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The mathematical concept of the currents' asymmetrical components in three-phase four-wire systems with sinusoidal and asymmetric voltage supply

Z. SOŁJAN*, G. HOŁDYŃSKI, and M. ZAJKOWSKI

Faculty of Electrical Engineering, Bialystok University of Technology, 45D Wiejska St., 15-351 Białystok, Poland

Abstract. The proper description of circuits supplied from an asymmetrical and sinusoidal voltage source, in which line parameters are included, requires an adequate mathematical concept or theory. The authors of the publication present the mathematical concept of the currents' asymmetrical components for three-phase four-wire systems, taking into account the impedance of the neutral conductor and the impedance of power transmission lines. In the new approach, four orthogonal current components were proposed in charge of its flow between the source and the load. The introduced distribution shows, regardless of the type of the voltage asymmetry (amplitude or phase), it is possible to set down the symmetrical active current and other components, i.e. reactive current, negative current and zero current, which will allow determining the reference current of the active filter.

Key words: currents' asymmetrical components (CAC), mathematical concept, asymetric supply.

1. Introduction

Transmission of electricity from sources to loads through the power systems, where it is transformed adequately to the needs of the consumer, is described by power theories or mathematical concepts [1–7, 17]. Over 100 years of electrical energy transmission, many different approaches have been created.

The description of the power theory is divided into two domains, i.e. time and frequency domain. The description of the time domain, in view of the rapidity of calculations, is mainly used to control semiconductor devices in active or hybrid power filters [4–6, 11–18]. The most common time domain methods are [4–6, 17]. The frequency domain description [1, 3], using the Fourier transform, causes delays in measuring circuit, but the approaches based on frequency description are more precise methods and also are used to generate the reference current of an active power filter [8–10].

In the power theories or mathematical concepts cited above, the mathematical description, and, by extension, the obtained results are correct, on the assumption that the voltage supply is symmetrical.

This publication proposes a mathematical concept of currents' asymmetrical components (CAC) for sinusoidal asymmetric three-phase four-wire systems, taking into account the impedance of all four wires.

The concept assumes the decomposition of the line current into four components, i.e.: the active current, reactive current, negative current and the zero current. Each current component can be presented in the form of the instantaneous waveform or in the form of the norm.

In addition, the shown components are mutually orthogonal, and therefore their existence in the system is independent.

The authors of the publication demonstrate the possibility of using the mathematical concept of the currents' asymmetrical components for three-phase four-wire systems to determine the reference current of an active power filter or active part of a hybrid power filter.

2. Currents' asymmetrical components in three-phase four-wire systems at sinusoidal and asymmetric voltages

In compliance with the mathematical concept of the currents' asymmetrical components (CAC), three voltage vectors can be defined in the supply system. The locations of these vectors are marked in Fig. 1.

According to Fig. 1, the vector of sinusoidal and asymmetrical instantaneous voltages $\mathbf{u}(t)$ on generator output terminals is described as follows [1]:

$$\mathbf{u}(t) = \begin{bmatrix} u_{L1}(t) \\ u_{L2}(t) \\ u_{L3}(t) \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\left\{\mathbf{U}e^{j\omega_1 t}\right\} = \left\{\begin{matrix} U_{L1}\sqrt{2}\cos\left(\omega_1 t + 0^\circ\right) \\ U_{L2}\sqrt{2}\cos\left(\omega_1 t - 120^\circ\right) \\ U_{L3}\sqrt{2}\cos\left(\omega_1 t + 120^\circ\right) \end{matrix}\right\} \tag{1}$$

*e-mail: z.soljan@pb.edu.pl

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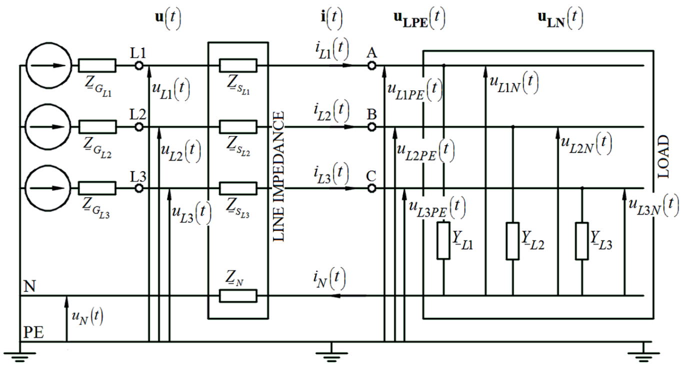

Fig. 1. Power-supply system with indicated asymmetrical sinusoidal voltage vectors

where: U_{L1PE}, U_{L2PE}, U_{L3PE} – rms values of the phase voltages, phases L1, L2 and L3 respectively, $u_{L1}(t)$, $u_{L2}(t)$, $u_{L3}(t)$ – instantaneous values of the voltages, phases L1, L2 and L3 respectively, ω_1 – fundamental harmonic angular speed, $\mathbf{U}$ – complex phase voltages vector, defined with the formula:

$$\mathbf{U} = \left[\underline{U}_{L1} \quad \underline{U}_{L2} \quad \underline{U}_{L3}\right]^T, \tag{2}$$

where: $\underline{U}_{L1}$, $\underline{U}_{L2}$, $\underline{U}_{L3}$ – complex values of the phase voltages.

