

MODELLING OF POSTPRODUCTION SUSPENSIONS' CONCENTRATION PROCESSES BY "BATCH" MEMBRANE MICROFILTRATION

Maciej Szwast^{1*}, Zbigniew Szwast¹, Marian Grądkowski², Wojciech Piątkiewicz¹

¹Warsaw University of Technology, Faculty of Chemical and Process Engineering,
ul. Waryńskiego 1, 00-645 Warsaw, Poland

²Institute for Sustainable Technologies – National Research Institute in Radom,
ul. K. Pułaskiego 6/10, 26-600 Radom, Poland

The mathematical model of postproduction suspension concentration by microfiltration has been developed. This model describes a process conducted in a batch system with membrane washing by reverse flow of permeate. The model considerations concern filtration pseudocycles consisting of the filtration period and the membrane washing period. The balances of continuous phase volume, dispersed phase mass and energy, for each period of pseudocycle respectively, have been presented.

Keywords: microfiltration, mathematical modelling, membrane processes, filtration pseudocycles, postproduction suspensions

1. INTRODUCTION

The results presented in this paper relate to membrane filtration modelling carried out using the suspensions' microfiltration technique created in a variety of industrial processes.

The considerations of this study cover a wide spectrum of suspensions created in industrial processes and are not encumbered with any restrictions. However, to focus the attention of the reader some examples of such suspensions can be shown. They are suspensions generated during the treatment of surfaces of metal parts, acid and alkaline processes of washing baths used in the food industry and cooling and lubricating processes carried out with the use of synthetic liquids. The continuous phase created in such suspensions is pure solvent or a solution consisting of solvent and dissolved therein a variety of chemical substances. The dispersed phase consists of solid particles or liquid droplets insoluble in the used solvent. The purpose of filtering such postproduction suspensions is their division into clear liquid (filtrate) and concentrated suspension. The filtrate is then returned to the treatment, washing, cooling or other processes. A reduction in the volume of raw liquid supplied to the respective process is thus obtained due to the filtration of postproduction suspensions. A reduction of the volume of discharged wastewater is also obtained as discharged wastewater is already present in a concentrated suspension state.

Membrane filtration using the microfiltration technique is described by general equations of convective transport, that are assumed to be valid regardless of what suspension is subject to filtration. In these equations there are parameters related to physicochemical properties of the liquid and the dispersed phase. Specific values of these parameters are adjusted in a particular model equation to the assigned

*Corresponding author, e-mail: mszwast@ichip.pw.edu.pl

suspension. Therefore, common and uniform modelling of membrane filtration using the suspensions' microfiltration technique arising in processes covered by particular interest of this paper, as in other industrial processes, is possible.

In the literature, some papers about microfiltration process modelling can be found. Most of them, like our model, start from the simple Darcy equation. However, these models differ in further development of this relationship. Silva et al. (2000) develop their model by describing changes in the denominator of the Darcy equation, that is, by the description of changes in resistance that the membrane makes. This resistance is divided by them into the resistance of the membrane, the resistance derived from fouling and the resistance derived from the filtration cake. These first two resistances are dependent on the viscosity, while the resistance of the filtration cake has been given a particular attention. Subsequent transformations of the equation lead to a model equation that contains, *inter alia*, the compressibility factor of filtration cake. Makardij et al. (2002) argue that the decline in the filtrate flux during the microfiltration process is caused by the concentration polarisation phenomenon, which is an effect of the accumulation of substances retained by the membrane on the surface. Starting from this assumption, they propose an equation describing filtration flux change in time as a function of concentration of the substance in the feed, the current flux and the Reynolds number, which is a measure of washing the sediment from the surface of the membrane. Finally, this leads to a multiparameter exponential equation describing flow changes during filtration.

Hasan et al. (2013) in their work devoted to microfiltration in a cross-flow system obtained a good model agreement with experiments. This model takes into account the Darcy equation as well, although, it starts from the empirical formula describing the relationship between filtrate flux and process time. This model assumes the aging process of the liquid elements on the membrane surface. Aging is described by the age-distribution function of Danckwerts. The other approach to the modelling of microfiltration process is the use of neural networks (Chellam, 2005). In this approach, the starting point for the model are not equations describing the phenomenology of the process but the results of many experimental tests which allow for network learning. In the above mentioned paper, the feedforward network, where the input layer includes such parameters as time, pressure changes over time, the initial concentration of the feed and the shear stress, is used.

The literature does not pay enough attention to processes conducted with periodic backwashing. This is also a feature of the microfiltration process model presented by us.

The modelling presented in the paper concerns microfiltration processes implemented in a periodical manner, i.e. "batch" method. The microfiltration process carried out this way is not accompanied by raw material (feed) supplied to the system. The entire volume of the feed is supplied to the system at one go, before the start of the filtration process. During the filtration process the flow of the part of the suspension continuous phase is removed from the system. Thus, from the viewpoint of the raw postproduction suspension, it is the process of its concentration. The "batch" system for concentrating wastewater using microfiltration method with simultaneous recovery of part of the suspension continuous phase is shown in Fig. 1. In addition, Fig. 1 also shows a system for reverse flow of filtrate (permeate) used for membrane washing.

