quality. The next information structures for (N Z N) and (N N Z) expands the geologist's room for manoeuvre. Up to 16 additional variant moves are possible here (Table 5). The highest income is being guaranteed by scheme EE EE EB, also scheme EE EB EE is better than EB EE EE. The last change of the information structure for (Z Z Z) includes all previously described structures, as well as (Z Z N), (Z N Z) and (N Z Z) (Table 6). Unfortunately, all are worse than scheme EE EE EB from the (N N Z) structure.

TABLE 4

Expected average incomes for assigned information structure (Z N N)

Behaviour scheme	**Expected average income ($/t)**
1	**2**
EE EE EE = sch. 1	303,30
EE EE BB = sch. 2	297,97
EE BB EE = sch. 3	301,39
BB EE EE = sch. 4	307,23
EE BB BB = sch. 5	196,06
BB EE BB = sch. 6	201,90
BB BB EE = sch. 7	205,33
BB BB BB = sch. 8	0,00

1	2
EB EE EE	307,49
BE EE EE	303,04
EB EE BB	259,66
BE EE BB	240,21
EB BB EE	263,08
BE BB EE	243,64
EB BB BB	115,25
BE BB BB	80,81

TABLE 5

Expected average incomes for assigned information structure (N Z N) and (N N Z)
(average incomes for structure N N N were omitted in the table)

Behaviour scheme	**Expected average income ($/t)**	**Behaviour scheme**	**Expected average income ($/t)**
EE EB EE	309.85	EE EE EB	310.52
EE BE EE	294.84	EE EE BE	290.75
EE EB BB	262.02	EE BB EB	266.11
EE BE BB	232.01	EE BB BE	231.34
BB EB EE	271.28	BB EE EB	271.96
BB BE EE	241.28	BB EE BE	237.18
BB EB BB	123.45	BB BB EB	127.55
BB BE BB	78.45	BB BB BE	77.78

TABLE 6

Expected average incomes for assigned information structure (Z Z Z)
(average incomes for previously presented structures were omitted in the table)

Behaviour scheme	**Expected average income ($/t)**	**Behaviour scheme**	**Expected average income ($/t)**	**Behaviour scheme**	**Expected average income ($/t)**
EB EB EE	299.04	EB EE EB	299.71	EE EB EB	302.07
BE EB EE	282.09	BE EE EB	282.77	EE BE EB	254.57
EB BE EE	271.54	EB EE BE	267.44	EE EB BE	269.79
BE BE EE	264.59	BE EE BE	260.49	EE BE BE	252.29
EB EB BB	208.71	EB BB EB	212.81	BB EB EB	221.00
BE EB BB	176.76	BE BB EB	180.86	BB BE EB	178.50
EB BE BB	166.20	EB BB BE	165.53	BB EB BE	173.73
BE BE BB	91.97	BE BB BE	143.59	BB BE BE	141.23
EB EB EB	276.26	EB BE EB	236.26	EB EB BE	231.48
BE EB EB	246.81	BE BE EB	216.81	BE EB BE	212.04
EB BE BE	201.48				
BE BE BE	192.04				

Scheme EE EE EB refers to exploitation of division A and B under both good and poor ore quality, with exploitation of division C only under good ore quality. This scheme is a solution for any information structure. Achievable income is higher by $3.29/t compared to that achieved in the structure of ignorance. Should it turn out that additional evaluation of the ore's quality parameters costs no more than \$3.29/t, then such evaluation should be conducted in division C. Additional evaluation is also possible in divisions A and B provided that will not cost more than \$0.26/t and \$2.62/t, respectively. If this difference failed to cover the additional costs of ore quality evaluation, it might be rejected out of hand, so that decisions would be made as in conditions of ignorance.

More accurate knowledge relating to the ore quality is significant in case of planned development of three divisions only. For two planned divisions one should behave like in the structure of the ignorance and not to incur extra costs for evaluation. Then, optimal decision refers to development of divisions B and C (scheme BB EE EE). Choice of single division is connected with application scheme BB BB EE referring to the division C development.

5. Conclusions

The question of information availability is a broad problem, concerning many aspects of reality. Acquiring new information used to decrease risk is usually expensive. Additional information is required as long as the expected advantages of obtaining it exceed the costs of its acquisition. Information appropriate to the problem for which it is being acquired increases the store of knowledge of the decision-maker; on the other hand, acquiring information in excess often leads to misinformation. Adaptation of the games involving incomplete information for the modeling of information issues ensures optimal utility and indicates the most effective solutions.

In the simplified analyzed example, use of payoff matrix is revealed as an effective tool to support decision-making in operational geological-mining activity. Thanks to this, the geologist (decision-maker) can make decisions based on a far more complete model of information. Certainly, the presented model does not cover all possible geological courses of action; however it is easily possible to include additional aspects in the model which will involve only extra arithmetical operations. Moreover, assumptions about maximization of average incomes may not always match the policy of the mine. Establishing this policy is essential here, because the motives behind the actions of company owners are not always known. Moreover, information modeling with game theory approach allows the geologist to establish the price (cost) of additional information. This leads to potential further research related to the evaluation of deposit quality parameters. Instead of the quality of ore, other aspects can be evaluated, e.g. geological-mining conditions, based on one hand on potential profit, and on the other on work and industrial safety.

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