On account of the voltage drop $(\Delta u_S(t))$ at the impedance of the power transmission line, the vector of the instantaneous values of the phase voltages $\mathbf{u_{LPE}}(t)$ with respect to the protective earthing conductor (PE) is:

$$\mathbf{u_{LPE}}(t) = \begin{bmatrix} u_{L1}(t) - \Delta u_{S_{L1}}(t) \\ u_{L2}(t) - \Delta u_{S_{L2}}(t) \\ u_{L3}(t) - \Delta u_{S_{L3}}(t) \end{bmatrix} = \begin{bmatrix} u_{L1PE}(t) \\ u_{L2PE}(t) \\ u_{L3PE}(t) \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\left\{\mathbf{U_{LPE}} e^{j\omega_1 t}\right\} = \begin{Bmatrix} U_{L1PE}\sqrt{2}\cos\left(\omega_1 t \pm \varphi_{U_{L1PE}}\right) \\ U_{L2PE}\sqrt{2}\cos\left(\omega_1 t \pm \varphi_{U_{L2PE}}\right) \\ U_{L3PE}\sqrt{2}\cos\left(\omega_1 t \pm \varphi_{U_{L3PE}}\right) \end{Bmatrix} \tag{3}$$

where: U_{L1PE}, U_{L2PE}, U_{L3PE} – rms values of the phase voltages at the terminals of the load to the protecting earthing conductor, phases L1, L2 and L3 respectively, $u_{L1}(t)$, $u_{L2}(t)$, $u_{L3}(t)$ – instantaneous values of the same voltage, phases L1, L2 and L3 respectively, $\varphi_{U_{L1PE}}$, $\varphi_{U_{L2PE}}$, $\varphi_{U_{L3PE}}$ – values of the voltages phase angle at the terminals of the load, phases L1, L2 and L3 respectively, $\mathbf{U_{LPE}}$ – complex phase voltages vector at the terminals of the load, defined with the equation:

$$\mathbf{U_{LPE}} = \left[\underline{U}_{L1PE} \quad \underline{U}_{L2PE} \quad \underline{U}_{L3PE}\right]^T, \tag{4}$$

where: $\underline{U}_{L1PE}$, $\underline{U}_{L2PE}$, $\underline{U}_{L3PE}$ – complex values of the phase voltages at the terminals of the load to the protecting earthing conductor.

Taking into consideration the voltage drop at the impedance of the neutral conductor, which results in the occurrence of the voltage at the load's neutral point relative to ground $u_N(t)$, the vector of instantaneous values of phase voltages with respect to the neutral conductor $\mathbf{u_{LN}}(t)$ is defined as follows:

$$\mathbf{u_{LN}}(t) = \begin{bmatrix} u_{L1}(t) - \Delta u_{S_{L1}}(t) - u_N(t) \\ u_{L2}(t) - \Delta u_{S_{L2}}(t) - u_N(t) \\ u_{L3}(t) - \Delta u_{S_{L3}}(t) - u_N(t) \end{bmatrix} = \begin{bmatrix} u_{L1N}(t) \\ u_{L2N}(t) \\ u_{L3N}(t) \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\left\{\mathbf{U_{LN}} e^{j\omega_1 t}\right\} = \begin{Bmatrix} U_{L1N}\sqrt{2}\cos\left(\omega_1 t \pm \varphi_{U_{L1N}}\right) \\ U_{L2N}\sqrt{2}\cos\left(\omega_1 t \pm \varphi_{U_{L2N}}\right) \\ U_{L3N}\sqrt{2}\cos\left(\omega_1 t \pm \varphi_{U_{L3N}}\right) \end{Bmatrix} \tag{5}$$

where: U_{L1N}, U_{L2N}, U_{L3N} – rms values of the phase voltages at the load to the neutral conductor, phases L1, L2 and L3 respectively, $\varphi_{U_{L1N}}$, $\varphi_{U_{L2N}}$, $\varphi_{U_{L3N}}$ – values of the voltages phase angle at the load, phases L1, L2 and L3 respectively, $\mathbf{U_{LN}}$ – complex phase voltages vector at the load, defined with the equation:

$$\mathbf{U_{LN}} = \left[\underline{U}_{L1N} \quad \underline{U}_{L2N} \quad \underline{U}_{L3N}\right]^T, \tag{6}$$

where: $\underline{U}_{L1N}$, $\underline{U}_{L2N}$, $\underline{U}_{L3N}$ – complex values of the phase voltages at the load to the neutral conductor.