The filter cake building up on the membrane surface causes the increase of filtration resistances, and thus a decrease in the flow of filtration. Thus, the issue of minimising the effects of this adverse event is important. The easiest way for this is to start flushing the membrane after a certain filtration period. The industrial process usually consists of subsequent two-element cycles. In each cycle the filtration (working time) precedes membrane flushing (regeneration time). Due to irreversibility of the membrane clogging process (imperfection of the cleaning process) each subsequent filtration cycle takes place in more difficult conditions, leading to ever smaller values of the flow of filtration in relation to previous cycles. For this reason, we should talk about pseudocycles rather than cycles. This issue is taken into account in the considerations of this paper.

The purpose of this paper is the analysis of postproduction suspensions' concentration processes by "batch" membrane microfiltration. The considerations assume that the reverse flow of permeate (filtrate) is used for membrane washing. The analysis applies both to working time and regeneration time of the filtration system. Regarding the working time, the analysis covers the preparation of balances of continuous and dispersed phase mass of any postproduction suspensions and the balance of energy, and then solving the system of equations describing these balances. This analysis is also to provide answers to the question of what experiment should be carried out and what values should be measured so that the resulting solution of the balance equations could be used for specific suspensions and membranes. Similarly, regarding the regeneration time, the analysis covers the preparation of balances of continuous and dispersed phase mass of any postproduction suspensions and the balance of energy, and then solving the system of equations describing these balances. In this case, the analysis is also to provide an answer to the question of what experiment should be carried out and what values should be measured to determine the degree of membrane regeneration for specific suspensions and membranes used expressed by the value of the flow of filtrate in the first moment of the subsequent pseudocycle working time, depending on the flow of filtrate at the last moment of the previous pseudocycle working time and membrane washing time.

2. TRANSPORT OF LIQUID THROUGH FRESH POROUS MEMBRANES

Similarly to the terminology used in the description of processes involving a deactivating catalyst, terminology defining the concept of fresh membrane can be introduced with regard to membranes. In this paper it has been assumed that a fresh membrane is a membrane that has not worked yet. The catalyst and membrane cease to be fresh at the first moment of their work. A fresh catalyst exhibits the highest activity that is possible, whereas the fresh membrane exhibits the highest possible filtrate permeability; resistances posed by the membrane to the flowing filtrate are then the lowest. At the first moment of filtration, and therefore filtration on fresh membrane, the filtrate has to overcome only resistances posed by the membrane. In each subsequent moment of the process the filtrate must additionally overcome resistances posed by the building up layer of residue (fouling). A regeneration (rest) period must take place after the work period both in processes involving a deactivating catalyst and membrane processes.

The basic equation describing the transport of liquid through fresh porous membranes is the Darcy's equation:

$$Q_p = Q_p(T) = \frac{KF}{\mu(T)L} \Delta p = \frac{\Delta p}{R(T)} \quad (1)$$

Darcy's equation is commonly used to describe microfiltration process. One can find that some models describing microfiltration process start from this equation (Makardij et al., 2002; Silva et al., 2000; Wang et al, 2009).

The continuous phase viscosity is an important physicochemical parameter of filtered suspensions. This value is highly dependent on temperature. The liquid viscosity, depending on its temperature, is presented by the Arrhenius equation, also used to describe microfiltration processes (Gan et al., 2006):

$$\mu = A \exp(B/T) \quad (2)$$

where A, B – constants characteristic for the particular liquid.

Knowing the liquid viscosity at a particular temperature, for example, at a standard temperature T_s , $\mu_s = \mu[T_s]$, the liquid viscosity at any temperature T can be, based Equation (2), presented by the following formula:

$$\mu = \mu[T] = \mu_s \exp \left[B \left(\frac{1}{T} - \frac{1}{T_s} \right) \right] \quad (3)$$

After using dependencies (3) in Equation (1), the volumetric flow of filtrate $Q_p = Q_p(T)$ obtained at a temperature T can be expressed as follows:

$$Q_p = Q_p(T) = \frac{KF}{\mu(T)L} \Delta p = \frac{KF}{\mu_s L} \Delta p \exp \left[-B \left(\frac{1}{T} - \frac{1}{T_s} \right) \right] \quad (4)$$

For a standard temperature, $T = T_s$, Equation (4) is simplified to the following form:

$$Q_p = Q_p(T_s) = \frac{KF}{\mu(T_s)L} \Delta p = \frac{KF}{\mu_s L} \Delta p \quad (5)$$

Equations (4) and (5) describe the volumetric flow of filtrate Q_p obtained by using fresh membrane working at any temperature T and at standard temperature T_s , respectively. However a description of the variability of the flow of permeate during the membrane work must also take into account the variability of situations above the membrane surface and its pores. The latter variability is generally characteristic of a specific method of execution of the membrane process. Therefore, further considerations will focus on the selected method of execution, namely on “batch” microfiltration processes for concentrating wastewater with simultaneous recovery of part of the suspension continuous phase.

