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Short-term forecasting of accidental oil spill movement in harbours

Keywords

harbour, accident, oil spill, stochastic modelling, Monte Carlo prediction

Abstract

The general model of oil spill movement forecasting based on a probabilistic approach is proposed. A semi-Markov model of the process of changing hydro-meteorological conditions is constructed. The method of oil spill domain determination for various hydro-meteorological conditions is recommended. Moreover, Monte Carlo simulation procedure for predicting the oil spill domain movement is proposed. The procedure is practically applied for Gdynia and Karlskrona ports' water areas.

1. Introduction

Water ecosystems are in danger nowadays because of the negative influence of chemical releases in seas, oceans or inland waters. The important issue is to prevent the oil spills and mitigate their consequences. Thus, there is a need for methods reducing the water pollution and increasing the effectiveness of port and sea environment protection (Bogalecka, 2020; Fingas, 2016). One of the important and basic ways to fulfill this need is the method of quick and exact determining the oil spill domain movement.

The oil spill central point drift trend, the oil spill domain shape and its random position fixed for changing different hydro-meteorological conditions allow us to construct the model of determination of the area in which, with the fixed probability, the oil spill domain is placed (Dąbrowska & Kołowrocki, 2019).

First in Chapter 2, a semi-Markov model of the process of changing hydro-meteorological conditions is defined and its parameters and characteristics are introduced. Next in Chapter 3, a theoretical background of oil spill domain movement is presented. After that in Chapter 4, Monte Carlo simulation approach general procedure is created and applied to generating the process of changing hydro-meteorological conditions at oil spill area and to the prediction of oil spill domain in varying hydro-meteorological conditions. This approach is used in Chapter 6 for oil spill movement forecasting at Gdynia and Karlskrona ports' water areas. The domains can be predicted on the base of statistical data coming from experiments performed at the sea.

2. Process of changing hydro-meteorological conditions

Let $A(t)$ denote the process of varying hydro-meteorological conditions in the sea water area (where the oil spill happened) and let $A = \{1,2,...,m\}$, be the set of all possible states of $A(t)$ in which it may stay at the moment t, $t \in \langle 0,T \rangle$, where $T > 0$. Further, we assume a semi-Markov model (Grabski, 2014; Kołowrocki, 2014) of the process $A(t)$ and denote by θ_{ij} its conditional sojourn time in state i while its next transition will be done to state j, where $i, j \in \{1,2,...,m\}$, $i \neq j$. Under these assumptions, the process of changing hydro-meteorological conditions $A(t)$ is completely described by the following parameters (Dąbrowska, 2020; Kołowrocki, 2014; Limnios & Oprisan, 2001; Xue, 1985; Xue & Yang, 1995a-b):

- the vector of probabilities of its initial states at the moment $t = 0$

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$$[p(0)] = [p_1(0), p_2(0),..., p_m(0)], \quad (1)$$

- the matrix of probabilities of its transitions between the particular states

$$[p_{ij}] = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{bmatrix}, \quad (2)$$

where $\forall\ i = 1,2,...,m,\ p_{ii} = 0$,

- the matrix of distribution functions of its conditional sojourn times θ_{ij} at the particular states

$$[W_{ij}(t)] = \begin{bmatrix} W_{11}(t) & W_{12}(t) & \cdots & W_{1m}(t) \\ W_{21}(t) & W_{22}(t) & \cdots & W_{2m}(t) \\ \vdots & \vdots & \ddots & \vdots \\ W_{m1}(t) & W_{m2}(t) & \cdots & W_{mm}(t) \end{bmatrix}, \quad (3)$$

where $\forall\ i = 1,2,...,m,\ W_{ii}(t) = 0$,

- the expected values (mean values) of its conditional sojourn times θ_{ij} at the particular states

$$M_{ij} = E[\theta_{ij}] = \int_0^\infty t dW_{ij}(t),\ i, j = 1,2,...,m,\ i \neq j, \quad (4)$$

- the variances of its conditional sojourn times θ_{ij} at the particular states

$$V_{ij} = D[\theta_{ij}] = \int_0^\infty (t - E[\theta_{ij}])^2 dW_{ij}(t),$$

$$i, j = 1,2,...,m,\ i \neq j. \quad (5)$$

Using the above defined parameters, we can determine the following characteristics of the considered process $A(t)$, $t \in \langle 0,T \rangle$, $T > 0$ (Kołowrocki, 2014; Kuligowska, 2018; Torbicki, 2018):

- the distribution functions of the unconditional sojourn time θ_i of the process of changing hydro-meteorological conditions at the particular states i, $i = 1,2,...,m$

$$W_i(t) = \sum_{j=1}^{m} p_{ij} W_{ij}(t),\ i = 1,2,...,m, \quad (6)$$

- the mean values of the unconditional sojourn time θ_i of the process of changing hydro-meteorological conditions at the particular states i, $i = 1,2,...,m$

$$M_i = E[\theta_i] = \sum_{j=1}^{m} p_{ij} E[\theta_{ij}],\ i = 1,2,...,m, \quad (7)$$

- the variances of the unconditional sojourn time θ_i of the process of changing hydro-meteorological conditions at the states i, $i = 1,2,...,m$

$$V_i = D[\theta_i] = E[\theta_i^2] - (E[\theta_{ij}])^2,\ i = 1,2,...,m, \quad (8)$$

- the limit values of the process of changing hydro-meteorological conditions transient probabilities at the particular states

$$p_i(t) = P(W(t) = i),\ t \in \langle 0, T \rangle,\ i = 1,2,...,m, \quad (9)$$

given by

$$p_i = \lim_{t \to \infty} p_i(t) = \frac{\pi_i M_i}{\sum_{j=1}^{m} \pi_j M_j},\ i = 1,2,...,m, \quad (10)$$

where M_i, $i = 1,2,...,m$, are given by (7), while the steady probabilities π_i of the vector $[\pi_i]_{1xm}$ satisfy the system of equations

$$\begin{cases} [\pi_i][p_{ij}] = [\pi_i] \\ \sum_{i=1}^{m} \pi_i = 1 \end{cases} \quad (11)$$

which in the case of a periodic process of changing hydro-meteorological conditions, are the long term proportions of this process sojourn times at the particular operation states i, $i = 1,2,...,m$,

- the total sojourn times $\hat{\theta}_i$ of the process at the particular hydro-meteorological states i, $i = 1,2,...,m$, during the fixed time θ, that have approximately normal distributions with the expected value given by

$$\hat{M}_i = E[\hat{\theta}_i] = p_i\theta,\ i = 1,2,...,m, \quad (12)$$

where p_i, $i = 1,2,...,m$, are given by (10).

3. Modelling oil spill domain – theoretical background

Assuming that the experiment takes place in the time interval $\langle 0,T\rangle$, we are interested in finding the oil spill domain $D^k(t)$, $t \in \langle 0,T\rangle$, $k = 1,2,\dots,m$, such that the central point of oil spill domain is placed in it with a fixed probability p. From (Dąbrowska & Kołowrocki, 2019) we have

$$P((X^k(t),Y^k(t)) \in D^k(t)) = \iint_{D^k(t)} \varphi_t^k(x,y)dxdy = p,$$

$$t \in \langle 0,T\rangle,\ k = 1,2,\dots,m, \quad (13)$$

where

$$D^k(t) = \{(x,y): \frac{1}{1-(\rho_{XY}^k(t))^2}[\frac{(x-m_X^k(t))^2}{(\sigma_X^k(t))^2}$$

$$-2\rho_{XY}^k(t)\frac{(x-m_X^k(t))(y-m_Y^k(t))}{\sigma_X^k(t)\sigma_Y^k(t)}$$

$$+\frac{(y-m_Y^k(t))^2}{(\sigma_Y^k(t))^2}] \le c^2\},$$

$$t \in \langle 0,T\rangle,\ k = 1,2,\dots,m. \quad (14)$$

is the domain bounded by an ellipse being the projection on the plane $0xy$ of the curve resulting from the intersection presented in Figure 3 in (Dąbrowska & Kołowrocki, 2019) of the density function surface

$$\pi_1^k = \{(x,y,z): z = \varphi_t^k(x,y), (x,y) \in R^2\}, \quad (15)$$

and the plane

$$\pi_2^k = \{(x,y,z):$$

$$z = \frac{1}{2\pi\sigma_X^k(t)\sigma_Y^k(t)\sqrt{1-(\rho_{XY}^k(t))^2}}\exp[-\frac{1}{2}c^2],$$

$$(x,y) \in R^2\},\ t \in \langle 0,T\rangle,\ k = 1,2,\dots,m. \quad (16)$$

It was shown in (Dąbrowska & Kołowrocki, 2019) that the inequality in (14) holds if $c^2 = -2\ln(1-p)$.

