

# APPLICATION OF 3-D FRACTURE CRITERIA FOR ASSESSMENT OF FATIGUE STRENGTH OF COMPOSITE PLATE WITH INTERNAL DELAMINATION

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## **Abstract**

*The fracture analysis of the composite plates with circular delaminations working in a regime of cyclic multi-axial stresses is the main object of the paper. Such structures with internal damage are mainly exposed for local or global buckling and delamination propagation under service loadings. Analyses of described phenomena are complex and require application of fracture mechanics and nonlinear analysis. The literature review of experimental and theoretical investigations of composite plates with circular delaminations is presented in the paper. The essential papers including experimental fatigue tests and theoretical techniques for studying of damage propagation growth are cited and crucial conclusions are given. In composite structures with internal delamination, the cyclic compression-tension and compression-compression stresses are the most dangerous. Moreover, the rapid increase of damage propagation occurs in the final stage of structure degradation. Such problem is investigated and illustrated on the example of rectangular plate with the circular delamination. The calculations are made for bi-axial compression and AS4/3501-6 graphite/epoxy material. The fracture analysis is made using different criteria based on the linear elastic fracture mechanics. The criteria are compared with the multiaxial experimental tests made for AS4/3501-6 graphite/epoxy material. It is observed that in composite plate with the internal delamination subjected for compressive loading the complex form of the gap opening occur. Because of this, the 3-D fracture criteria are applied in the investigations. Presented methodology allow for suitable selection of the multiaxial fracture model for analysis of such structures including interactions between different forms of the gap opening. The critical loadings for different layers orientation are estimated and given in the paper.*

**Keywords:** Aircraft Engineering, Mechanical Engineering

## **1. Introduction**

Composite materials are widely used in many engineering structures. They have excellent engineering parameters such as high strength-to-weight ratio, high stiffness-to-weight ratio, corrosion resistance, etc. High strength-to-weight ratio is essential in many low-weight aerospace, aeronautical, marine, military, civil and automotive applications. The scale of application of composite materials can be observed on the example of the Boeing 787 passenger aircraft, in which weight of composite parts on its major components is about 50% [8]. However, structures made of laminated composite plates are vulnerable to different forms of damages such as delamination, debonding, interface failure, matrix cracking fibre breakage, transverse-ply cracking, etc. Above form of damages have significant influence on safety, especially in aerospace applications in which they are exposed to changing weather conditions, fatigue loadings or foreign object impacts.

Delamination is one of the most frequent forms of degradation of structures made of multi-layered composite materials at high-cycle fatigue. It can be originated by imperfections, manufacturing defects [16], and mismatch between different fibre orientations, object impacts or high stress concentrations from geometrical discontinuities. Generally, two stages during delamination propagation can be observed. The first one is related with slight increase of buckling

deformation. The second stage typically occurs after 90% of number of cycles to failure. During this final stage, the rapid damage growth can be observed and delamination starts growing outside the introduced zone (i.e. by foreign object impact). The presence of delamination in composite plates may lead to local or global buckling, kink band formation or delamination growth. In consequence, it can significantly reduce the stiffness and strength of laminate plates. Because of this, the detection of such damages and analysis their influence on the mechanical behaviour of delaminated composite plate become important practical issues [12].

## **2. Fatigue of structures made of graphite/epoxy composite with circular delamination – a review**

The overview of experimental delamination fracture and fatigue testing was described by Ireman et al. [7]. The basic delamination fracture tests, the compression tests of structures with single or multiple delaminations, the compression tests of samples weakened by open holes or impact damages, the indentation tests, etc. can be distinguished. The fatigue behaviour of 4 mm thick samples with stacking pattern assumed as  $[+45/0/-45/90]_{2s}$  made of AS4/8552 material was investigated by Butler et al. [4]. The experimental compressive fatigue tests were performed for composite plates with the small circular delaminations.