3. “BATCH” SYSTEM FOR CONCENTRATING WASTEWATER

As mentioned above, the “batch” system for concentrating wastewater using microfiltration method with simultaneous recovery of part of the suspension continuous phase is shown in Fig. 1. It has also been stated that the industrial process usually consists of subsequent two-element cycles. In each cycle the filtration (working time) precedes membrane flushing (regeneration time).

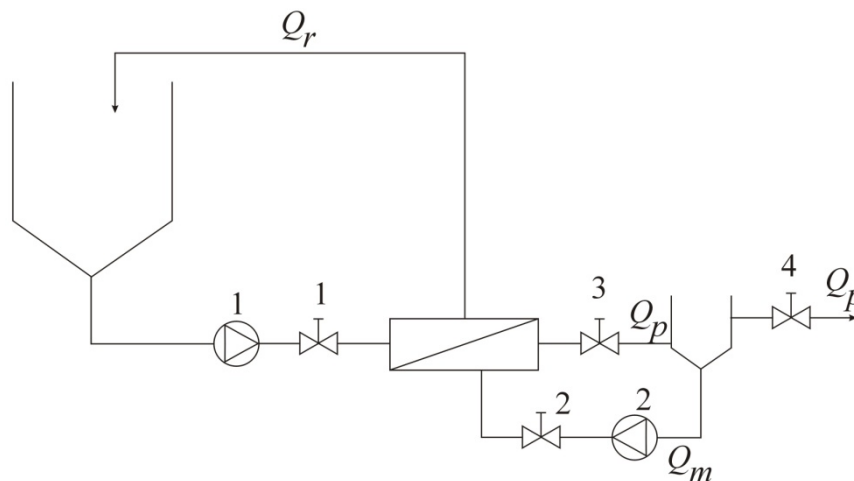


Fig. 1. “Batch” system for concentrating wastewater using microfiltration method with simultaneous recovery of part of the suspension continuous phase together with the system for the reverse flow of permeate used for membrane washing

In each pseudocycle, the pump no. 1 is working and valves no. 1, no. 3 and no. 4 are open during filtration (working time). At the same time pump no. 2 is switched off and the valve no. 2 is closed. The postprocess suspension contained in the tank is pumped to a membrane. A separation of the flow of this

suspension into a flow of clear permeate (filtrate) and the flow of retentate (concentrated suspension) takes place there. The latter flow is returned to the tank. As a result the passage of filtration time in the system decreases the continuous phase volume (clear liquid), and the dispersed phase mass in the system remains constant (the considerations have been limited to the case where the dispersed phase consists of particles of the solid body), which causes an increase in the suspension concentration in the tank. Regarding each pseudocycle, the filtration process (working time) carried out in the "batch" system is an undetermined process due to the fact of variation in time of the continuous phase volume (clear liquid) in the tank, the variation of the suspension concentration in the tank, the variation of temperature (power supplied to the pump) and the viscosity and, consequently, the variation of the flow of filtrate (permeate).

During each pseudocycle, pump no. 2 is operating and the valve no. 2 is open during membrane washing (regeneration time). During that time pump no. 1 is switched off and valves no. 1, no. 3 and no. 4 are closed. The permeate (filtrate) located in a small buffer tank is pumped to the membrane as a reverse flow washing the membrane, and then to the main tank. As a result with the passage of membrane washing time the continuous phase volume (clear liquid) increase in the tank, and the dispersed phase mass in the system remains constant, which causes a decrease in the suspension concentration in the tank. Regarding each pseudocycle, the washing process (regeneration time) carried out in a batch system is an undetermined process due to the fact of variation in time of the continuous phase volume (clear liquid) in the tank, the variation of the suspension concentration in the tank, the variation of temperature (power supplied to the pump).

The following text presents an analysis of the behaviour of the filtration system during filtration time (working time) of each pseudocycle. The analysis of the behaviour of the filtration system at the time of membrane washing (regeneration time) during each pseudocycle will be presented in the final section of this paper's chapter.

After determining the momentary liquid continuous phase volume in the system (tank) by $V = V(\tau)$, with the initial condition $V_0 = V(0)$, and after taking into account the fact that the dispersed phase mass in the system remains constant, $w(\tau) = w(0) = w_0$, the suspension concentration was defined $c = c(\tau)$. It was assumed that the suspension concentration is expressed in kilograms of constant dispersed phase per cubic meter of the liquid continuous phase and is defined by Equation (6)

$$c(\tau) = \frac{w_0}{V(\tau)} \quad (6)$$

with the following initial condition:

$$c_0 = c(0) = \frac{w_0}{V_0} \quad (7)$$

As mentioned above, Equations (4) and (5) describe the volume of the flow of filtrate Q_p obtained by using fresh membrane working at any temperature T and at standard temperature T_s , respectively. Whereas a description of the variation of this flow at the membrane's working time must also take into account the variation of situation above the membrane surface and its pores. The latter variability is generally characteristic of a specific method of execution of the membrane process. At this stage of research the best step is, as far as the technical capacity allows it, to conduct filtration tests on the pilot installation of a specific method of execution of the process, and then to try to correlate the obtained results of experiments. It turns out that for the considered method of execution of the membrane process, the dependency of the reduced flow of filtration (flow of permeate volume obtained at a standard temperature from the unit membrane surface at a unit transmembrane pressure) from the natural logarithm of suspension concentration, c/c_0 , is practically a decreasing straight linear dependency (Piątkiewicz, 2012). The slope obtained as a result of experiments of the straight line assumes a value characteristic of the system, including the membrane and suspension properties. After determining the

gradient by $-\alpha'$, the analysis of the results of experiments can be summarised as the following dependency:

$$q_p(T_s) = q_{p0}(T_s) - \alpha' \ln \frac{c}{c_0} \quad (8)$$

where $q_p(T_s)$ – reduced flow of permeate volume obtained on the pilot installation with a unit membrane surface working at a unit transmembrane pressure at standard temperature, $q_{p0}(T_s)$ is reduced flow of permeate volume obtained on the pilot installation with a unit fresh membrane surface (concerns the first moment of the process, and thus for $c/c_0 = c_0/c_0 = 1$) working at a unit transmembrane pressure at a standard temperature – the value read from the prepared graph $q_p(T_s) = f(\ln(c/c_0))$, $-\alpha'$ is the slope of a straight line – the value read from the same graph.

The meaning given in this study to the term “transmembrane pressure” needs to be clearly defined at this point. It is an overpressure coming from the pump, equal to the total of pressure drops on the membrane and residue collected on its surface (filtration cake). Thus, the understanding of the term “transmembrane pressure” adopted in this paper is the same as the common understanding of the term when talking about the fresh membrane, not yet having the residue on its surface.

Due to the fact that the industrial filter has a surface F and works at a transmembrane pressure Δp , in order to adjust Equation (8) to the industrial filter, this equation should be multiplied by sides by $F\Delta p$, and thus

$$q_p(T_s) F \Delta p = q_{p0}(T_s) F \Delta p - \alpha' F \Delta p \ln \frac{c}{c_0} \quad (9)$$

or

$$Q_p(T_s) = Q_{p0}(T_s) - \alpha \ln \frac{c}{c_0} \quad (10)$$

where:

$$\alpha = \alpha' F \Delta p \quad (11)$$

In order to adjust Equation (10), applicable to a process carried out at a standard temperature T_s , to the description of the process carried out at any temperature T , it is necessary to conduct an operation leading from Equation (5) to Equation (4). And thus:

$$Q_p(T) = \left[Q_{p0}(T_s) - \alpha \ln \frac{c}{c_0} \right] \exp \left[-B \left(\frac{1}{T} - \frac{1}{T_s} \right) \right] \quad (12)$$

Preparing the balance of the continuous phase volume in the system shown in Fig. 1, the flow of permeate (filtrate) can be described with the following dependency

$$-\frac{dV}{d\tau} = Q_p(T) \quad (13)$$

or after using Equation (12):

$$\frac{dV}{d\tau} = \left[-Q_{p0}(T_s) + \alpha \ln \frac{c}{c_0} \right] \exp \left[-B \left(\frac{1}{T} - \frac{1}{T_s} \right) \right] \quad (14)$$

Equation (14) is accompanied by the already mentioned initial condition $V(0) = V_0$.

In view of the stability of the dispersed phase mass in the system shown in Fig. 1 and the definition (6), the following equation can be written as

$$\frac{d(Vc)}{d\tau} = 0 \quad (15)$$

which after further transformations takes the following form:

$$V \frac{dc}{d\tau} + c \frac{dV}{d\tau} = 0 \quad (16)$$

$$\frac{dc}{d\tau} = -\frac{c}{V} \frac{dV}{d\tau} \quad (17)$$

After using Equation (14) in Equation (17) the final form of dependency describing the variation in time of the suspension concentration contained in the tank is obtained:

$$\frac{dc}{d\tau} = \frac{c \left[Q_{p0}(T_s) - \alpha \ln \frac{c}{c_0} \right] \exp \left[-B \left(\frac{1}{T} - \frac{1}{T_s} \right) \right]}{V} \quad (18)$$

Equation (18) is accompanied by the already discussed initial condition (7).