The vector of the instantaneous value of the line currents $\mathbf{i}(t)$ is defined in the same way as [1]:

$$\mathbf{i}(t) = \begin{bmatrix} i_{L1}(t) \\ i_{L2}(t) \\ i_{L3}(t) \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\left\{\mathbf{I} e^{j\omega_1 t}\right\} = \begin{Bmatrix} I_{L1}\sqrt{2}\cos\left(\omega_1 t \pm \varphi_{I_{L1}}\right) \\ I_{L2}\sqrt{2}\cos\left(\omega_1 t \pm \varphi_{I_{L2}}\right) \\ I_{L3}\sqrt{2}\cos\left(\omega_1 t \pm \varphi_{I_{L3}}\right) \end{Bmatrix} \tag{7}$$

where: I_{L1}, I_{L2}, I_{L3} – rms values of the line currents, phases L1, L2 and L3 respectively, $i_{L1}(t)$, $i_{L2}(t)$, $i_{L3}(t)$ – instantaneous values of the line currents, phases L1, L2 and L3 respectively, $\varphi_{I_{L1}}$, $\varphi_{I_{L2}}$, $\varphi_{I_{L3}}$ – values of the phase angle of the line currents, phases L1, L2 and L3 respectively, $\mathbf{I}$ – complex values vector of the line currents, defined with the formula:

$$\mathbf{I} = \left[\underline{I}_{L1} \quad \underline{I}_{L2} \quad \underline{I}_{L3}\right]^T, \tag{8}$$

where: $\underline{I}_{L1}$, $\underline{I}_{L2}$, $\underline{I}_{L3}$ - complex values of the line currents.

In accordance with the mathematical concept of the currents' asymmetrical components the instantaneous value of the line current can be divided into four orthogonal components as follows [1, 2]:

$$\mathbf{i}(t) = \mathbf{i_a}(t) + \mathbf{i_r}(t) + \mathbf{i_n}(t) + \mathbf{i_z}(t) \tag{9}$$

where: $\mathbf{i_a}(t)$ – vector of the instantaneous values of the active current, $\mathbf{i_r}(t)$ – vector of the instantaneous values of the reactive current, $\mathbf{i_n}(t)$ – vector of the instantaneous values of the negative current, $\mathbf{i_z}(t)$ – vector of the instantaneous values of the zero current.

On the basis of the proposed concept, the total value of the current norm $\|\mathbf{i}\|$ expressed as follows:

$$\|\mathbf{i}\| = \sqrt{\|\mathbf{i_a}\|^2 + \|\mathbf{i_r}\|^2 + \|\mathbf{i_n}\|^2 + \|\mathbf{i_z}\|^2} \tag{10}$$

where: $\|\mathbf{i_a}\|$ – norm value of the active current vector, $\|\mathbf{i_r}\|$ – norm value of the reactive current vector, $\|\mathbf{i_n}\|$ – norm value of the negative current vector, $\|\mathbf{i_z}\|$ – norm value of the zero current vector.

The value of each of the four current components can be presented in the form of a vector norm or in the form of an instantaneous value vector.

For generating the active current $\mathbf{i_a}$ the equivalent conductance is responsible, which in circuits supplied from an asymmetric voltage source (5) in accordance with the mathematical concept of CAC assumes the form of an equation system, since the equivalent conductance must be calculated for each phase separately as follows:

$$\begin{cases} G_{e_{L1}} = \frac{1}{3}\mathrm{Re}\{1\alpha_{L1N}\underline{Y}_{L1}\} \\ G_{e_{L2}} = \frac{1}{3}\mathrm{Re}\{\alpha\alpha_{L2N}\underline{Y}_{L2}\} \\ G_{e_{L3}} = \frac{1}{3}\mathrm{Re}\{\alpha^*\alpha_{L3N}\underline{Y}_{L3}\} \end{cases} \tag{11}$$

where: 1, α, α^* – symmetrical rotation coefficient equal to 0°, 240°, 120° respectively, α_{L1N}, α_{L2N}, α_{L3N} – actual voltage phase shift coefficients for L1, L2 and L3 phases respectively, $\underline{Y}_{L1}$, $\underline{Y}_{L2}$, $\underline{Y}_{L3}$ – complex value of the phase admittance of the load.

On the grounds of the equivalent phases conductance (11), the value of the active current vector norm $\|\mathbf{i_a}\|$ is given as follows:

$$\|\mathbf{i_a}\| = \sqrt{3\cdot\left(G_{e_{L1}}U_{L1N}^2 + G_{e_{L2}}U_{L2N}^2 + G_{e_{L3}}U_{L3N}^2\right)} \tag{12}$$

and a vector of the instantaneous complex values of the active current $\mathbf{i_a}(t)$ is:

$$\mathbf{i_a}(t) = \sqrt{2}\,\mathrm{Re} \left\{\left(G_{e_{L1}}U_{L1N} + G_{e_{L2}}U_{L2N} + G_{e_{L3}}U_{L3N}\right)\mathbf{1^p}\,e^{j\omega_1 t}\right\} \tag{13}$$

where: $\mathbf{1^p}$ – coefficients rotation vector of the positive sequence is equal:

$$\mathbf{1^p} = \begin{bmatrix}1 & \alpha^* & \alpha\end{bmatrix}^T \tag{14}$$

The presence of reactive current $\mathbf{i_r}$ in the system is the result of the occurrence of equivalent susceptance in each phase in accordance with the system of equations:

$$\begin{cases} B_{e_{L1}} = \frac{1}{3}\mathrm{Im}\{1\alpha_{L1N}\underline{Y}_{L1}\} \\ B_{e_{L2}} = \frac{1}{3}\mathrm{Im}\{\alpha\alpha_{L2N}\underline{Y}_{L2}\} \\ B_{e_{L3}} = \frac{1}{3}\mathrm{Im}\{\alpha^*\alpha_{L3N}\underline{Y}_{L3}\} \end{cases} \tag{15}$$