Considering the varying hydro-meteorological conditions, we assume that for a fixed time-step Δt at the successive states $k_1, k_2, ..., k_{n+1}$, s_i is a number of steps such that

$$(s_i - 1)\Delta t < \sum_{j=1}^{i} E[\theta_{k_j k_{j+1}}] \le s_i\Delta t,$$

$$i = 1,2,...,n,\ s_n\,\Delta t \le T. \quad (17)$$

Therefore, assuming oil spill central point drift trend

$$K^{k_i}: \begin{cases} x^{k_i} = x^{k_i}(t) \\ y^{k_i} = y^{k_i}(t), \end{cases} \quad t \in \langle 0,T\rangle,$$

at each state, we obtain the sequences of oil spill domains

$$\overline{D}^{k_i}((s_{i-1}+1)\Delta t),\ \overline{D}^{k_i}((s_{i-1}+2)\Delta t),\dots,\ \overline{D}^{k_i}(s_i\Delta t),$$

$$i = 1,2,...,n, \quad (18)$$

where $\overline{D}^{k_i}(t)$, for t equals to

$(s_{i-1}+1)\Delta t$, $(s_{i-1}+2)\Delta t$, ..., $s_i\Delta t$,

are given by (14) with:

- $c^2 = -2\ln(1-p)$,
- expected values:

$$m_X^{k_i}(t) := m_X^{k_{i-1}}(s_{i-1}\Delta t) + m_X^{k_i}(a_i\Delta t),$$
$$m_Y^{k_i}(t) := m_Y^{k_{i-1}}(s_{i-1}\Delta t) + m_Y^{k_i}(a_i\Delta t),$$

- standard deviations:

$$\overline{\sigma}_X^{k_i}(t) := \sigma_X^{k_i}((s_{i-1}+a_i)\Delta t) + \sum_{j=1}^{i} r^{k_j}(b_j\Delta t),$$

$$\overline{\sigma}_Y^{k_i}(t) := \sigma_Y^{k_i}((s_{i-1}+a_i)\Delta t) + \sum_{j=1}^{i} r^{k_j}(b_j\Delta t),$$

with radiuses $r^{k_j}(t) := r^{k_j}(b_j\Delta t)$, $j = 1,2,...,i$, correlation coefficients $\rho_{XY}^{k_i}(t)$,

for $a_i = 1,2,...,b_i$, $b_i = 1,2,...,s_i - s_{i-1}$,

$i = 1,2,...,n$.

The oil spill domain in the experiment is described by the sum of domains determined by (18).

4. Monte Carlo simulation approach

In Figure 1 we present a general procedure for predicting oil spill domain movement impacted by changing hydro-meteorological conditions. The procedure is based on a probabilistic approach and proposed to create a simulation procedure for short term prediction of oil spill movement (Dąbrowska & Kołowrocki, 2019; Kuligowska, 2018; Marseguerra & Zio, 2002).
Generally, this procedure consists of the following steps:

- we select the initial state at the moment $t = 0$, by generating realizations from the distribution defined by the vector (1) using formula $k_i := k_i(q)$, $i \in \{1,2,...,m\}$, where q is a randomly generated number from the uniform distribution on the interval $\langle 0,1)$,
- we can fix the next operation state of the process of changing hydro-meteorological conditions at oil spill area and denote by $k_{i+1} = k_{i+1}(g)$, $i \in \{1,2,...,m\}$, $i \neq i+1$, the sequence of the realizations of the operation process' consecutive states generated from the distribution defined by the matrix (2), where g is a randomly generated number from the uniform distribution on the interval $\langle 0,1)$,
- we can use several methods generating draws from a given probability distribution from the matrix (3), e.g. an inverse transform method, a Box-Muller transform method, Marsaglia and Tsang's rejection sampling method (Rao & Naikan, 2016); using the inverse transform method, the realization is generated from $t^{(i)}_{k_i k_{i+1}}(h) := W^{-1}_{k_i k_{i+1}}(h)$,
- we substitute i := j and repeat drawing another randomly generated numbers g and h (selecting the states ki+1 and generating realizations $t^{(i)}_{k_i k_{i+1}}(h)$, of the conditional sojourn time), until the sum $\sum_{j=1}^{i} t_{k_j k_{j+1}}$ of all generated realisations $t^{(i)}_{k_i k_{i+1}}(h)$ reach a fixed experiment time T,
- we calculate the necessary parameters and get (18),
- we obtain the sequences of oil spill domains for varying hydro-meteorological conditions.

INPUT DATA:

- step of time Δt, $t > 0$;
- experiment time T, $T > 0$;
- initial state $k_i := k_i(q)$;
- next state $k_{i+1} := k_{i+1}(g)$;
- $K^{k_i}: \begin{cases} x^{k_i} = x^{k_i}(t) \\ y^{k_i} = y^{k_i}(t), \end{cases}$
- expected values $m_X^{k_i}(t)$, $m_Y^{k_i}(t)$;
- standard deviations $\sigma_X^{k_i}(t)$, $\sigma_Y^{k_i}(t)$;
- correlation coefficients $\rho_{XY}^{k_i}(t)$;
- radiuses $r^{k_i}(t)$;
- conditional sojourn time $\theta_{k_j k_{j+1}}$ realisation: $t^{(i)}_{k_i k_{i+1}}(h) := W^{-1}_{k_i k_{i+1}}(h)$;
- $M_{k_j k_{j+1}} = E[\theta_{k_j k_{j+1}}]$.

FIX: $t^{(i)}_{k_i k_{i+1}}(h) := 0$, $i := 1$, $\tau = 0$, $s_0 = 0$,
$m_X^{k_0}(s_0 \Delta t) = 0$, $m_Y^{k_0}(s_0 \Delta t) = 0$,
$k_i \in \{1,2,\ldots,m\}$, $m \in \mathrm{N}$, $a_i = 1$, $b_i = 1$;
FOR $i = 1$ TO n,

- *GENERATE*: q; *FIX*: $k_i \in \{1,2,\ldots,m\}$, $m \in \mathrm{N}$;
- *GENERATE*: g; *FIX*: $k_{i+1} \in \{1,2,\ldots,m\}$, $i \neq i + 1$;
- *GENERATE*: h; *FIX*: $t^{(i)}_{k_i k_{i+1}}(h)$;
- *CHECK*: $(s_i - 1)\Delta t < \sum_{j=1}^{i} t_{k_j k_{j+1}} \le s_i \Delta t$;
- *SELECT*: $\sum_{j=1}^{i} t_{k_j k_{j+1}}$;
- *CALCULATE*: $s_i \Delta t - s_{i-1} \Delta t$;
  FOR $b_i = 1$ TO $b_i = s_i \Delta t - s_{i-1} \Delta t$,
  FOR $a_i = 1$ TO $a_i = b_i$,
- $m^{k_i}(t) := m^{k_{i-1}}(s_{i-1}\Delta t) + m^{k_i}(a_i \Delta t)$;
- $\bar{\sigma}^{k_i}(t) := \bar{\bar{\sigma}}^{k_i}(s_{i-1} + a_i \Delta t)$
  $= \sigma^{k_i}(s_{i-1} + a_i \Delta t) + \sum_{j=1}^{i} r^{k_j}(b_j \Delta t)$;

PRINT $\bigcup_{a_i=1}^{b_i} \bar{\bar{D}}^{k_i}(s_{i-1} + a_i \Delta t)$.

PRINT $\bar{\bar{D}}^{k_i}(s_{i-1} + a_i \Delta t) := \bar{D}^{k}(s_{i-1} + a_i \Delta t)$;

WHILE $\sum_{j=1}^{i} t_{k_j k_{j+1}} \le T$;

OUTPUT: $\bar{\bar{\boldsymbol{D}}}^{k_1,k_2,\ldots,k_n}(b_i) = \bigcup_{i=1}^{n} \bigcup_{a_i=1}^{b_i} \bar{\bar{D}}^{k_i}(s_{i-1} + a_i \Delta t)$

Figure 1. Procedure of oil spill domain movement for varying hydro-meteorological conditions.

5. Short-term forecast application

5.1. Identification of process of varying hydro-meteorological conditions at Gdynia Port water area

Taking into account expert opinions on the process of changing hydro-meteorological conditions $A(t)$ for the Gdynia port water area at the Baltic Sea, we distinguished $m = 6$ following states of this process (GMU Safety Interactive Platform):

- state 1 – the wave height from 0 up to 2 m and the wind speed from 0 m/s up to 17 m/s,
- state 2 – the wave height from 2 m up to 5 m and the wind speed from 0 m/s up to 17 m/s,
- state 3 – the wave height from 5 m up to 14 m and the wind speed from 0 m/s up to 17 m/s,
- state 4 – the wave height from 0 up to 2 m and the wind speed from 17 m/s up to 33 m/s,
- state 5 – the wave height from 2 m up to 5 m and the wind speed from 17 m/s up to 33 m/s,
- state 6 – the wave height from 5 m up to 14 m and the wind speed from 17 m/s up to 33 m/s.