The balance of thermal energy shown by Equation (19) includes respectively the flow of energy supplied to the system through a pump (pump motor power), the flow of enthalpy (in relation to ambient temperature T_e) supplied together with the flow of permeate and the rate of energy storage in the tank, which is the total of the rate of the energy storage in the continuous and dispersed phase (with a mass constant in time w_0), and thus:

$$N - Q_p(T) \rho_c c_c (T - T_e) = \frac{d[V \rho_c c_c (T - T_e)]}{d\tau} + \frac{d[w_0 c_s (T - T_e)]}{d\tau} \quad (19)$$

After expanding derivatives, Equation (19) takes the following form:

$$N - Q_p(T) \rho_c c_c (T - T_e) = \rho_c c_c (T - T_e) \frac{dV}{d\tau} + V \rho_c c_c \frac{dT}{d\tau} + w_0 c_s \frac{dT}{d\tau} \quad (20)$$

or after additionally taking into account Equation (13)

$$N - Q_p(T) \rho_c c_c (T - T_e) = -Q_p(T) \rho_c c_c (T - T_e) + V \rho_c c_c \frac{dT}{d\tau} + w_0 c_s \frac{dT}{d\tau} \quad (21)$$

and thus

$$N = V \rho_c c_c \frac{dT}{d\tau} + w_0 c_s \frac{dT}{d\tau} \quad (22)$$

and finally:

$$\frac{dT}{d\tau} = \frac{N}{c_c \rho_c V + w_0 c_s} \quad (23)$$

Equation (23) is accompanied by the initial condition (24).

$$T(0) = T_0 \quad (24)$$

The initial temperature of the suspension in the tank is usually the ambient temperature. Then the initial condition (24) can be written in the following form:

$$T(0) = T_0 = T_e \quad (25)$$

Equation (23) demonstrates that in relation to the continuous phase volume decreasing in time in the system, together with the lapse of time, the suspension's temperature increases faster and faster.

Equations (14), (18) and (23) allow to calculate and graphically present microfiltration process trajectories of technological model liquids. These trajectories show time courses of changes in the continuous phase volume in the system $V(\tau)$, changes in the suspension concentration $c(\tau)$ and its temperature $T(\tau)$. Of course, the system of Equations (14), (18) and (23) allow to determine time courses $V(\tau)$, $c(\tau)$ and $T(\tau)$ only after the experimental designation of the parameter B values appearing in Equation (2) and the parameters $Q_{p0}(T_s)$ and α appearing in Equation (12), respectively. In addition, after designating dependencies determining time courses $V(\tau)$, $c(\tau)$ and $T(\tau)$ and after using Equations (12), (13) and (14) the variation course $Q_p(\tau)$ can be determined.

In Fig. 2, above the x-axis τ , a draft of courses of variabilities of the flows of permeate (filtrate) $Q_p(\tau)$ for subsequent pseudocycles (exact courses are obtained after carrying out calculations described in the previous paragraph and starting by taking into account the system of Equations (14), (18) and (23)), is presented. The filtration time (working time) is usually the same for all pseudocycles and it is marked here as τ_f . Similarly, membrane washing time (regeneration time) is the same for all pseudocycles and it is marked here as τ_m . Equality of filtration times and equality of washing times, respectively, for all pseudocycles, is mainly due to the ease of setting such a time switch. Further pseudocycles are marked as i and $(i+1)$, respectively. The flow of permeate at the beginning of the work period in the pseudocycle i is marked as Q_{pip} , and at the end of this period as Q_{pik} . Similarly, the flow of permeate at the beginning of the work period in the pseudocycle $(i+1)$ is marked as $Q_{p(i+1)p}$, and at the end of this period as $Q_{p(i+1)k}$. The analysis of Fig. 2 easily leads to the conclusion that a decrease in permeability of the membrane during its operation (at the time of filtration) is the result of alignment of two phenomena. The first one is a reversible decrease in permeability, the effects of which can be removed by membrane washing; the second one is an irreversible decrease in permeability, the effects of which cannot be removed. The same phenomenon can be observed when analysing the work of a catalyst undergoing deactivation. During the catalyst work it undergoes reversible deactivation, the effects of which can be removed by regeneration and irreversible deactivation, the effects of which cannot be removed. Le Goff (1983) analysing the deactivation of the catalyst introduced the concept that has long been known regarding living organisms. He compared the reversible deactivation, removable in the process of regeneration, to fatigue of living organisms, removable during regeneration (rest), and he compared the irreversible deactivation to the aging of living organisms, the effects of which cannot be removed. It turns out, therefore, that the concepts of fatigue and aging may also be useful to describe a decrease in permeability of membranes during their work.

Fig. 3, Fig. 4 and Fig. 5 show, respectively, sketches of courses of variability in time of the continuous phase volume in the tank $V(\tau)$, suspension concentrations in the tank $c(\tau)$ and the suspension temperature in the tank $T(\tau)$. In these figures, filtration periods in subsequent pseudocycles take place at time intervals marked with the symbol τ_f . During filtration the continuous phase volume in the tank decreases due to discharge of the flow of permeate, Fig. 3, which increases the suspension concentration in the tank, Fig. 4. Due to the energy supplied by the pump into the system, the suspension temperature increases, Fig. 5.

The exact courses of curves in Fig. 3, Fig. 4 and Fig. 5 for filtration (work) periods satisfy the equations introduced earlier, i.e. Equation (14) with respect to Fig. 3, Equation (18) with respect to Fig. 4 and Equation (23) with respect to Fig. 5.

