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Preliminary results of fractal analysis of the poligonal survey from cave: case study of Małołączniak area (Tatra Mts.)

KEY WORDS:
fractal dimension, cave survey, box counting technique, Tatra Mts.

ABSTRACT

Traverses (polygons) from two caves have been tested: Wielka Śnieżna Cave System (2858 vectors) and Śnieżna Studnia cave (742). The box counting technique was applied to evaluate the fractal analysis of spatial orientation of traverses. The polygonal survey of Wielka Śnieżna Cave, Śnieżna Studnia and both caves merged together have a fractal geometry. It may be concluded that these caves are close to the full recognition of passages forming by a hydrological system and they could have close relation with geological structures. The usual explanation of fractal dimension D higher than 1 indicates that caves with such dimension fill more space than those with ideal dimension of 1.00 (for example a straight line), and the geological constraints limit the dimension to be lower than 2 (Verbovsek 2007). It may suggest that systems can be developped into more complicated passages in future. The fact that both caves merged together also have showed fractal geometry indicates that they are belonging to the same hydrological system. It was noticed that D-value of merged caves is slightly larger than individual cave. It can be explained by "occupying" more space in rock mass.

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Introduction

Caves which are the underground drainage systems of mountain massifs has a geometry close to the river systems. A pattern of cave passages are strong relative to geological plane structures: bedding, fractures, fault etc. Hydrological (Kusumayudha *et al*. 2000) and discontinuity systems (Teper 1998) have a fractal geometry which suggests that polygons should also have it. In the speleology, fractal analysis was used inter alia to the shape of passages (Curl 1986) or distribution of caves length investigation (Verbovsek 2007).

The aim of this study was to calculate the fractal dimension of caves geometry on the base of polygonal survey. Demonstration of the fractal dimension of the cave passages pointed to the possibility of full recognition (discovery) cave passages as hydrological system.

Study Area

Polygonal surveys of two caves has been analyzed: Wielka Śnieżna Cave System and Śnieżna Studnia Cave (Fig.1B). Both caves are developed in northern slopes of the Małołączniak in Czerwone Wierchy massif (Fig.1A). From geological point of view, study area is composed of Czerwone Wierchy Nappe, autochtonous sedimentary cover and Giewont Nappe (Fig.1A).

Both caves are drained by a system of Lodowe Spring, located in Kościeliska Valley. This is a classic example of independence between the surfaces valleys system and karstic drainage system (Małecka 1993; Fig.1A).

Wielka Śnieżna Cave System is developed in Ździary sub-unit (a part of the Czerwone Wierchy Nappe) and Autochtonous Unit (Grodzicki & Kardaś 1989). Cave is 23 723 m long and its denivelation is 824 m (Kardaś 2002).

Śnieżna Studnia is located in Ździary and Organy sub-units (Szczygieł 2013). Cave is 12 050 m long and its denivelation is 763 m (Fuja et al. 2002).

Material and methods

Traverses (polygons) from two caves have been tested (Fig.1). 2858 vectors of traverse were carried out in the first cave, and 742 in the second one. The box counting technique was applied to evaluate the fractal dimension of spatial orientation of traverses. The box counting algorithm is often applied to fault systems, lineaments, river systems and coast line geometry analysis. In this paper an attempt was made to use the box counting technique in cave system analysis.

The box counting algorithm based on dividing a map with cave polygons into square boxes with the side length, r_o, and counting number N_o of boxes contained polygon lines. In the next steps, box side, r_o, is divided by natural numbers, k, and number $N_i = N(r_i)$ of boxes with polygon lines is counted each time. It can be expressed by the following relation:

$$N(r_i) = N_0 \left(\frac{r_i}{r_0} \right)^{-D}$$

(1)

where r_i is new box side and D is fractal dimension. Plotted function (1) is called "the box curve" (Sasaki 1994, Falconer 2003).

For the both side of equation (1) logarithms can be calculated and then a linear function with D-value as slope is obtained:

$$\log_{10} N(r_i) = \log_{10} N_0 - D \log_{10} \left(\frac{r_i}{r_0} \right)$$

(2)

The slope (D-value) is fractal dimension of spatial distribution of analyzed system (Sasaki 1994, Falconer 2003). For box-counting technique NARO software has been used.