On the basis of the statistical data collected in Marches (the process depends of the season and is a periodic one) in the years 1988-1993 (GMU Safety Interactive Platform) and the identification method given in (Torbicki, 2018), it is possible to evaluate the following unknown basic parameters of the semi-Markov model of the process of changing hydro-meteorological conditions at Gdynia port area:

- the vector

  $$[p(0)] = [0.403, 0.027, 0.42, 0.005, 0.145, 0], \quad (19)$$

  of initial state probabilities $[p_1(0), p_2(0), ..., p_6(0)]$, where $p_i(0)$, $i = 1,2,…,6$, is the probability that the initial state of $A(t)$ at $t = 0$ is equal to i,
- the matrix

  $$[p_{ij}] = \begin{bmatrix} 0 & 0.04 & 0.87 & 0.01 & 0.08 \\ 0.58 & 0 & 0.25 & 0.17 & 0 \\ 0.87 & 0.01 & 0 & 0.01 & 0.11 \\ 0.17 & 0 & 0.83 & 0 & 0 \\ 0.51 & 0 & 0.49 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (20)$$

  of transition probabilities, where p_{ij}, $i, j \in \{1,2,…,6\}$, is the probability that $A(t)$ changes its state from i to j.

According to (Kołowrocki, 2014), we may verify the hypotheses on the distributions of this process' conditional sojourn times at the particular states. To do this, we need a sufficient number of realizations of these variables, namely, the sets of their realizations should contain at least 30 realizations coming from the experiment. Unfortunately, this condition is not satisfied for all sets of the statistical data we have at our disposal.

The sets of the realizations of the conditional sojourn times θ_{13} and θ_{31} of the process $A(t)$ of changing hydro-meteorological conditions were sufficiently large and we verified that they have chimney distributions with the following density functions, respectively:

$$w_{13}(t) = \begin{cases} 0, & t < 0 \\ 0.039, & 0 \le t \le 19.5 \\ 0.002, & 19.5 \le t \le 126.75 \\ 0, & t > 126.75, \end{cases} \quad (21)$$

$$w_{31}(t) = \begin{cases} 0, & t < 0 \\ 0.037, & 0 \le t \le 20 \\ 0.002, & 20 \le t \le 130 \\ 0, & t > 130. \end{cases} \quad (22)$$

The sets of sample conditional remaining sojourn times of $A(t)$ in particular states contain less than 30 realizations. Thus, we assumed that the distribution functions of the process conditional sojourn times θ_{12}, θ_{14}, θ_{15}, θ_{21}, θ_{23}, θ_{24}, θ_{32}, θ_{34}, θ_{35}, θ_{41}, θ_{43}, θ_{51}, θ_{53} have the empirical distribution functions, given as follows:

$$W_{12}(t) = \begin{cases} 0, & t \le 3 \\ 0.125, & 3 < t \le 6 \\ 0.375, & 6 < t \le 9 \\ 0.625, & 9 < t \le 12 \\ 1, & t > 12, \end{cases} \quad (23)$$

$$W_{14}(t) = \begin{cases} 0, & t \le 3 \\ 1, & t > 3, \end{cases} \quad (24)$$

$$W_{15}(t) = \begin{cases} 0, & t \le 3 \\ 0.533, & 3 < t \le 6 \\ 0.8, & 6 < t \le 9 \\ 1, & t > 9, \end{cases} \quad (25)$$

$$W_{21}(t) = \begin{cases} 0, & t \le 3 \\ 0.286 & 3 < t \le 6 \\ 0.429 & 6 < t \le 9 \\ 0.571 & 9 < t \le 15 \\ 0.714 & 15 < t \le 24 \\ 1, & t > 24, \end{cases} \quad (26)$$

$$W_{23}(t) = \begin{cases} 0, & t \le 3 \\ 0.333, & 3 < t \le 6 \\ 0.667, & 6 < t \le 12 \\ 1, & t > 12, \end{cases} \quad (27)$$

$$W_{24}(t) = \begin{cases} 0, & t \le 9 \\ 0.5, & 9 < t \le 21 \\ 1, & t > 21, \end{cases} \quad (28)$$

$$W_{32}(t) = \begin{cases} 0, & t \le 3 \\ 0.5, & 3 < t \le 12 \\ 1, & t > 12, \end{cases} \quad (29)$$

$$W_{34}(t) = \begin{cases} 0, & t \le 6 \\ 0.5, & 6 < t \le 9 \\ 1, & t > 9, \end{cases} \quad (30)$$

$$W_{35}(t) = \begin{cases} 0, & t \le 3 \\ 0.286, & 3 < t \le 12 \\ 0.333, & 12 < t \le 15 \\ 0.429, & 15 < t \le 18 \\ 0.524, & 18 < t \le 21 \\ 0.571, & 21 < t \le 27 \\ 0.619, & 27 < t \le 33 \\ 0.714, & 33 < t \le 42 \\ 0.762, & 42 < t \le 51 \\ 0.81, & 51 < t \le 60 \\ 0.857, & 60 < t \le 75 \\ 0.952, & 75 < t \le 87 \\ 1, & t > 87, \end{cases} \quad (31)$$

$$W_{41}(t) = \begin{cases} 0, & t \le 6 \\ 1, & t > 6, \end{cases} \quad (32)$$

$$W_{43}(t) = \begin{cases} 0, & t \le 3 \\ 0.4, & 3 < t \le 6 \\ 0.8, & 6 < t \le 15 \\ 1, & t > 15, \end{cases} \quad (33)$$

$$W_{51}(t) = \begin{cases} 0, & t \le 3 \\ 0.08, & 3 < t \le 6 \\ 0.192, & 6 < t \le 9 \\ 0.231, & 9 < t \le 12 \\ 0.269, & 12 < t \le 15 \\ 0.385, & 15 < t \le 18 \\ 0.423, & 18 < t \le 21 \\ 0.462, & 21 < t \le 24 \\ 0.5, & 24 < t \le 30 \\ 0.615, & 30 < t \le 36 \\ 0.692, & 36 < t \le 42 \\ 0.769, & 42 < t \le 45 \\ 0.808, & 45 < t \le 48 \\ 0.846, & 48 < t \le 54 \\ 0.885, & 54 < t \le 60 \\ 0.923, & 60 < t \le 66 \\ 0.962, & 66 < t \le 72 \\ 1, & t > 72, \end{cases} \quad (34)$$

$$W_{53}(t) = \begin{cases} 0, & t \le 3 \\ 0.2, & 3 < t \le 6 \\ 0.28, & 6 < t \le 9 \\ 0.4, & 9 < t \le 12 \\ 0.48, & 12 < t \le 15 \\ 0.52, & 15 < t \le 18 \\ 0.6, & 18 < t \le 21 \\ 0.64, & 21 < t \le 24 \\ 0.72, & 24 < t \le 30 \\ 0.76, & 30 < t \le 33 \\ 0.8, & 33 < t \le 42 \\ 0.88, & 42 < t \le 54 \\ 0.92, & 54 < t \le 78 \\ 0.96, & 78 < t \le 96 \\ 1, & t > 96. \end{cases} \quad (35)$$

The distribution functions of the process conditional sojourn times θ_{16}, θ_{25}, θ_{26}, θ_{36}, θ_{42}, θ_{45}, θ_{46}, θ_{52}, θ_{54}, θ_{56}, θ_{61}, θ_{62}, θ_{63}, θ_{64}, θ_{65}, could not be evaluated because of the lack of data.

Considering the conditional distributions given by (21)–(35), according to (4), the conditional mean values $M_{ij} = E[\theta_{ij}]$, of the sojourn times at the particular states measured in hours are fixed as follows:

$$M_{12} \cong 8.63,\ M_{13} \cong 23.99,\ M_{14} = 3,\ M_{15} = 5,$$
$$M_{21} \cong 12,\ M_{23} \cong 7,\ M_{24} = 15,$$
$$M_{31} \cong 24.3,\ M_{32} = 7.5,\ M_{34} = 7.5,$$
$$M_{35} \cong 28.58, \quad (36)$$
$$M_{41} = 6,\ M_{43} = 6.6,$$
$$M_{51} \cong 28.61,\ M_{53} \cong 22.92.$$

5.2. Process of changing hydro-meteorological conditions at Karlskrona Port water area identification

Based on the experts' opinions considering the process $A(t)$ for the Karlskrona port area, we distinguish $m = 6$ states of $A(t)$ (GMU Safety Platform). These states, as in Section 5.1, are related to wave height and wind speed.