Proceeding to the analysis of the behaviour of the filtration system during membrane washing time (regeneration time), let us assume that the contemplated method of washing membranes is back flushing. The same pump is usually used for back flushing and filtration. Therefore, the power supplied by the pump to the system is the same during filtration as during membrane washing.

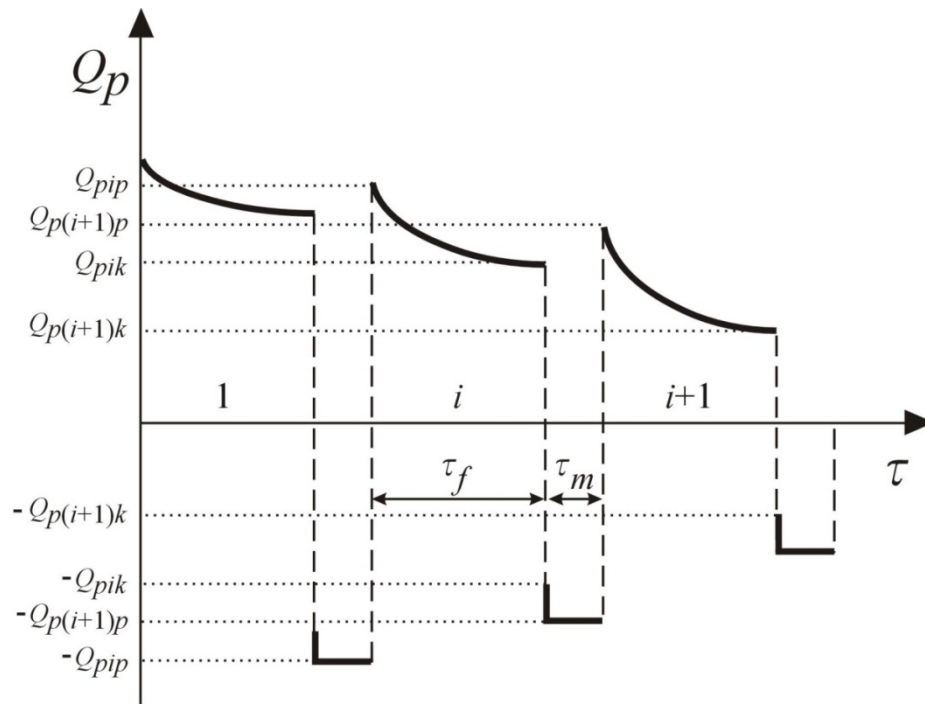


Fig. 2. Flows of filtration in the microfiltration process and flows of back flushing

The results of the discussion conducted below on reverse flows of washing liquid are shown in Fig. 2, below the x-axis τ .

While considering any pseudocycle i it is easy to notice that the last moment of filtration is also the first moment of membrane washing. Thus, it can be assumed that at this moment the flow of filtrate encounters the same resistances, regardless of its flow direction. Therefore, it can also be assumed that the reverse flow of membrane washing filtrate in the first moment of washing is equal to the flow of filtration in its last moment, and hence the following equation

$$Q_{mip} = Q_{pik} \quad (26)$$

Natural is the further conclusion that the reverse flow of membrane washing filtrate quickly dumps the residue collected on the membrane surface during its work and equally quickly removes from membrane pores these particles, which may be removed with this flow. Due to this process the membrane quickly reaches permeability, which is maintained until the last moment of washing, and thus to the first moment of filtration of the next pseudocycle, and thus

$$Q_{p(i+1)p} = Q_{mik} \quad (27)$$

The above terms "quickly dumps the residue" and "quickly reaches permeability" require clarification. In the extreme case it can be assumed that these processes occur in an infinitely short time in the first moment of membrane washing. This in turn requires an additional explanation as to why washing lasts for a finite time τ_m , and is not limited to an infinitely short impulse. In fact, the purpose of membrane washing with filtrate back flushing is to regenerate the membrane surface and pores, but also move away from the vicinity of the membrane particles removed during the washing, so that these do not settle again too quickly on the membrane surface. Assuming an infinitely quick membrane regeneration in the first moment of washing is shown by a vertical section in Fig. 2, regarding this very moment.

Courses of dependencies in Fig. 3, Fig. 4 and Fig. 5 for membrane washing (regeneration) periods satisfy the following equations derived.

The balance of the continuous phase volume in the tank for membrane washing periods is described by the following equation:

$$\frac{dV}{d\tau} = Q_{mik} \quad (28)$$

In view of the flow stability Q_{mik} , it results from Equation (28) that dependencies presented in Fig. 3 for membrane washing periods grow as straight linear dependencies.