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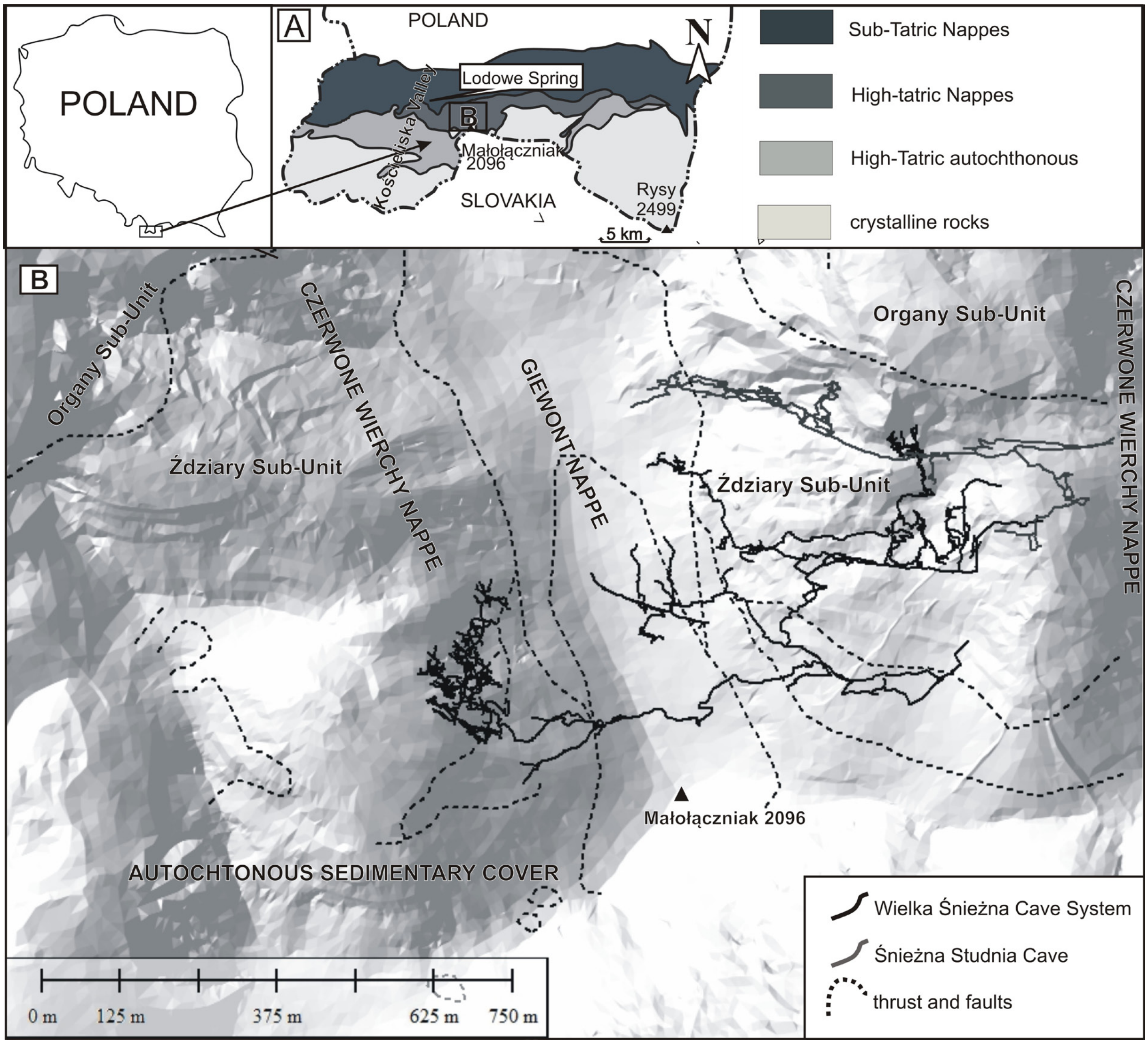

Fig.1. A – study area in relation to the main tectonic units of the Tatra Mts (after Bac-Moszaszwili et al., 1979); B – surface geological structures of study area (Piotrowska et al., 2008; modify) on the background of DEM;

Results

Three sets of data has been analyzed by box counting technique: Wielka Śnieżna cave system, Śnieżna Studnia cave and the both caves merged together. The square grid was laid on vertical projection of caves survey. The number of squares ("boxes") contained the polygonal vectors has been counted as No. Then the sides ri of the squares were gradually reduced (in the following steps $r_1=r_0/k$, $r_2=r_1/k_2$,..., for k = 2). At each step the total number of squares Pi, squares containing vectors, Ni, and "empty" squares, P_i - N_i, were counted. When all boxes were counted, the logarithms values $\log(r_0/r)$ and $\log(N_i)$ were also estimated:. Results were shown in Tab. 1 and as "box curve" in Fig. 2.

Data from Tab.1 were used to estimate fractal dimension and its statistical assessment. Linear regression were applied to calculate D together with its standard deviation (ΔD) and box function fitting were assessed by determination coefficient R2 and standard error of estimation SEE (Tab.2).

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Tab.1. Results of the box counting for each data set and different box side length.

r_1/r_0	$log(r_1/r_0)$	Wielka Śnieżna		Śnieżna Studnia		both caves	
		N_i	$log(N_i)$	N_i	$log(N_i)$	N_i	$log(N_i)$
1	0	37876	4,578364	22927	4,360347	43988	4,643334
2	0,301029996	21872	4,339888	13128	4,118199	25278	4,402743
4	0,602059991	11134	4,046651	6861	3,836387	12789	4,106837
8	0,903089987	5265	3,721398	3383	3,529302	6028	3,780173
16	1,204119983	2342	3,369587	1580	3,198657	2672	3,426836
32	1,505149978	984	2,992995	716	2,854913	1114	3,046885
64	1,806179974	414	2,617	302	2,480007	457	2,659916
128	2,10720997	165	2,217484	118	2,071882	176	2,245513
256	2,408239965	62	1,792392	47	1,672098	69	1,838849
512	2,709269961	24	1,380211	19	1,278754	25	1,39794