On the basis of the statistical data, it was possible to evaluate the following unknown basic parameters of the semi-Markov model of the process of changing hydro-meteorological conditions at the considered area, where the oil spill happened:

- the vector of the initial probabilities:

$$[p_i(0)] = [0.324, 0.018, 0.447, 0.029, 0.182, 0], \quad (37)$$

- the matrix of the probabilities of transitions from the state i into the state j:

$$[p_{ij}] = \begin{bmatrix} 0 & 0.12 & 0.67 & 0.03 & 0.18 & 0 \\ 0.25 & 0 & 0.11 & 0.64 & 0 & 0 \\ 0.6 & 0.01 & 0 & 0.15 & 0.23 & 0.01 \\ 0.01 & 0.03 & 0.95 & 0 & 0 & 0.01 \\ 0.37 & 0 & 0.63 & 0 & 0 & 0 \\ 0 & 0 & 0.67 & 0 & 0.33 & 0 \end{bmatrix}. \quad (38)$$

The hypotheses on the distributions of this process' conditional sojourn times at the particular states were verified for the sets containing at least 30 realizations coming from the experiment. The random samples of the conditional sojourn times θ_{12}, θ_{13}, θ_{15}, θ_{24}, θ_{31}, θ_{34}, θ_{35}, θ_{43}, θ_{51} and θ_{53} were sufficiently large, while the remaining ones, i.e. θ_{14}, θ_{21}, θ_{23}, θ_{32}, θ_{36}, θ_{41}, θ_{42}, θ_{46}, θ_{63} and θ_{65} were assumed to have the following empirical CDF-s:

$$W_{12}(t) = 1 - \exp[-0.0317t],\ t > 0, \quad (39)$$

$$W_{13}(t) = \begin{cases} 0, & t < 0 \\ 0.028t, & 0 \le t \le 23.08 \\ 0.002t + 0.596, & 23.08 \le t \le 161.56 \\ 1, & t > 161.56, \end{cases} \quad (40)$$

$$W_{14}(t) = \begin{cases} 0, & t \le 3 \\ 0.375, & 3 < t \le 6 \\ 0.625, & 6 < t \le 9 \\ 0.875, & 9 < t \le 18 \\ 1, & t > 18, \end{cases} \quad (41)$$

$$W_{15}(t) = 1 - \exp[-0.0277t],\ t > 0 \quad (42)$$

$$W_{21}(t) = \begin{cases} 0, & t \le 3 \\ 0.571, & 3 < t \le 9 \\ 0.714, & 9 < t \le 12 \\ 0.857, & 12 < t \le 18 \\ 1, & t > 18, \end{cases} \quad (43)$$

$$W_{23}(t) = \begin{cases} 0, & t \le 3 \\ 0.333, & 3 < t \le 6 \\ 1, & t > 6, \end{cases} \quad (44)$$

$$W_{24}(t) = \begin{cases} 0, & t \le 3 \\ 0.278, & 3 < t \le 6 \\ 0.5, & 6 < t \le 9 \\ 0.722, & 9 < t \le 12 \\ 0.889, & 12 < t \le 15 \\ 0.944, & 15 < t \le 24 \\ 1, & t > 24 \end{cases} \quad (45)$$

(6)

$$W_{31}(t) = \begin{cases} 0, & t < 0 \\ 0.032t, & 0 \le t \le 12.4 \\ 0.007t + 0.317, & 12.4 \le t \le 99.2 \\ 1, & t > 99.2, \end{cases} \quad (46)$$

$$W_{32}(t) = \begin{cases} 0, & t < 3 \\ 0.333, & 3 \le t \le 12 \\ 0.667, & 12 < t \le 30 \\ 1, & t > 30, \end{cases} \quad (47)$$

$$w_{34}(t) = 0.048 \cdot t^{0.131} \cdot \exp[-0.064t],\ t \ge 0, \quad (48)$$

$$w_{35}(t) = 0.028 \cdot t^{0.156} \cdot \exp[-0.043t],\ t \ge 0, \quad (49)$$

$$W_{36}(t) = \begin{cases} 0, & t \le 21 \\ 0.167, & 21 < t \le 45 \\ 0.333, & 45 < t \le 69 \\ 0.5, & 69 < t \le 93 \\ 0.667, & 93 < t \le 117 \\ 0.833, & 117 < t \le 120 \\ 1, & t > 120, \end{cases} \quad (50)$$

$$W_{41}(t) = \begin{cases} 0, & t \le 3 \\ 1, & t > 3, \end{cases} \quad (51)$$

$$W_{42}(t) = \begin{cases} 0, & t \le 3 \\ 1, & t > 3, \end{cases} \quad (52)$$

$$W_{43}(t) = \begin{cases} 0, & t < 0 \\ 0.052t, & 0 \le t \le 13.5 \\ 0.006t + 0618, & 13.5 < t \le 60.75 \\ 1, & t > 60.75, \end{cases} \quad (53)$$

$$W_{46}(t) = \begin{cases} 0, & t \le 3 \\ 1, & t > 3, \end{cases} \quad (54)$$

$$w_{51}(t) = 0.007 \cdot t^{0.492} \cdot e^{-0.034t},\ t \ge 0, \quad (55)$$

$$w_{53}(t) = 0.04 \cdot t^{0.138} \cdot e^{-0.056t},\ t \ge 0, \quad (56)$$

$$W_{63}(t) = \begin{cases} 0, & t \le 3 \\ 0.5, & 3 < t \le 6 \\ 1, & t > 6, \end{cases} \quad (57)$$

$$W_{65}(t) = \begin{cases} 0, & t \le 6 \\ 1, & t > 6. \end{cases} \quad (58)$$

The remaining distribution functions of the process conditional sojourn times could not be evaluated because of the lack of data. Considering the above conditional distributions (39)–(58), according to (4), the conditional mean values of the sojourn times at the particular states measured in hours are fixed as follows:

$$\begin{aligned} &M_{12} \cong 31.55,\ M_{13} \cong 39.49,\ M_{14} \cong 7.12, \\ &M_{15} \cong 36.1, \\ &M_{21} \cong 7.29,\ M_{23} = 5,\ M_{24} \cong 8.33, \\ &M_{31} \cong 35.86,\ M_{34} \cong 17.56,\ M_{35} \cong 26.75, \\ &M_{36} \cong 77.5, \\ &M_{41} = 3,\ M_{42} = 3,\ M_{43} \cong 15.77,\ M_{46} = 3, \\ &M_{51} \cong 43.45,\ M_{53} \cong 20.45, \\ &M_{63} \cong 4.5,\ M_{65} = 6. \end{aligned} \quad (59)$$

5.3. Generating formulae necessary for Monte Carlo simulation approach for Gdynia Port water area

The simulation is performed according to the data given in Section 5.1. The first step is to select the initial state k_1 at the moment $t = 0$, generated according to the distribution of the initial states given by the vector of probabilities (19) which is given by

$$k_1(q) = \begin{cases} 1, & 0 \le q < 0.403 \\ 2, & 0.403 \le q < 0.43 \\ 3, & 0.43 \le q < 0.85 \\ 4, & 0.85 \le q < 0.855 \\ 5, & 0.855 \le q < 1, \end{cases} \quad (60)$$

where q is a randomly generated number from the uniform distribution on the interval $\langle 0,1)$.
The next state $k_2 = k_2(g)$, is generated according to the distribution given by the matrix of transition probabilities (20), using the procedure defined as follows

$$k_2(g) = \begin{cases} 2, & 0 \le g < 0.04 \\ 3, & 0.04 \le g < 0.91 \\ 4, & 0.91 \le g < 0.92 \\ 5, & 0.92 \le g < 1, \end{cases} \quad (61)$$

if $k_1(q) = 1$;

$$k_2(g) = \begin{cases} 1, & 0 \le g \le 0.58 \\ 3, & 0.58 \le g \le 0.83 \\ 4, & 0.83 \le g < 1, \end{cases} \quad (62)$$

if $k_1(q) = 2$;

$$k_2(g) = \begin{cases} 1, & 0 \le g < 0.87 \\ 2, & 0.87 \le g < 0.88 \\ 4, & 0.88 \le g < 0.89 \\ 5, & 0.89 \le g < 1, \end{cases} \quad (63)$$

if $k_1(q) = 3$;

$$k_j(g) = \begin{cases} 1, & 0 \le g < 0.17 \\ 3, & 0.17 \le g < 1, \end{cases} \quad (64)$$

if $k_1(q) = 4$;

$$k_j(g) = \begin{cases} 1, & 0 \le g < 0.51 \\ 3, & 0.51 \le g < 1, \end{cases} \quad (65)$$

if $k_1(q) = 5$, and so on.

Using the inverse transform method to the distribution functions, we can get the formulae to determine the realizations of the empirical conditional sojourn times. For instance, if $k_1(q) = 1$ and $k_2(g) = 3$, $k_3(g) = 1$, the sets of sample conditional sojourn times θ_{13} and θ_{31} of $A(t)$ are obtained from the following chimney distributions:

$$W_{13}(t) \cong \begin{cases} 0, & t \le 0 \\ 0.04t, & 0 < t \le 19.5 \\ 0.002t + 0.741, & 19.5 < t \le 126.75 \\ 1, & t > 126.75, \end{cases} \quad (66)$$

and

$$W_{31}(t) \cong \begin{cases} 0, & t \le 0 \\ 0.039t, & 0 < t \le 20 \\ 0.002t + 0.74, & 20 < t \le 130 \\ 1, & t > 130. \end{cases} \quad (67)$$

by generating them according to the formulae:

$$t_{13}(h) = \begin{cases} 25h, & 0 \le h \le 0.78 \\ 500h - 370.5, & 0.78 < h < 1, \end{cases} \quad (68)$$

and

$$t_{31}(h) = \begin{cases} 25.641h, & 0 \le h \le 0.78 \\ 500h - 370, & 0.78 < h < 1. \end{cases} \quad (69)$$