The influence of residual thermal stresses on the circular delamination initiation and growth in graphite/epoxy laminated FRP composites was investigated by Ramesh Babu and Pradhan [16]. The damage level was estimated using FEM with application of the Modified Crack Closure Integral technique. They observed that the residual thermal stresses increases of delamination growth at the interface of cross-ply  $[0/90]$  and angle-ply  $[+45/-45]$  laminates in regime of in-plane tension.

The experimental compressive fatigue tests of quasi-isotropic composite plates (250x50 mm, stacking sequence  $[(0/45/-45/90)_3]_s$ ) with the hole (5 mm diameter) and the circular delamination (20 mm diameter) at a centre of plate were made by Shen et al. [22]. The laminates were made from unidirectional graphite-epoxy Fibredux 924 prepregs and damage was located between different layers. In the experimental and numerical analyses, it was observed that the direction of damage growth generally agrees with the direction of the maximal strain energy release rate. Nilsson et al. [14] investigated HTA/6376C carbon fibre/epoxy composite 35 plies plates  $[(90/0)_{17}/90]$ , thickness 4.58 mm) with the circular delamination after 3rd, 5th or 7th ply. Obtained results indicate that failure occurs slightly below the global buckling load of undamaged panel due to the damage propagation.

The experimental fatigue tests for impact-damaged carbon-fibre epoxy (HTA/6376) plates under constant- amplitude loading were performed Gunnar Melin and Schön [10, 11]. The samples had a thickness 6.24 mm and size 450x156 mm. They used specimens with different plies orientations and the fatigue tests were made after object impact. The authors investigated delamination propagation during cycling compression loading and they demonstrated that the fatigue life depend on delamination growth in the transverse direction to the load. However, the final failure can be caused by another damage mechanism.

Influence of loading ratio on fatigue life of impact damaged AS/3501-6 graphite/epoxy composite samples was experimentally studied by Ramkumar [17]. The tests were made for tension-tension ( $R \approx 0$ ), tension-compression ( $R = -1$ ), and compression-compression ( $R \rightarrow -\infty$ ) regime. It was observed that samples loaded in stress ratio  $R = 0$  had the longest fatigue life. On the other hand specimens loaded in  $R = -1$  and  $R \rightarrow -\infty$  had almost the same fatigue life [3, 11, 17]. Beheshty & Harris [3] studied influence of varying load ratios  $R$  of impact damaged CFRP HTA/982A composite. It is worth to mention, that introduced damage had not effect on fatigue strength in purely tension regime. However, with increasing compression stress, the fatigue life was strongly reduced. Tension compression ( $R = -1$ ) tests were also made by Symons and Davis [23] and Rosenfeld and Gause [21].

The experimental tests of delamination growth and buckling behaviour for carbon fibre/epoxy cross-ply composite plates with the circular delaminations were made by Asp et al. [1]. In their tests, the damage growth direction was perpendicular to the load direction. Symons and Davis [23] studied propagation of the circular damage zone in T300/914 carbon/epoxy composite plate using ultrasonic C-scan. Other methods of monitoring of composite structures are based on acoustic emission, non-destructive inspection, ultrasonic scanning or Lamb wave analysis [2, 13].

### 3. Analytical techniques of damage propagation analysis

Delamination propagation can be experimentally investigated using different techniques such as analysis of the stiffness degradation or designation of delamination zone [23]. The problem of the delamination initiation and propagation in layered composite structures (such as beams, plates, etc.) can be studied using different techniques such as the multi-axial criteria, the cohesive damage models and the fracture mechanics. Application of above techniques to the analysis of the damage growth in composite plates can be found in Ref. [15, 19, 20, 22].

The fatigue life of composite laminates can be predicted using the fatigue life model based at S-N curves, the residual strength or the residual stiffness model (cumulative damage model) and the progressive damage models. Some of them (such as the fatigue life model based at S-N curves, the Tsai-Hill or the Tsai-Wu criteria) are empirical and do not take into account the details of damage mechanism. Due to this fact, they require many experimental tests.