On the grounds of the equivalent phases susceptance (15) the value of the reactive current vector norm $\|\mathbf{i_r}\|$ is:

$$\|\mathbf{i_r}\| = \sqrt{3\cdot\left|\left(jB_{e_{L1}}U_{L1N}^2 + jB_{e_{L2}}U_{L2N}^2 + jB_{e_{L3}}U_{L3N}^2\right)\right|} \tag{16}$$

and a vector of the instantaneous complex values of the reactive current $\mathbf{i_r}(t)$ is:

$$\mathbf{i_r}(t) = \sqrt{2}\,\mathrm{Re} \left\{\left(jB_{e_{L1}}U_{L1N} + jB_{e_{L2}}U_{L2N} + jB_{e_{L3}}U_{L3N}\right)\mathbf{1^p}\,e^{j\omega_1 t}\right\} \tag{17}$$

For the presence of the currents' asymmetry two consecutive components are responsible. For the component of the negative current $\mathbf{i_n}$ responds the negative admittance, which is described as follows:

$$\begin{cases} \underline{Y}_{L1}^n = \frac{1}{3}\{1\alpha_{L1N}\underline{Y}_{L1}\} \\ \underline{Y}_{L2}^n = \frac{1}{3}\{\alpha^*\alpha_{L2N}\underline{Y}_{L2}\} \\ \underline{Y}_{L3}^n = \frac{1}{3}\{\alpha\alpha_{L3N}\underline{Y}_{L3}\} \end{cases} \tag{18}$$

Based on (18) the value of the negative current vector norm $\|\mathbf{i_n}\|$ is given as:

$$\|\mathbf{i_n}\| = \sqrt{3\cdot\left|\left(\underline{Y}_{L1}^n U_{L1N}^2 + \underline{Y}_{L2}^n U_{L2N}^2 + \underline{Y}_{L3}^n U_{L3N}^2\right)\right|} \tag{19}$$

and a vector of the instantaneous complex values of the negative current $\mathbf{i_n}(t)$ is:

$$\mathbf{i_n}(t) = \sqrt{2}\,\mathrm{Re} \left\{\left(\underline{Y}_{L1}^n U_{L1N} + \underline{Y}_{L2}^n U_{L2N} + \underline{Y}_{L3}^n U_{L3N}\right)\mathbf{1^n}\,e^{j\omega_1 t}\right\} \tag{20}$$

where: $\mathbf{1^n}$ – coefficients rotation vector of the negative sequence is equal:

$$\mathbf{1^n} = \begin{bmatrix} 1 & \alpha & \alpha^* \end{bmatrix}^T. \tag{21}$$

The zero current $\mathbf{i_z}$ is created as a consequence of the zero admittance, which is defined as follows:

$$\begin{cases} \underline{Y}_{L1}^z = \frac{1}{3}\{1\alpha_{L1N}\underline{Y}_{L1}\} \\ \underline{Y}_{L2}^z = \frac{1}{3}\{1\alpha_{L2N}\underline{Y}_{L2}\}. \\ \underline{Y}_{L3}^z = \frac{1}{3}\{1\alpha_{L3N}\underline{Y}_{L3}\} \end{cases} \tag{22}$$

Due to (22) the value of the zero current vector norm $\|\mathbf{i_z}\|$ is given as:

$$\|\mathbf{i_z}\| = \sqrt{3 \cdot \left|\left(\underline{Y}_{L1}^z U_{L1N}^2 + \underline{Y}_{L2}^z U_{L2N}^2 + \underline{Y}_{L3}^z U_{L3N}^2\right)\right|} \tag{23}$$

and a vector of the instantaneous complex values of the zero current $\mathbf{i_z}(t)$ is:

$$\mathbf{i_z}(t) = \sqrt{2}\,\mathrm{Re} \left\{\left(\underline{Y}_{L1}^z U_{L1N} + \underline{Y}_{L2}^z U_{L2N} + \underline{Y}_{L3}^z U_{L3N}\right)\mathbf{1^z}\, e^{j\omega_1 t}\right\} \tag{24}$$

where: $\mathbf{1^z}$ – coefficients rotation vector of the zero sequence is equal:

$$\mathbf{1^z} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T. \tag{25}$$

3. Three-phase four-wire circuit powered by sinusoidal and asymmetrical voltages – the calculation example

The authors of the publication in the calculation example presented in Fig. 2 assume an unbalanced linear load supplied from a sinusoidal asymmetric voltage source. The impedance of the power transmission line wires is $(0{,}1 + j0{,}015)\Omega$. The impedance of the power transmission line gives rise to the asymmetry of the voltage at the load's terminals and the voltage asymmetry on the load's elements – Table 1.