In the next steps, the realizations of other conditional sojourn times given by (21)–(35) are generated respectively according to the following formulae:

$$t_{12}(h) = \begin{cases} 3, & 0 \le h \le 0.125 \\ 6, & 0.125 < t \le 0.375 \\ 9, & 0.375 < t \le 0.625 \\ 12, & 0.625 < t < 1, \end{cases} \quad (70)$$

$$t_{14}(h) = 3, \quad (71)$$

$$t_{15}(h) = \begin{cases} 3, & 0 \le h \le 0.533 \\ 6, & 0.533 \le h \le 0.8 \\ 9, & 0.8 < h < 1, \end{cases} \quad (72)$$

$$t_{21}(h) = \begin{cases} 3, & 0 \le h \le 0.286 \\ 6, & 0.286 < t \le 0.429 \\ 9, & 0.429 < t \le 0.571 \\ 15, & 0.571 < t \le 0.714 \\ 24, & 0.714 < t < 1, \end{cases} \quad (73)$$

$$t_{23}(h) = \begin{cases} 3, & 0 \le h \le 0.333 \\ 6, & 0.333 \le h \le 0.667 \\ 12, & 0.667 \le h < 1, \end{cases} \quad (74)$$

$$t_{24}(h) = \begin{cases} 9, & 0 \le h \le 0.5 \\ 21, & 0.5 < h < 1, \end{cases} \quad (75)$$

$$t_{32}(h) = \begin{cases} 3, & 0 \le h \le 0.5 \\ 12, & 0.5 < h < 1, \end{cases} \quad (76)$$

$$t_{34}(h) = \begin{cases} 6, & 0 \le h \le 0.5 \\ 9, & 0.5 < h < 1, \end{cases} \quad (77)$$

$$t_{35}(h) = \begin{cases} 3, & 0 \le h \le 0.286 \\ 12, & 0.286 < h \le 0.333 \\ 15, & 0.333 < h \le 0.429 \\ 18, & 0.429 < h \le 0.524 \\ 21, & 0.524 < h \le 0.571 \\ 27, & 0.571 < h \le 0.619 \\ 33, & 0.619 < h \le 0.714 \\ 42, & 0.714 < h \le 0.762 \\ 51, & 0.762 < h \le 0.81 \\ 60, & 0.81 < h \le 0.857 \\ 75, & 0.857 < h \le 0.952 \\ 87, & 0.952 < h < 1, \end{cases} \quad (78)$$

$$t_{41}(h) = 6, \quad (79)$$

$$t_{43}(h) = \begin{cases} 3, & 0 \le h \le 0.4 \\ 6, & 0.4 \le h \le 0.8 \\ 15, & 0.8 \le h < 1, \end{cases} \quad (80)$$

$$t_{51}(h) = \begin{cases} 3, & 0 \le h \le 0.08 \\ 6, & 0.08 < h \le 0.192 \\ 9, & 0.192 < h \le 0.231 \\ 12, & 0.231 < h \le 0.269 \\ 15, & 0.269 < h \le 0.385 \\ 18, & 0.385 < h \le 0.423 \\ 21, & 0.423 < h \le 0.462 \\ 24, & 0.462 < h \le 0.5 \\ 30, & 0.5 < h \le 0.615 \\ 36, & 0.615 < h \le 0.692 \\ 42, & 0.692 < h \le 0.769 \\ 45, & 0.769 < h \le 0.808 \\ 48, & 0.808 < h \le 0.846 \\ 54, & 0.846 < h \le 0.885 \\ 60, & 0.885 < h \le 0.923 \\ 66, & 0.923 < h \le 0.962 \\ 72, & 0.962 < h < 1, \end{cases} \quad (81)$$

$$t_{53}(h) = \begin{cases} 3, & 0 \le h \le 0.2 \\ 6, & 0.2 < h \le 0.28 \\ 9, & 0.28 < h \le 0.4 \\ 12, & 0.4 < h \le 0.48 \\ 15, & 0.48 < h \le 0.52 \\ 18, & 0.52 < h \le 0.6 \\ 21, & 0.6 < h \le 0.64 \\ 24, & 0.64 < h \le 0.72 \\ 30, & 0.72 < h \le 0.76 \\ 33, & 0.76 < h \le 0.8 \\ 42, & 0.8 < h \le 0.88 \\ 54, & 0.88 < h \le 0.92 \\ 78, & 0.92 < h \le 0.96 \\ 96, & 0.96 < h < 1. \end{cases} \quad (82)$$

5.4. Generating formulae necessary for Monte Carlo simulation approach for Karlskrona Port water area

The unknown basic parameters of the semi-Markov model of the process of changing hydro-meteorological conditions at this area were evaluated in Section 5.2. Thus, the generating formulae necessary for the Monte Carlo simulation approach are as follows (Kuligowska, 2018):

- initial state generating formula:

$$k_i(q) = \begin{cases} 1, & 0 \le q < 0.324 \\ 2, & 0.324 \le q < 0.342 \\ 3, & 0.342 \le q < 0.789 \\ 4, & 0.789 \le q < 0.818 \\ 5, & 0.818 \le q < 1, \end{cases} \quad (83)$$

- next states' generating formula:

$$k_2(g) = \begin{cases} 2, & 0 \le g < 0.12 \\ 3, & 0.12 \le g < 0.79 \\ 4, & 0.79 \le g < 0.82 \\ 5, & 0.82 \le g < 1, \end{cases} \quad (84)$$

if $k_1(q) = 1$;

$$k_2(g) = \begin{cases} 1, & 0 \le g < 0.25 \\ 3, & 0.25 \le g < 0.36 \\ 4, & 0.36 \le g < 1, \end{cases} \quad (85)$$

if $k_1(q) = 2$;

$$k_2(g) = \begin{cases} 1, & 0 \le g < 0.6 \\ 2, & 0.6 \le g < 0.61 \\ 4, & 0.61 \le g < 0.76 \\ 5, & 0.76 \le g < 0.99 \\ 6, & 0.99 \le g < 1, \end{cases} \quad (86)$$

if $k_1(q) = 3$;

$$k_2(g) = \begin{cases} 1, & 0 \le g < 0.01 \\ 2, & 0.01 \le g < 0.04 \\ 3, & 0.04 \le g < 0.99 \\ 6, & 0.99 \le g < 1, \end{cases} \quad (87)$$

if $k_1(q) = 4$;

$$k_2(g) = \begin{cases} 1, & 0 \le g < 0.37 \\ 3, & 0.37 \le g < 1, \end{cases} \quad (88)$$

if $k_1(q) = 5$;

$$k_2(g) = \begin{cases} 3, & 0 \le g < 0.67 \\ 5, & 0.67 \le g < 1, \end{cases} \quad (89)$$

if $k_1(q) = 6$;

and so on.

- the formulae generating the realizations of the conditional sojourn times of $A(t)$, $t \in \langle 0,\infty)$, having exponential (θ_{12}, θ_{15}), chimney (θ_{13}, θ_{31}, θ_{43}) and empirical (θ_{14}, θ_{21}, θ_{23}, θ_{24}, θ_{32}, θ_{36}, θ_{41}, θ_{42}, θ_{46}, θ_{63}, θ_{65}) distribution functions:

$$t_{12}(h) = -31.546 \ln[1-h],\ 0 \le h < 1, \tag{90}$$

$$t_{13}(h) = \begin{cases} 35.291h, & 0 \le h \le 0.654 \\ 400.231h - 238.671, & 0.654 < h < 1, \end{cases} \tag{91}$$

$$t_{14}(h) = \begin{cases} 3, & 0 \le h \le 0.375 \\ 6, & 0.375 < h \le 0.625 \\ 9, & 0.625 < h \le 0.875 \\ 18, & 0.875 < h < 1, \end{cases} \tag{92}$$

$$t_{15}(h) = -36.101 \ln[1-h],\ 0 \le h < 1, \tag{93}$$

$$t_{21}(h) = \begin{cases} 3, & 0 \le h \le 0.571 \\ 9, & 0.571 < h \le 0.714 \\ 12, & 0.714 < h \le 0.857 \\ 18, & 0.857 < h < 1, \end{cases} \tag{94}$$

$$t_{23}(h) = \begin{cases} 3, & 0 \le h \le 0.333 \\ 6, & 0.333 < h < 1, \end{cases} \tag{95}$$

$$t_{24}(h) = \begin{cases} 3, & 0 \le h \le 0.278 \\ 6, & 0.278 < h \le 0.5 \\ 9, & 0.5 < h \le 0.722 \\ 12, & 0.722 < h \le 0.889 \\ 15, & 0.889 < h \le 0.944 \\ 24, & 0.944 < h < 1, \end{cases} \tag{96}$$

$$t_{31}(h) = \begin{cases} 30.846h, & 0 \le h \le 0.402 \\ 145.151h - 45.951, & 0.402 < h < 1, \end{cases} \tag{97}$$

$$t_{32}(h) = \begin{cases} 3, & 0 \le h \le 0.333 \\ 12, & 0.333 < h \le 0.667 \\ 30, & h > 0.667, \end{cases} \tag{98}$$

$$t_{36}(h) = \begin{cases} 21, & 0 \le h \le 0.167 \\ 45, & 0.167 < h \le 0.333 \\ 69, & 0.333 < h \le 0.5 \\ 93, & 0.5 < h \le 0.667 \\ 117, & 0.667 < h \le 0.833 \\ 120, & 0.833 < h < 1, \end{cases} \tag{99}$$

$$t_{41}(h) = t_{42}(h) = t_{46}(h) = 3, \tag{100}$$

$$t_{43}(h) = \begin{cases} 19.203h, & 0 \le h \le 0.703 \\ 159.091h - 98.341, & 0.703 < h < 1, \end{cases} \tag{101}$$

$$t_{63}(h) = \begin{cases} 3, & 0 \le h \le 0.5 \\ 6, & 0.5 < t < 1, \end{cases} \tag{102}$$

$$t_{65}(h) = 6; \tag{103}$$

- the formulae generating the realizations of the conditional sojourn times of $A(t)$ having gamma (θ_{34}, θ_{35}, θ_{51}, θ_{53}) density functions

$$w_{34}(t) = 0.048 \cdot t^{0.131} \cdot e^{-0.064t}, \tag{104}$$

$$w_{35}(t) = 0.028 \cdot t^{0.156} \cdot e^{-0.043t}, \tag{105}$$

$$w_{51}(t) = 0.007 \cdot t^{0.492} \cdot e^{-0.034t}, \tag{106}$$

$$w_{53}(t) = 0.04 \cdot t^{0.138} \cdot e^{-0.056t}, \tag{107}$$

for $t \ge 0$, are given in (Kuligowska, 2018).