### 4. Multi-axial fracture criteria

The numerical study of the delamination propagation was performed using the linear mixed mode [25] and Hashemi interaction [6] fracture criteria. The linear multi-axial model is the most commonly used in the fracture analysis of composite materials and is expressed as:

$$\frac{G_I}{G_{IC}} + \frac{G_{II}}{G_{IIC}} + \frac{G_{III}}{G_{IIIC}} \geq 1, \quad (1)$$

where:

$G_I, G_{II}, G_{III}$  – the strain energy release rates,

$G_{IC}, G_{IIC}, G_{IIIC}$  – the critical strain energy release rates.

The linear hypothesis is simple in use and requires only three fracture parameters  $G_{IC}, G_{IIC}$  and  $G_{IIIC}$ . This model does not take into account interaction between particular fracture modes and can describe only linear failure curve. However, for composite materials different shapes (from convex to concave) of material behaviour are observed. Thus, application of this model for such materials may leads to inaccurate estimation of the damage parameter.

The model proposed by Hashemi [6] is the modification of Williams [24] criterion and have additional parameter  $\varphi$ . It makes the fracture curve more flexible than in the Williams model. It is assumed, that fracture will occur when

$$\frac{G_I}{G_{IC}} + \frac{G_{II}}{G_{IIC}} + \left( \kappa - 1 + \varphi \frac{G_I}{G_I + G_{II}} \right) \frac{G_I G_{II}}{G_{IC} G_{IIC}} \geq 1, \quad (2)$$

where  $\kappa, \varphi$  – interaction parameters.

The interaction parameters are used to fit the fracture curve to the multi-axial experimental tests. They describe how the mode I and mode II affect each other.

In the 3D analysis, it is necessary to include both shear modes: the in-plane shear (the mode II) and the anti-plane shear (the mode III). Due to this fact, the author proposed the modification of the Hashemi formula expressed as:

$$\frac{G_I}{G_{IC}} + \frac{G_{II}}{G_{IIC}} + \frac{G_{III}}{G_{IIIC}} + \left( \kappa - 1 + \varphi \frac{G_I}{G_I + G_{II} + G_{III}} \right) \frac{G_I (G_{II} + G_{III})}{G_{IC} G_{IIC}} \geq 1. \quad (3)$$

In the above formula, it was assumed that anti-plane shear mode III affect mode I in similar way like mode II (in-plane shear).

Validation of the interaction parameters for Hashemi interaction model is presented in Fig. 1. The multi-axial experimental tests [18] were made for AS4/3501-6 graphite/epoxy material and for different ratio of the mode I and the mode II gap opening. In such composite material, the characteristic “overshoot” phenomenon can be observed (Fig. 1). Introducing small the mode II parameter ( $G_{II}$ ) results in slight increase of the maximal admissible mode I parameter ( $G_I$ ). It should be noted, that in such material  $G_I$  can slightly exceed the critical strain energy release rate  $G_{IC}$ . Such “overshoot” phenomenon cannot be investigated using the linear mixed-mode criterion, which in this case is more conservative than the Hashemi hypothesis.