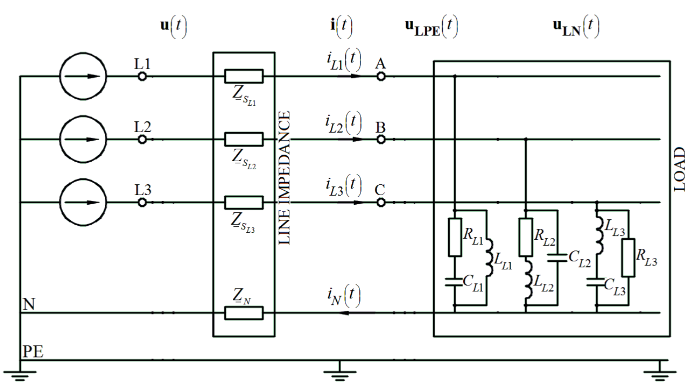

Fig. 2. Circuit diagram for the calculation example

All computations were performed for the frequency of the power grid equal to $f = 50$ Hz.

Table 1 lists the voltage values at the terminals of the generator (1), at the terminals of the load (2), and at the load (3).

Table 1
The list of the phase voltages in analyzed system

Voltage	Phase L1	Phase L2	Phase L3
$\underline{U}$	$230e^{j3°}$	$240e^{-j116°}$	$220e^{j127°}$
$\underline{U}_{LPE}$	$227{,}27e^{j3{,}88°}$	$234{,}77e^{-j116{,}39°}$	$201{,}7e^{j119{,}42°}$
$\underline{U}_{LN}$	$259{,}12e^{j5{,}34°}$	$214{,}43e^{-j109{,}93°}$	$196{,}49e^{j110{,}19°}$

The waveform of the instantaneous value of the voltage at the terminals of the generator (1) is shown in Fig. 3.

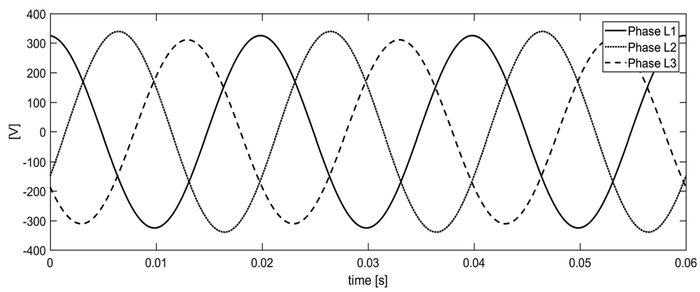

Fig. 3. The waveform of the instantaneous phase voltage at the terminals of the generator

Figure 4 shows the waveform of the instantaneous value of the voltage at the terminals of the load (2).

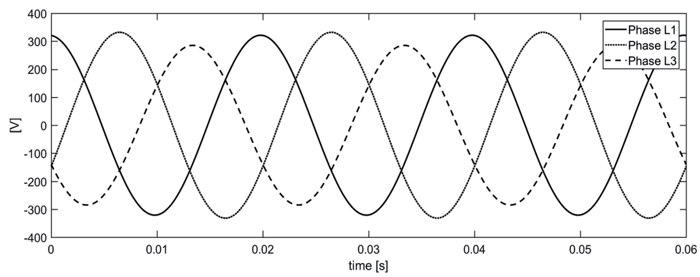

Fig. 4. The waveform of the instantaneous phase voltage at the terminals of the load

The waveform of the instantaneous value of the voltage at the load (3) is shown in Fig. 5.

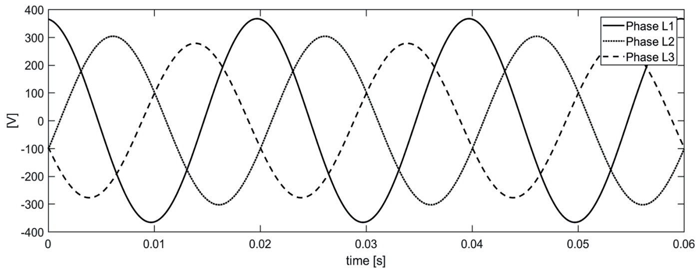

Fig. 5. The waveform of the instantaneous phase voltage at the load

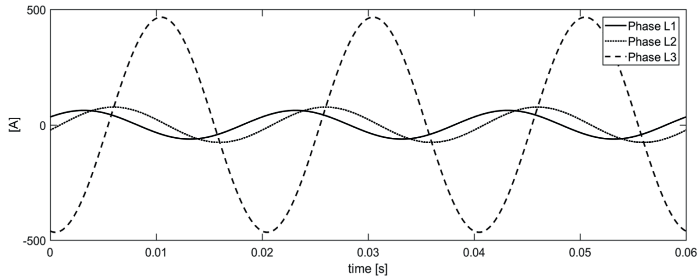

Fig. 6. The waveform of the instantaneous current value of the line

The load parameters, used in the circuit shown in Fig. 2, are summarized in Table 2.

Table 2
The list of the parameters of the load

Load	Phase L1	Phase L2	Phase L3
R	1,08 Ω	1,35 Ω	1,24 Ω
X_L	2,45 Ω	1,88 Ω	3,25 Ω
X_C	3,55 Ω	2,77 Ω	3,03 Ω

As a result of the calculations, in Table 3 the value of the admittance has been listed in two cases, i.e.:

1) the phase admittance without impedance of the power transmission line – $\underline{Y}$,
2) the phase admittance with impedance of the power transmission line – $\underline{Y}_S$.