5.5. Determination of oil spill domain in varying hydro-meteorological conditions at Gdynia Port water area

We assume that the process of changing hydro-meteorological conditions $A(t)$ is taking six different states, earlier marked by 1, 2, 3, 4, 5, 6. Moreover, we assume, that the experiment time is equal to 2 days, i.e. 48 h and the points $(m_X^{k_i}(t), m_Y^{k_i}(t))$, $t \in \langle 0,48 \rangle$, $k_i \in \{1,2,...,6\}$, $i = 1,2,...,n$, create a curve K^{k_i} called an oil spill central point drift trend that varies at different states of the process $A(t)$. It may be described in the parametric form, according to Section 3:

$$K^{k_i}: \begin{cases} x^{k_i} = t^2 \\ y^{k_i} = k_i \cdot t, \end{cases}$$

$$t \in \langle 0, 48 \rangle,\ k_i \in \{1,2,\ldots,6\},\ i = 1,2,\ldots,n, \quad (108)$$

where x and y are measured in meters.
Taking the fixed time-step $\Delta t = 1\ h$, and sequentially applying the procedure from Figure 1, we obtain the oil spill domain $\overline{D}^{k_1,k_2,\ldots,k_n}$, $k_1, k_2, \ldots, k_n \in \{1,2,\ldots,6\}$, determined for arbitrarily assumed radiuses

$$r^{k_i}(t) = 0.5 + 0.5t,\ t \in \langle 0,48 \rangle,\ k_i \in \{1,2,\ldots,6\},$$

$$i = 1,2,\ldots,n. \quad (109)$$

On the base of data from Section 5.4, we can apply the Monte Carlo simulation method to oil spill domain movement prediction in varying hydro-meteorological conditions at Gdynia Port water area.
First, a randomly generated number q drawn from the uniform distribution on the interval $\langle 0,1)$, equals

$$q \cong 0.07. \quad (110)$$

Next, we select the initial state k_1, $k_1 \in \{1,2,\ldots,6\}$, according to (60) and we receive

$$k_1(0.07) = 1. \quad (111)$$

Further, we draw another randomly generated number g from the uniform distribution on the interval $\langle 0,1)$ and we get

$$g \cong 0.34. \quad (112)$$

For the fixed state $k_1 = 1$, we select the next state k_2, $k_2 \in \{1,2,\ldots,6\}$, according to (61)–(65), i.e.

$$k_2(0.34) = 3. \quad (113)$$

Subsequently, we draw from the uniform distribution on the interval $\langle 0,1)$ a randomly generated number

$$h \cong 0.07. \quad (114)$$

For the fixed state $k_1 = 1$ and $k_2 = 3$, we generate the first realization $t_{ij} = t_{13}^{(1)}$ of the conditional sojourn time θ_{13} from a given probability distribution, according to (68)

$$t_{ij}^{(1)} = t_{13} = 1.68. \quad (115)$$

Consequently, assuming the step of time $\Delta t = 1h$, we have

$$(s_1 - 1) < 1.68 \le s_1. \quad (116)$$

Hence, $s_1 = 2$ and $s_0 = 0$, $s_1 - s_0 = s_1 = 2$. Then, we compare s_1 with the experiment time $T = 48$. We notice, that $s_1 = 2 << 48 = T$, thus we draw the sequence of domains $\overline{\overline{D}}^{k_1}(b_1\Delta t)$, for $b_1 = 2$, $\Delta t = 1$, using the appropriate formula from Section 4, i.e. two ellipses shown in Figure 2.

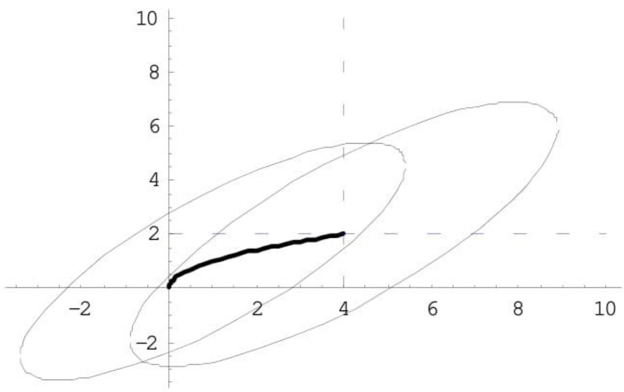

Figure 2. Oil spill domain at the moment $a_1 = 2$, for the time interval $t \in \langle 0,2h \rangle$.

Further, we substitute $k_2 = 3$ and repeat drawing another randomly generated numbers $g \cong 0.51$ and $h \cong 0.14$, selecting the state

$$k_3(0.51) = 1, \quad (117)$$

and generating another realization

$$t_{ij}^{(2)} = t_{31} = 3.69, \quad (118)$$

of the conditional sojourn time. Having these realizations, we find their sum

$$t_{ij}^{(1)} + t_{ij}^{(2)} = t_{13} + t_{31} \cong 1.68 + 3.69 = 5.37. \quad (119)$$

Since

$$(s_2 - 1) < 5.37 \leq s_2, \quad (120)$$

then $s_2 = 6$ and $s_2 - s_1 = 6 - 2 = 4$. We compare $s_2 = 6$ with the experiment time and we notice, that $s_2 = 6 < 48 = T$, thus we get the sequence of domains $\overline{\overline{D}}^{k_1,k_2}(b_2\Delta t) = \overline{\overline{D}}^{1,3}(b_2\Delta t)$, for $b_2 = 4$, $\Delta t = 1$. The oil spill domain in the time interval $\langle s_1, s_2) = (2, 6\rangle$ is illustrated in Figure 3. We can notice, that it consists of 4 ellipses with bigger radiuses than those shown in Figure 2.

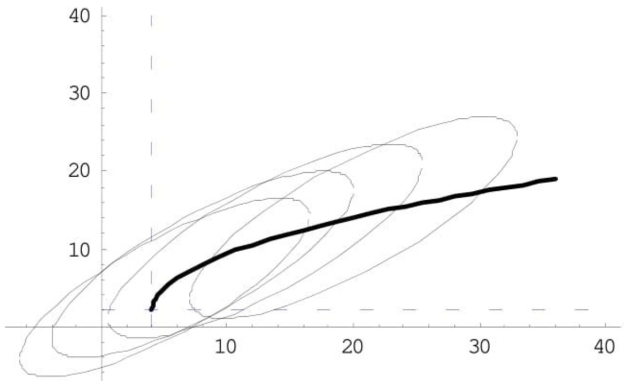

Figure 3. Oil spill domain at the moment $a_2 = 4h$, for the time interval $t \in \langle 2h, 6h\rangle$.

In the next step, we substitute $k_3 := 1$ and draw randomly generated numbers $g \cong 0.19$ and $h \cong 0.82$, selecting the state

$$k_4(0.19) = 3, \quad (121)$$

and realization

$$t_{13}^{(3)} = 39.50, \quad (122)$$

of the conditional sojourn time. The entire sojourn time is

$$t_{ij}^{(1)} + t_{ij}^{(2)} + t_{ij}^{(3)} = t_{13} + t_{31} + t_{13}$$

$$\cong 1.68 + 3.69 + 39.5 = 52.50, \quad (123)$$

and

$$(s_3 - 1) < 13.64 \leq s_3. \quad (124)$$

Since s_3 is greater than the experiment time $T = 48$, we substitute and find the difference $s_3 - s_2 = 14 - 6 = 8$ and we draw the sequence of domains $\overline{\overline{D}}^{k_1,k_2,k_3}(b_3\Delta t) = \overline{\overline{D}}^{1,3,1}(b_3\Delta t)$, for $b_3 = 40$, $\Delta t = 1$. The oil spill domain at the moment $t = 48h$ for the time interval $\langle s_3, s_4) = (14, 48\rangle$ is illustrated in Figure 4.