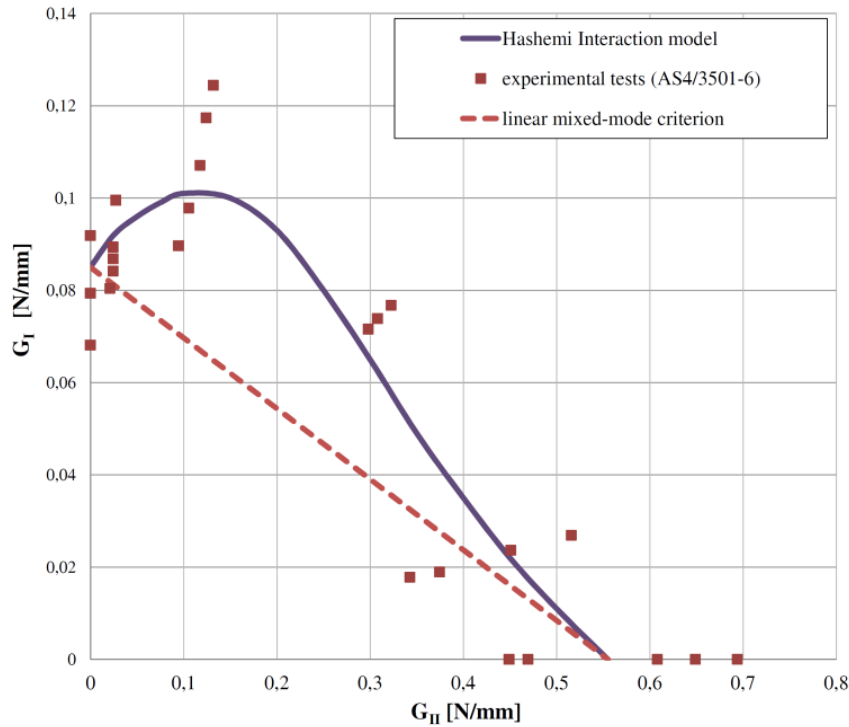


Fig. 1. Validation and adjusting of the Hashemi Interaction model (with parameters  $\kappa=0.8$  and  $\varphi=-3$ ) and linear mixed-mode criterion for the experimental multi-axial tests of graphite/epoxy AS3/3501-6 composite material

## 5. Numerical analysis of square plate with inter-ply circular delamination

The numerical study of the composite plate with circular delamination was the main aim of the presented study. The geometry of the analysed structure is presented in Fig. 2a. The thickness  $h$  of the plate was 1.5 mm. The circular delamination, with radius  $R = 15$  mm was located at the centre of the plate between layers with different orientations. The damaged square plate, with length  $a$ , was subjected to bi-axial compression. In the presented pictures, the stresses were given in relation to the longitudinal compressive strength –  $F_{1c}$ .

The calculations were performed for AS4/3501-6 graphite/epoxy material. The mechanical properties of a single layer were as follows [5]:

- the Young’s modules:  $E_1 = 1.5 \cdot 10^5$  MPa,  $E_2 = E_3 = 1.1 \cdot 10^4$  MPa,
- the Kirchhoff shear modules:  $G_{12} = G_{13} = 6.0 \cdot 10^3$  MPa,  $G_{23} = 3.6 \cdot 10^3$  MPa,
- the Poisson’s ratio:  $\nu_{12} = \nu_{13} = 0.3$ ,  $\nu_{23} = 0.45$ ,
- the longitudinal and the transverse tensile strength:  $F_{1t} = 2300$  MPa,  $F_{2t} = 57$  MPa,
- the longitudinal and the transverse compressive strength:  $F_{1c} = 1725$  MPa,  $F_{2c} = 228$  MPa,
- the critical strain energy release rates:  $G_{IC} = 0.085$  N/mm,  $G_{IIC} = G_{IIIC} = 0.085$  N/mm.

The numerical calculations were performed using ANSYS software. The SOLID185 8-node structural/layered solid elements were used in the analysis. The numerical model and boundary condition are presented in Fig. 2b. At the end of plate in nodes at the interface between layers the vertical displacement  $u_y = 0$  are also assumed.

The release strain energy rates  $G_I$ ,  $G_{II}$ ,  $G_{III}$  for investigated damage were estimated using the Virtual Crack Closure Technique (VCCT) [9]. Using this technique it is possible to calculate all three particular parameters of the gap opening. The application of the VCCT technique requires designation of the displacements between corresponding nodes of the crack faces and forces necessary to close the existing crack. In order to estimate such forces the spring element COMBIN14 were used in the numerical model.

The numerical studies were performed for plates made of layers orientated in two different directions:  $[0/90]$  and  $[0/60]$ . The orientation of the layers is given in relation to the x-z local coordinate system (Fig. 2a).