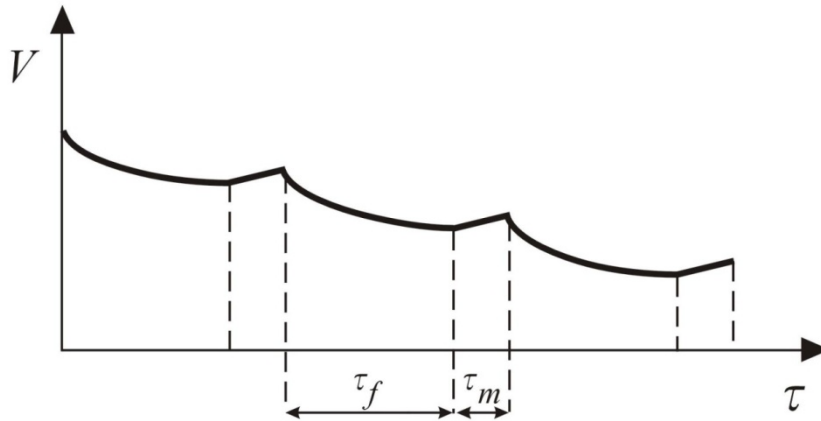


Fig. 3. Change in time in the liquid volume (of the continuous phase) in the tank

For a complete analysis of membrane washing processes using the reverse flow of permeate, the theoretical considerations outlined above must be supplemented by the results of experiments carried out for specific suspensions and membranes used. Experiments must provide information on membrane permeability at the beginning of the pseudocycle $i+1$. This information will allow to design the flow of permeate at this very moment, i.e. $Q_{p(i+1)}$. It could be recommended that the discussed membrane permeability should be dependent on the membrane “age” and its average “life” conditions. The membrane “age” is a time of its work so far calculated as the product of the number of already worked pseudocycles i and filtration time during each pseudocycle τ_f . Whereas the average “life” conditions can be expressed, for example, as a function of the initial (raw) suspension concentration and applied transmembrane pressure. Once again terms generally used in relation to living organisms describe technological processes.

To determine the variation course of the suspension concentration in the tank at the time of washing during the pseudocycle, it should be noted that the following dependency between the rate of increase in the continuous phase volume in the tank and the reverse flow of washing liquid (cf. Equation (27))

$$\frac{dV}{d\tau} = Q_{mik} = Q_{p(i+1)p} \quad (29)$$

and that the dispersed phase mass in the system is constant, and thus:

$$\frac{d(Vc)}{d\tau} = 0 \quad (30)$$

The solution of the system of Equations (29) and (30) leads to the following dependencies

$$\frac{dc}{d\tau} = -\frac{c}{V} Q_{mik} \quad (31)$$

indicating that during membrane washing the suspension concentration in the tank decreases.

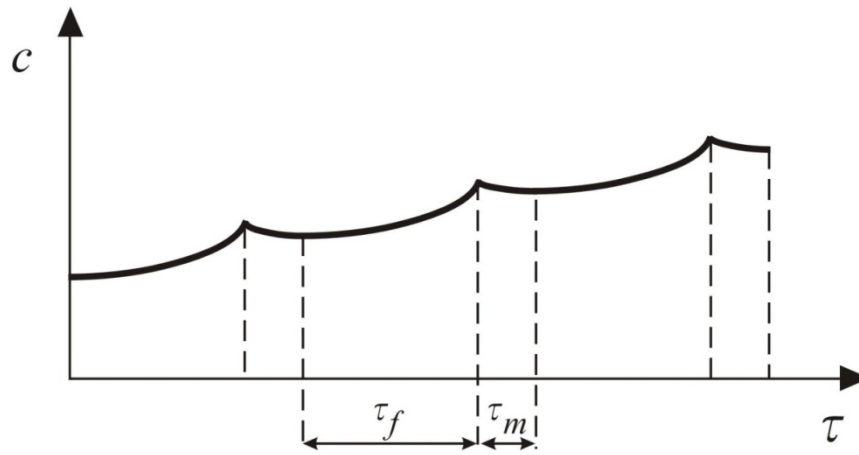


Fig. 4. Change in time of the dispersed phase concentration in the tank

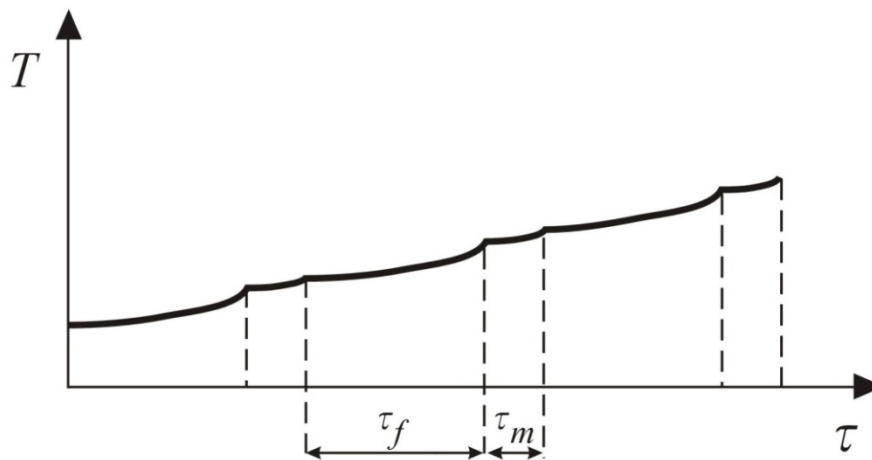


Fig. 5. Change in time in the suspension temperature in the tank

When washing the membrane, the total energy supplied to the system by the pump is accumulated in this system. In addition, there are no flows of mass supplied to or discharged from the system, and thus:

$$N = \frac{d[V\rho_c c_c (T-T_e)]}{d\tau} + \frac{d[w_0 c_s (T-T_e)]}{d\tau} \quad (32)$$