Table 3
The list of the admittance of the phase

Condition	Phase L1	Phase L2	Phase L3
$\underline{Y}$	$0{,}17e^{-j62{,}45°}$	$0{,}252e^{j2{,}29°}$	$1{,}677e^{j61{,}26°}$
$\underline{Y}_S$	$0{,}168e^{-j61{,}66°}$	$0{,}246e^{j2{,}02°}$	$1{,}567e^{j52{,}71°}$

Table 4 shows the list of the currents' line values and current line norm.

Table 4
The list of the currents' line values

Current	Phase L1	Phase L2	Phase L3
$\underline{I}$	$43{,}94e^{-j57{,}11°}$	$54{,}08e^{-j107{,}64°}$	$329{,}55e^{j171{,}45°}$
$\lVert\mathbf{i}\rVert$	336,838		

Figure 6 presents the waveform of the instantaneous current value of the line – Table 4.

In accordance with the mathematical concept of the currents' asymmetrical components for the obtained waveform of the line current – Table 4 – conform equivalent parameters (alternative) presented in Table 5, i.e.:

1) equivalent conductance – G_e – equation (11),
2) equivalent susceptance – $\underline{B}_e$ – equation (15),
3) negative admittance – $\underline{Y}^n$ – equation (18),
4) zero admittance – $\underline{Y}^z$ – equation (22).

Table 5
The list of the equivalent parameters (alternative)

Para.	Phase L1	Phase L2	Phase L3
G_e	0,0307	0,0821	0,3484
$\underline{B}_e$	$-j0{,}0475$	$j0{,}018$	$j0{,}4372$
$\underline{Y}^n$	$0{,}031-j0{,}048$	$-0{,}057+j0{,}062$	$0{,}205-j0{,}052$
$\underline{Y}^z$	$0{,}031-j0{,}048$	$-0{,}026-j0{,}08$	$-0{,}553+j0{,}083$

On the basis of equivalent parameters – Table 5 – the values of the current components occurring in the CAC concept were calculated. Table 6 lists the values of the currents' norms: active, passive, negative and zero, and the total norm of the current.

Table 6
The list of the norms of the currents' components and the total norm of the current

Norm of the currents	Value of the currents
$\lVert\mathbf{i_a}\rVert$	162,841
$\lVert\mathbf{i_r}\rVert$	134,187
$\lVert\mathbf{i_n}\rVert$	186,062
$\lVert\mathbf{i_z}\rVert$	185,249
$\lVert\mathbf{i}\rVert$	336,838

According to (13), (17), (20) and (24), the values of the currents' component are listed in Table 7.

Table 7
The list of the complex values of the currents' component

Cur.	Phase L1	Phase L2	Phase L3
$\underline{I}_a$	$94{,}02e^{j0°}$	$94{,}02e^{-j120°}$	$94{,}02e^{j120°}$
$\underline{I}_r$	$77{,}47e^{j90°}$	$77{,}47e^{-j30°}$	$77{,}47e^{-j150°}$
$\underline{I}_n$	$107{,}42e^{-j70{,}4°}$	$107{,}42e^{j49{,}6°}$	$107{,}42e^{j169{,}6°}$
$\underline{I}_z$	$106{,}95e^{-j172{,}9°}$	$106{,}95e^{-j172{,}9°}$	$106{,}95e^{-j172{,}9°}$

Based on the data presented in Table 7, the waveform of the instantaneous values has been prepared.

Figure 7 presents the waveform of the instantaneous value of the active current (13).

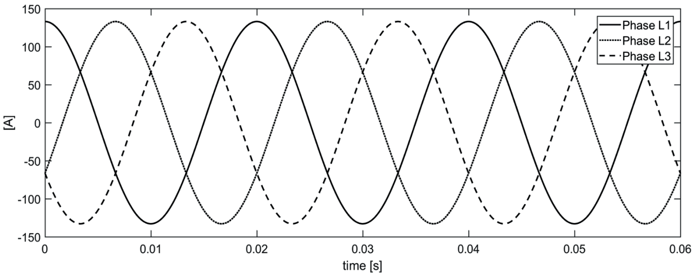

Fig. 7. The waveform of the instantaneous value of the active current

Figure 8 presents the waveform of the instantaneous value of the reactive current (17).

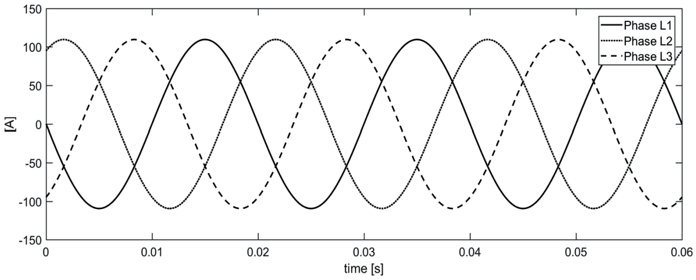

Fig. 8. The waveform of the instantaneous value of the reactive current

The waveform of the instantaneous value of the negative current (20) has been shown in Fig. 9.