Tab.2. Results of linear regression presented for each data set: fractal dimension (D), fractal dimension standard deviation (ΔD), determination coefficient R2 and standard error of estimation (SEE)

	Wielka Śnieżna	Śnieżna Studnia	both caves
D	1,196	1,151	1,212
ΔD	0,031	0,031	0,032
R^2	0,994	0,994	0,994
SEE	0,085	0,085	0,088

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Discussion and Conclusion

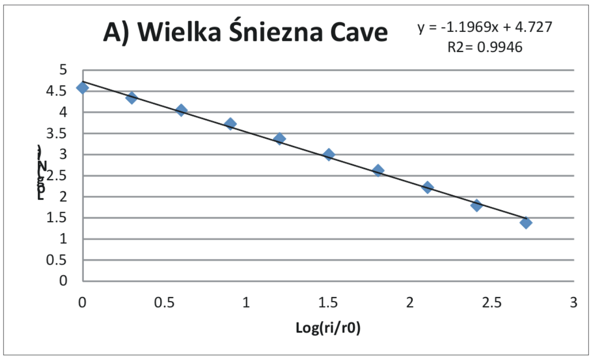
Linear regression results showed a very good approximation of data sets by linear function. The determination coefficient was very close to one what suggested that data point were almost ideal straight line. Also the standard error of estimation with value about 0,08 was satisfying.

It can be concluded that the Wielka Śnieżna,and Śnieżna Studnia caves separately as well as both caves merged together have fractal geometry. It may confirm the validity of the thesis that these caves are close to the full recognition of passages forming by a hydrological system and they could have close relation with faults and fractures systems in rock mass. Fractal dimension equals to 1 represents line structure, however fractal dimension equals to 2 represents plane (square). Obtained values for both caves are more than 1 but much less than 2. These characterize study area as cave passages which tend to "occupy" the entire plane The usual explanation of fractal dimension D higher than 1 indicates that caves with such dimension fill more space than those with ideal dimension of 1.00 (for example a straight line), and the geological constraints limit the dimension to be lower than 2 (Kortas 2003, Verbovsek 2007). It may also suggest that systems can be developing into more complicated passages in the future.

The fact, that both caves merged together also have showed fractal geometry indicates that they are belonging to the same hydrological system. To confirm this hypothesis ultimately a similar analysis should be carried out for the surface hydrological, faults and lineaments systems of the area under study. It was noticed that D-value of merged caves is slightly larger than individual cave. It can be explained by "occupying" more space in rock mass.

Caves with small fractal dimension have larger river flows. The hardness of mineral composition and grain size have influence the fractal dimension as well. If rock is harder, the fractal dimension is greater (Kusumayudha *et al.* 2000). Therefore, relative low values of D-value suggest that in the both caves strong river flows occur and surrounding rocks are not hard. This thesis confirms that Czerwone Wierchy Nappe is composed of limestones and dolomite in general. This type of rock are not hard and are easy to erode.

The lowest values can be found in the tectonic units of Tatra Mts rocks, which are comprised mostly of low-porosity and especially of low-permeability rocks. Some foreign study confirm this conclusions. The highest fractal dimensions (D=1.10) appear in the unit of External Dinarides. This unit is represented mostly by carbonates of Dinaric carbonate platform, which are intensely fractured and karstified. Similar explanation is valid for the unit of Southern Alps (D=1.00), also consisting of karstified and fractured carbonates (Verbovsek 2007).



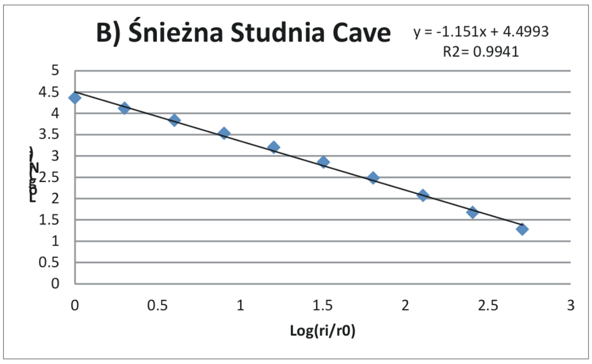
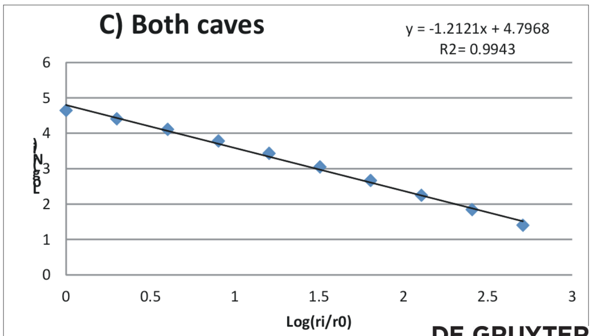


Fig.2. Logarithmic plots of box function: A) Wielka Śnieżna Cave, B) Śnieżna Studnia Cave, C) Both caves merge together



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