After expanding derivatives, Equation (32) takes the following form:

$$N = \rho_c c_c (T-T_e) \frac{dV}{d\tau} + V\rho_c c_c \frac{dT}{d\tau} + w_0 c_s \frac{dT}{d\tau} \quad (33)$$

or after using Equation (29):

$$N = -Q_{mik} \rho_c c_c (T-T_e) + V\rho_c c_c \frac{dT}{d\tau} + w_0 c_s \frac{dT}{d\tau} \quad (34)$$

and finally:

$$\frac{dT}{d\tau} = \frac{N + Q_{mik} \rho_c c_c (T-T_e)}{V\rho_c c_c + w_0 c_s} \quad (35)$$

Equation (35) indicates that the increase in time in the suspension temperature in the tank is described by curvilinear dependence.

4. SUMMARY

The mathematical model of the so-called “batch” microfiltration process working periodically with the use of membrane washing by reverse flow of permeate has been developed. Model equations are valid for various suspensions. The adjustment of sets of equations to the assigned suspension requires conducting an experiment using a pilot installation.

Knowledge of the mathematical model of the suspensions' microfiltration process is helpful in designing installations used to conduct this process, and in particular to design control systems of the process.

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SYMBOLS

A	coefficient appearing in Equation (2), Pa·s
B	coefficient appearing in Equation (2), K
c	suspension concentration in the system defined by Equation (6) and expressing the dispersed phase mass per unit of the continuous phase volume, $\text{kg}\cdot\text{m}^{-3}$
c_0	initial suspension concentration in the system, $\text{kg}\cdot\text{m}^{-3}$
c_w	specific heat of the suspension continuous phase, $\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$
c_s	specific heat of the suspension dispersed phase, $\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$
F	membrane filtration surface, m^2
i	number of the subsequent pseudocycle,
K	permeability coefficient, m^2
L	membrane thickness, m
m	liquid mass, kg
N	power transmitted to the liquid by the pump as the flow of heat, the flow of energy, W
Q_m	volumetric reverse flow of permeate for membrane washing, $\text{m}^3\cdot\text{s}^{-1}$
Q_n	volumetric flow of feed, $\text{m}^3\cdot\text{s}^{-1}$
Q_p	volumetric flow of permeate (filtrate), $\text{m}^3\cdot\text{s}^{-1}$
Q_{p0}	initial volumetric flow of permeate, $\text{m}^3\cdot\text{s}^{-1}$
Q_r	volumetric flow of retentate, $\text{m}^3\cdot\text{s}^{-1}$
Q_{pip}	the flow of permeate at the beginning of the work period in the pseudocycle i , $\text{m}^3\cdot\text{s}^{-1}$
Q_{pik}	the flow of permeate at the end of the work period in the pseudocycle i , $\text{m}^3\cdot\text{s}^{-1}$
$Q_{p(i+1)p}$	the flow of permeate at the beginning of the work period in the pseudocycle $(i+1)$, $\text{m}^3\cdot\text{s}^{-1}$
$Q_{p(i+1)k}$	the flow of permeate at the end of the work period in the pseudocycle $(i+1)$, $\text{m}^3\cdot\text{s}^{-1}$
$q_p(T_s)$	reduced flow of permeate defined by Equation (8), $\text{m}^3\cdot\text{s}^{-1}$
$q_{p0}(T_s)$	reduced flow of permeate defined by fresh membrane, $\text{m}^3\cdot\text{s}^{-1}$
p	pressure, Pa
Δp	transmembrane pressure, Pa
R	hydraulic resistance, $\text{Pa}\cdot\text{s}\cdot\text{m}^{-3}$
T	temperature, K
T_e	ambient temperature, K
T_s	standard temperature, K
T_0	initial liquid temperature in the tank, K
V	liquid (continuous phase) volume in the system (tank), m^3

w dispersed phase mass in the system, a constant value in time, kg

Greek symbols

α	coefficient defined by Equation (11), $\text{m}^3 \cdot \text{s}^{-1}$
α'	coefficient defined by Equation (8); its value is determined after conducting experiments for the tested membrane – suspension system, $\text{m}^3 \cdot \text{s}^{-1}$
μ	liquid viscosity, Pa·s
μ_s	liquid viscosity at a standard temperature, Pa·s
ρ_c	continuous phase density, $\text{kg} \cdot \text{m}^{-3}$
τ	time, s
τ_f	filtration time (working time), the same for all pseudocycles, s
τ_m	membrane washing time (regeneration time), the same for all pseudocycles, s

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