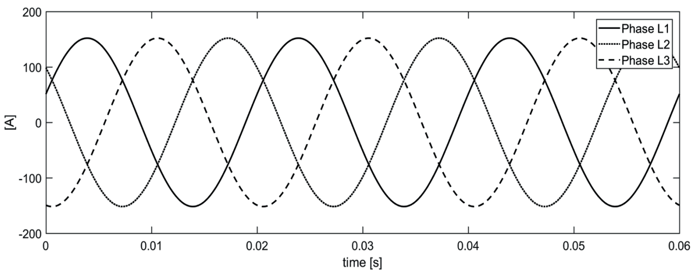

Fig. 9. The waveform of the instantaneous value of the negative current

Figure 10 shows the waveform of the instantaneous value of the zero current (24).

As has been shown in Figs. 7–10, and on the basis of the data presented in Table 7, the decomposition of the currents in the CAC concept for systems with sinusoidal asymmetric voltage supply is the symmetrical decomposition in terms of the current's amplitude and angular displacements between the individual phases.

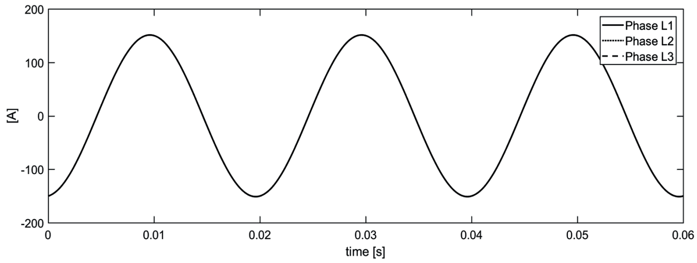

Fig. 10. The waveform of the instantaneous value of the zero current

4. Application of the currents' asymmetrical components concept

Mathematical concept of the currents' asymmetrical components could be used for the algorithm that generates the reference current of a potential active power filter or active part of a hybrid power filter. In order to obtain the reference current, use the following requirement:

$$\mathbf{i_{ref}}(t) = \begin{bmatrix} i_{ref_{L1}}(t) \\ i_{ref_{L2}}(t) \\ i_{ref_{L3}}(t) \end{bmatrix} = \begin{cases} -\sqrt{2}\left\{i_{r_{L1}}(t) + i_{n_{L1}}(t) + i_{z_{L1}}(t)\right\} \\ -\sqrt{2}\left\{i_{r_{L2}}(t) + i_{n_{L2}}(t) + i_{z_{L2}}(t)\right\} \\ -\sqrt{2}\left\{i_{r_{L3}}(t) + i_{n_{L3}}(t) + i_{z_{L3}}(t)\right\} \end{cases} \quad (26)$$

Using (26), in the analyzed case from Fig. 1, we obtained the waveform of the instantaneous value of the reference current shown in Fig. 11.

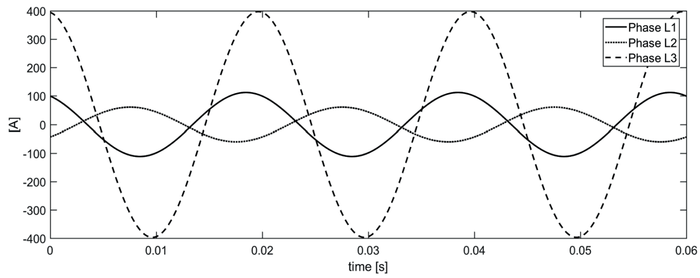

Fig. 11. The waveform of the instantaneous value of the reference current

The generated waveform of the instantaneous value of the reference current $\mathbf{i_{ref}}(t)$ should be added to the waveform of the instantaneous value of the line current $\mathbf{i}(t)$ as follows:

$$\mathbf{i_c}(t) = \begin{bmatrix} i_{c_{L1}}(t) \\ i_{c_{L2}}(t) \\ i_{c_{L3}}(t) \end{bmatrix} = \begin{cases} i_{L1}(t) + i_{ref_{L1}}(t) \\ i_{L2}(t) + i_{ref_{L2}}(t). \\ i_{L3}(t) + i_{ref_{L3}}(t) \end{cases} \quad (27)$$

Using (27), we obtained the waveform of the instantaneous value of the current after compensation, shown in Fig. 12.

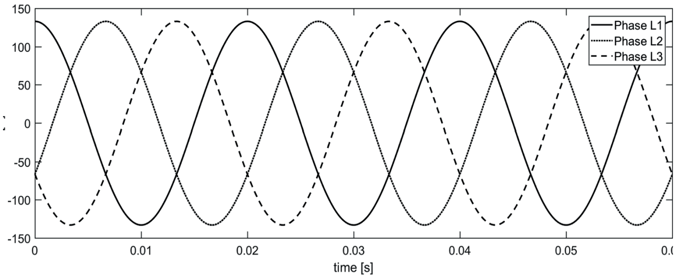

Fig. 12. The waveform of the instantaneous value of the current after compensation

As can be seen, the waveform of the instantaneous value of the current after compensation $\mathbf{i_c}(t)$ is equal to the waveform of the instantaneous value of the active current $\mathbf{i_a}(t)$.