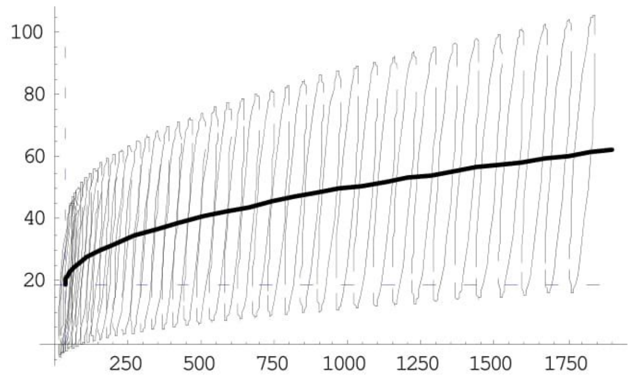

Figure 4. Oil spill domain at the moment $a_3 = 34h$, for the time interval $t \in \langle 14h, 48h\rangle$.

The oil spill domain movement at the moment $t = 48h$ is illustrated in Figure 5.

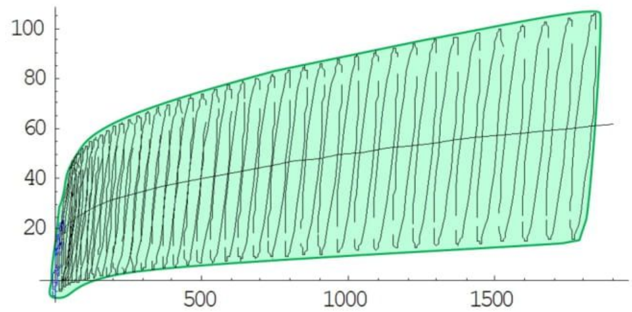

Figure 5. Oil spill domain movement at the moment $t = 48h$.

In Figure 5, the oil spill domain is illustrated for one simulation trial.

5.6. Determination of oil spill domain in varying hydro-meteorological conditions at Karlskrona Port water area

We arbitrarily assume, that the experiment time T is equal to 2 days, i.e. 48 h and the points $(m_X^{k_i}(t), m_Y^{k_i}(t))$, $t \in \langle 0,48\rangle$ for each state k_i, create a curve (108). Moreover, we arbitrarily assume

$$\overline{D}^{k_i}(t) = \{(x,y): \frac{1}{1-0.8^2}[\frac{(x-t^2)^2}{(\overline{\sigma}_X^{k_i}(t))^2}$$

$$-1.6\frac{(x-t^2)(y-k_i\cdot t)}{\overline{\sigma}_X^{k_i}(t)\overline{\sigma}_Y^{k_i}(t)}+\frac{(y-k_i\cdot t)^2}{(\overline{\sigma}_Y^{k_i}(t))^2}] \quad (125)$$

$$\le 5.99\},$$

where $\overline{\sigma}^{k_i}(t)$, $t \in \langle 0,48\rangle$, are defined in Section 3, substituting

$$\sigma^{k_i}(t) = 0.1 + 0.2t, \quad (126)$$

and

$$r^{k_i}(t) = 0.5 + 0.5t,\ t \in \langle 0,48\rangle,$$

$$k_i \in \{1,2,...,6\},\ i = 1,2,...,n. \quad (127)$$

Having all the parameters determined, we proceed with simulation procedure from Section 4:

- $i = 1$
  - $g \cong 0.456$, $k_1(0.456) = 3$, from (83),
  - $q \cong 0.88$, $k_2(0.88) = 5$, from (84)–(89),
  - $h_1 = 0.7$, $h_2 = 0.9$,
  - $t^{(1)}_{k_1k_2} = t^{(1)}_{35} = 5.593$, which is a realisation of the sojourn time θ_{35},
  - $(s_1 - 1) = s_0 = 0 < t^{(1)}_{35} = 5.593 \le s_1$,
  - $s_1 = 6$, which is the number of ellipses;
  - $s_1 = 6 < 48 = T$, thus, the sequence of the oil spill domains for $a_1 = 1,2,\dots,b_1$, $b_1 = 1,2,\dots,6$, given by (125) is illustrated in Figure 6.

- $i = 2$
  - $q \cong 0.217$, $k_3(0.217) = 1$, from (84)–(89),
  - $h_1 = 0.2$, $h_2 = 0.6$,
  - $t^{(2)}_{k_2k_3} = t^{(2)}_{51} = 5.593$, which is a realisation of the sojourn time θ_{51},
  - $t^{(1)}_{k_1k_2} + t^{(2)}_{k_2k_3} = t^{(1)}_{35} + t^{(2)}_{51} = 5.593 + 11.928 = 17.521$,
  - $(s_2 - 1) < 17.521 \le s_2$,
  - $s_2 = 18$, number of ellipses is $18 - 6 = 12$,
  - $s_2 = 18 < 48 = T$, thus, the sequence of domains for $a_2 = 1,2,\dots,b_2$, $b_2 = 1,2,\dots,12$, given by (125) is illustrated in Figure 7.

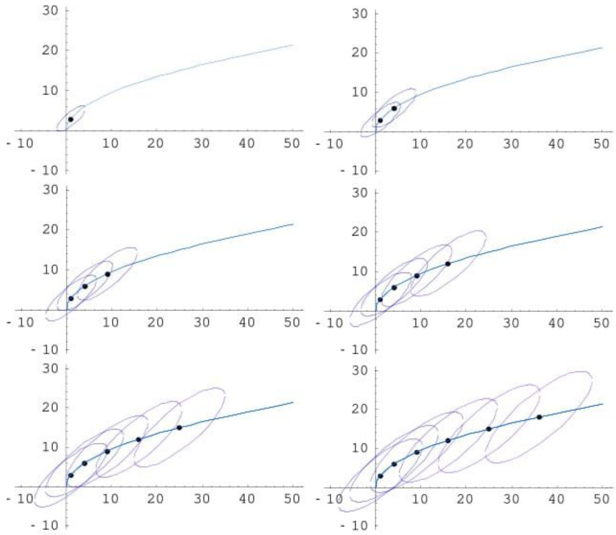

Figure 6. Oil spill domain at the moments a_1 = 1,2,...,6.

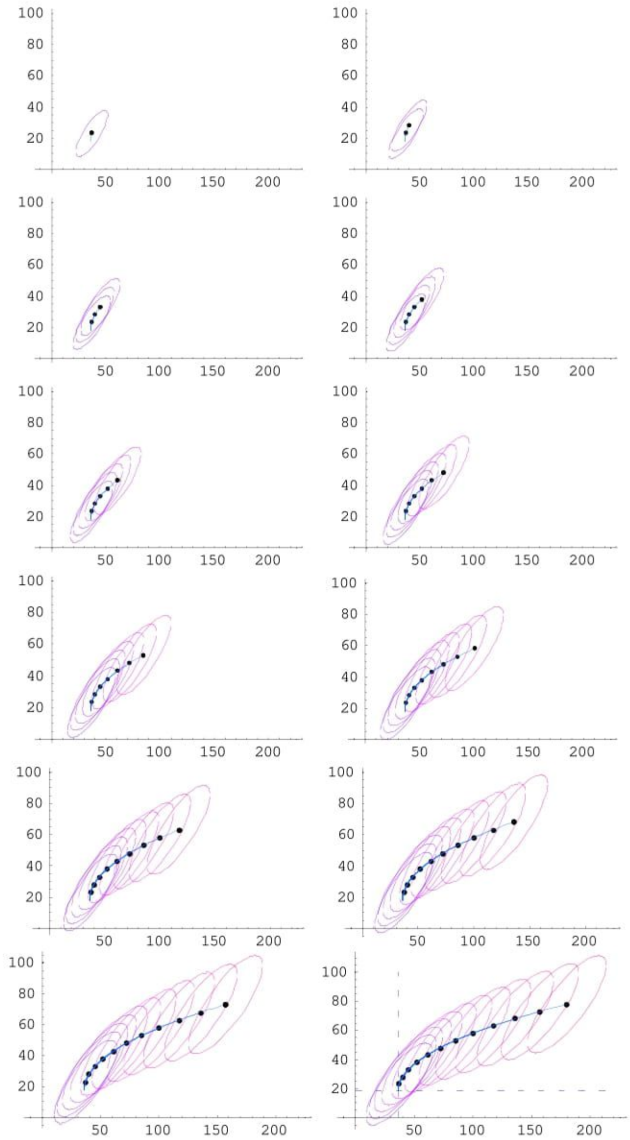

Figure 7. Oil spill domain at the moments a_2 = 1,2,...,12.

- $i = 3$
  - $q \cong 0.469$, $k_4(0.469) = 3$, from (84)–(89),
  - $h = 0.3$,
  - $t^{(3)}_{k_3k_4} = t^{(3)}_{13} = 10.587$, which is a realisation of θ_{13},
  - $t^{(1)}_{k_1k_2} + t^{(2)}_{k_2k_3} + t^{(3)}_{k_3k_4} = t^{(1)}_{35} + t^{(2)}_{51} + t^{(3)}_{13}$
    $= 5.593 + 11.928 + 10.587$,
    $= 28.108$,
  - $(s_3 - 1) < 28.108 \le s_3$,
  - $s_3 = 29$, number of ellipses is $29 - 18 = 11$,
  - $s_3 = 29 < 48 = T$, thus, the sequence of domains for $a_3 = 1,2,\ldots,b_3$, $b_3 = 1,2,\ldots,11$, given by (20) is illustrated in Figure 8.

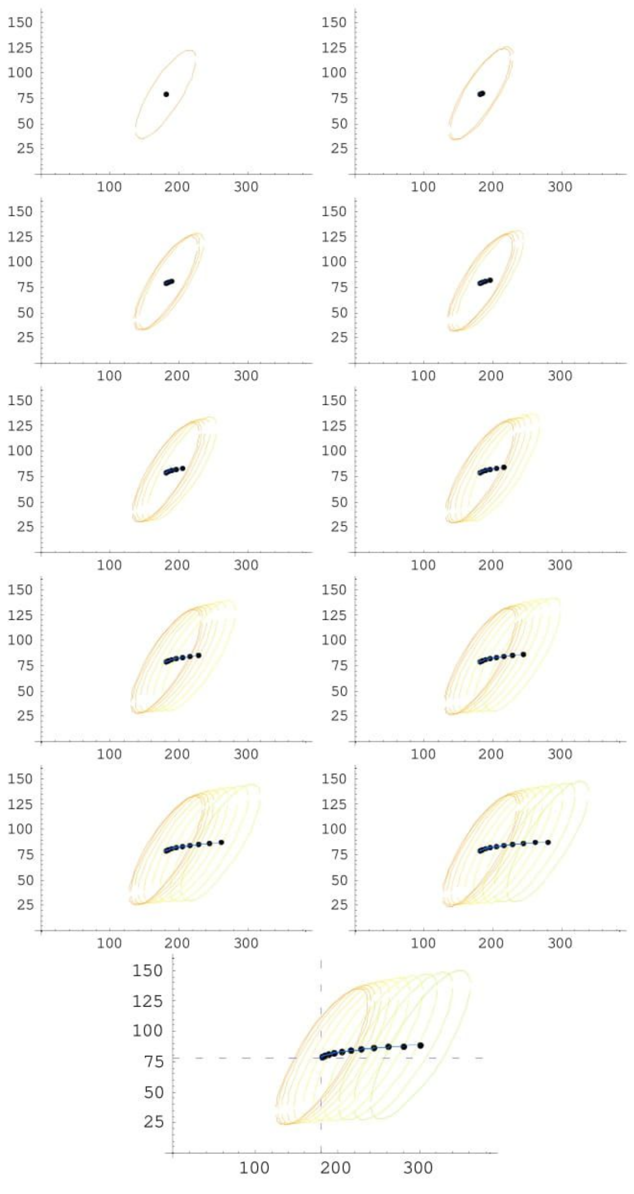

Figure 8. Oil spill domain at the moments $a_3 = 1,2,...,11$.

- $i = 4$
  - $q \cong 0.758$, $k_5(0.758) = 1$, from (84)–(89),
  - $h = 0.45$,
  - $t^{(4)}_{k_4k_5} = t^{(4)}_{31} = 19.367$, which is a realisation of θ_{31},
  - $t^{(1)}_{k_1k_2} + t^{(2)}_{k_2k_3} + t^{(3)}_{k_3k_4} + t^{(4)}_{k_4k_5}$
    $= t^{(1)}_{35} + t^{(2)}_{51} + t^{(3)}_{13} + t^{(4)}_{31}$
    $= 5.593 + 11.928 + 10.587 + 19.367$
    $= 47.475$,
  - $(s_4 - 1) < 47.475 \le s_4$,
  - $s_4 = 48$, number of ellipses is $48 - 29 = 19$,
  - $s_4 = 48 \ge 48 = T$, thus, the sequence of domains for $a_4 = 1,2,\ldots,b_4$, $b_4 = 1,2,\ldots,19$, given by (125) is illustrated in Figure 9.

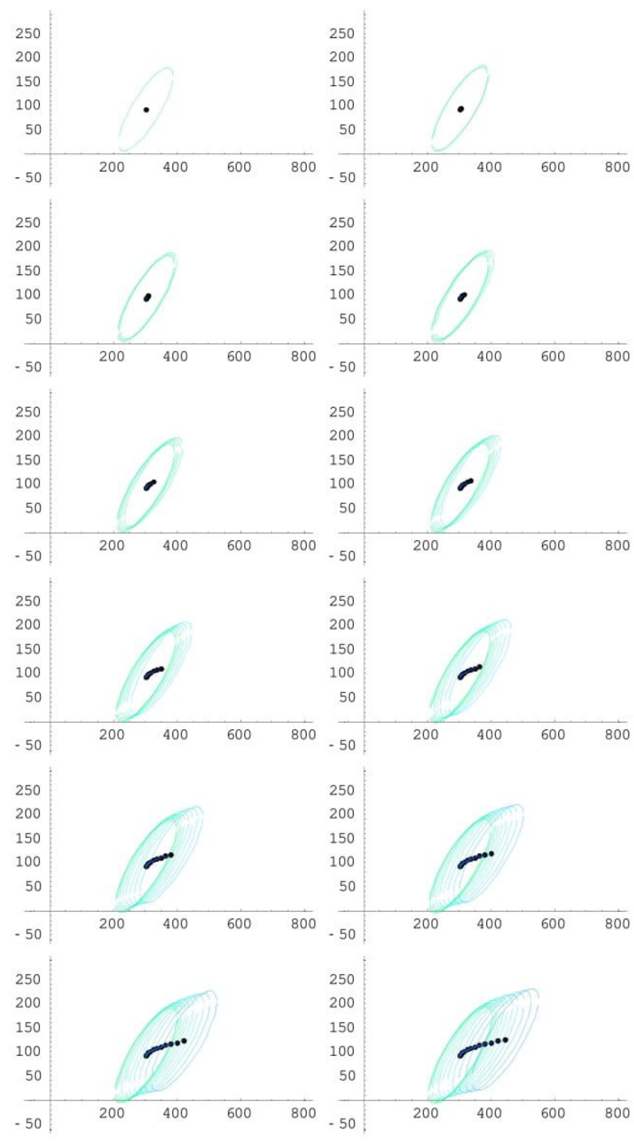

Figure 9. Oil spill domain at the moments $a_4 = 1,2,...,19$.

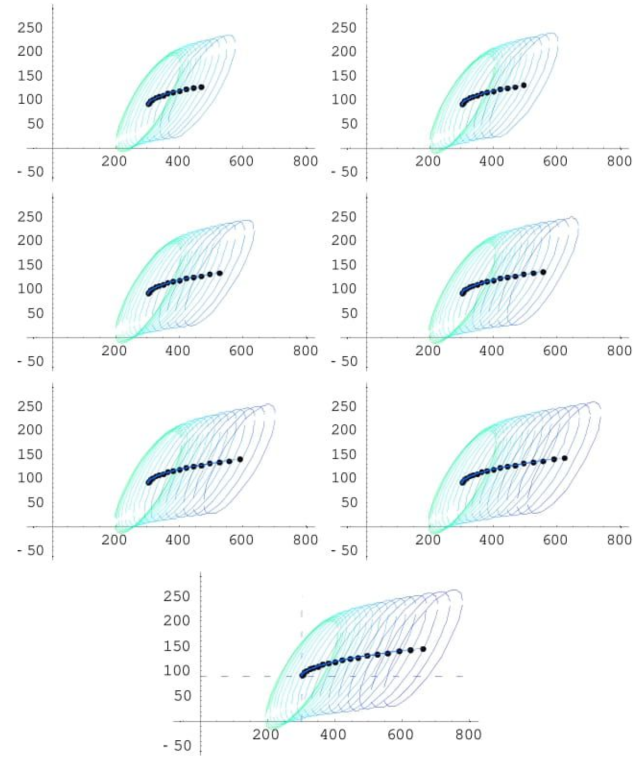

Figure 9 (continuation). Oil spill domain at the moments $a_4 = 1,2,...,19$.

The oil spill domain movement at the moment $t = 48h$ is illustrated in Figure 10.

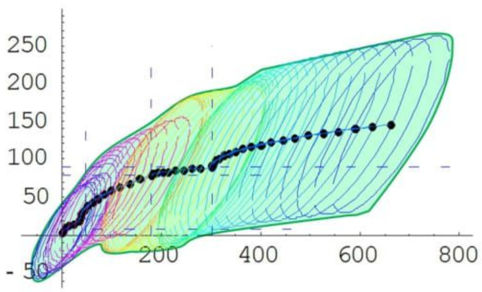

Figure 10. Oil spill domain movement at the moment $t = 48$.

6. Conclusion

The chapter presents Monte Carlo simulation method of oil spill domain movement prediction impacted by changing hydro-meteorological conditions, applied at Gdynia and Karlskrona port water areas. The following two significant parameters were considered: the wave height and the wind speed.

Author's current research is related to the further development of the simulation procedure to take into account more relevant factors, e.g. the density of chemicals and more hydro-meteorological parameters. The final effect of the research should be a model for rapid simulation of the situation at sea during a disaster.

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