5. Orthogonality of the components

Sinusoidal waveforms are orthogonal [1, 2] in two cases:

1) when, at any moment of the time, one of the existing waveforms has a zero value, while the other one has a non-zero value, thereby the product of their instantaneous values is always equal to zero:

$$x(t)\,y(t) \equiv 0, \tag{28}$$

2) when, the trigonometric waveforms (sinusoidal) of the same frequency are shifted from each other by a quarter period (T/4 or $\pi/2$):

$$\begin{cases} x(t) = \sqrt{2}X\sin(\omega t - \varphi) \\ y(t) = \sqrt{2}Y\sin(\omega t - \varphi \pm \pi/2) \end{cases}, \tag{29}$$

If it is impossible to determine orthogonality from the definition, the dependency on the scalar product is used:

$$\begin{aligned}(x,y) &= \frac{1}{T}\int_0^T x(t)\,y(t)\,dt \\ &= \frac{1}{T}\int_0^T \sqrt{2}\,\mathrm{Re}\left\{\mathbf{X}e^{j\omega_1 t}\right\}\cdot\sqrt{2}\,\mathrm{Re}\left\{\mathbf{Y}e^{j\omega_1 t}\right\}dt\end{aligned} \tag{30}$$

which, after the solution of the equation is equal:

$$(\mathbf{x},\mathbf{y}) = \frac{1}{T}\int_0^T \mathbf{x}^T(t)\,\mathbf{y}(t)\,dt = \mathrm{Re}\sum_{h\in N}\mathbf{X}^T\mathbf{Y}^*. \tag{31}$$

According to the information provided for [21, 22], i.e. including the non-zero impedance of the power source (condition of the supply voltage asymmetry), the currents are not orthogonal to each other and relative to the supply voltage. However, in publications [19, 20] it has been shown that having regard to the asymmetry of the supply voltage, it is possible to show the orthogonality of currents and voltages.

In the CAC mathematical concept, the currents are mutually orthogonal, but are not orthogonal in relation to the voltage.

Therefore, making use of (29) the active and reactive current components are mutually orthogonal $(\mathbf{i_a}, \mathbf{i_r}) = 0$

Scalar product (31) of the active current and the negative current is:

$$\begin{aligned}(\mathbf{i_a}, \mathbf{i_n}) &= \mathrm{Re}\Big\{\big[(G_{e_{L1}}U_{L1N} + G_{e_{L2}}U_{L2N} + G_{e_{L3}}U_{L3N})\cdot\mathbf{1^p}\big]^T\cdot \\ &\quad\cdot\big[(\underline{Y}^n_{L1}U_{L1N} + \underline{Y}^n_{L2}U_{L2N} + \underline{Y}^n_{L3}U_{L3N} +)\cdot\mathbf{1^n}\big]^*\Big\} = \\ &= \mathrm{Re}\Big\{\big[(G_{e_{L1}}U_{L1N} + G_{e_{L2}}U_{L2N} + G_{e_{L3}}U_{L3N})\cdot\mathbf{1^p}\big]^T\cdot \\ &\quad\cdot(\underline{Y}^n_{L1}U_{L1N} + \underline{Y}^n_{L2}U_{L2N} + \underline{Y}^n_{L3}U_{L3N})^*\cdot\mathbf{1^p}^T\cdot\mathbf{1^n}^*\Big\} = \\ &= \mathrm{Re}\Big\{\big[(G_{e_{L1}}U_{L1N} + G_{e_{L2}}U_{L2N} + G_{e_{L3}}U_{L3N})^T\cdot \\ &\quad\cdot(\underline{Y}^n_{L1}U_{L1N} + \underline{Y}^n_{L2}U_{L2N} + \underline{Y}^n_{L3}U_{L3N})^*\cdot(1+\alpha+\alpha^*)\Big\} = 0.\end{aligned}$$

The other components are also mutually orthogonal, so their scalar products (31) are equal:

$$(\mathbf{i_a}, \mathbf{i_z}) = 0,\ (\mathbf{i_n}, \mathbf{i_r}) = 0,\ (\mathbf{i_z}, \mathbf{i_r}) = 0,\ (\mathbf{i_n}, \mathbf{i_z}) = 0.$$

6. Conclusion

As has been shown by the authors of this publication, it is possible to decompose the current into four orthogonal components, i.e.: the active current, reactive current, negative current and the zero current, with asymmetrical sinusoidal voltage supply.

The mathematical concept of the currents' asymmetrical components for sinusoidal asymmetric three-phase four-wire systems takes into account the impedance of wires, i.e. power transmission line and neutral conductor, so that the used vector of the instantaneous voltages is not located at the terminals of the generator but is taken directly from the load's phase elements – it is a vector of the actual voltages.

The proposed current decomposition makes it possible to define the reference current of a potential active power filter or hybrid power filter. The reference current generated in this way, after entering to the system, allows obtaining a waveform of the symmetrical active current of the fundamental harmonic.

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