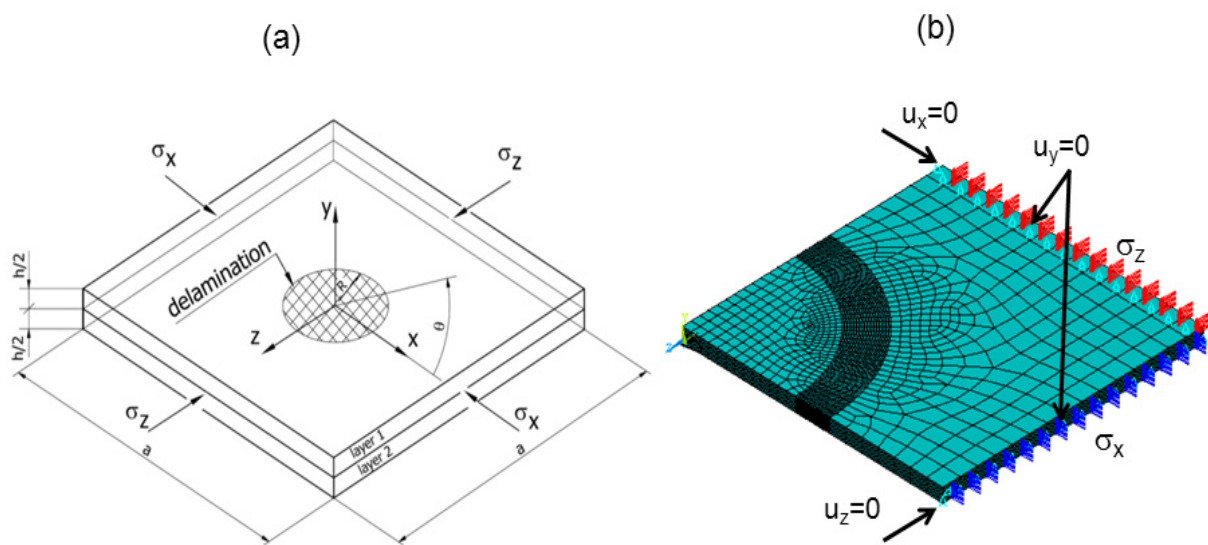


Fig. 2. Model of composite plate with internal circular delamination, angle  $\theta$  defines location of the calculated strain energy release rate on the crack face (a) and quarter-part of numerical model with finite element mesh and boundary conditions (b)

The application of the multi-axial fracture criteria allows for determine in which manner delamination or crack will propagate. The influence of the layer orientation on the  $RB_{del}$  parameter is given in Fig. 3.  $RB_{del}$  is the damage parameter and is equal to the left hand-side of the linear mixed mode fracture criterion (1) or Hashemi model (3). In the plate with circular delamination the mode I is the most dominant form of the gap opening. However, both shear modes (in plane shear – the mode II and anti-plane shear – the mode III) have significant influence on the damage parameter. For  $[0/60]^\circ$  configuration the damage will probably propagate in direction designated by  $\theta = 30^\circ$ , but for  $[0/90]^\circ$  the angle of further damage propagation is  $\theta = 45^\circ$ .

Application of the linear mixed-mode criterion gives the more conservative results in the investigated case of square plate with circular delamination (Tab. 1). For example for plate with layers orientation  $[0/90]$  the critical loading estimated using the linear mixed-mode model was  $\sigma_x=0.26F_{1c}$ ,  $\sigma_z=0.23F_{1c}$  but using Hashemi 3D interaction hypothesis  $\sigma_x=0.26F_{1c}$ ,  $\sigma_z=0.32F_{1c}$ .

Larger value of the critical loading in Hashemi model can be explained by so-called “overshoot” phenomenon. Introducing small levels of the mode II  $G_{II}$  and the mode III  $G_{III}$  strain energy release rate, whose sum  $G_{II} + G_{III} < 0.22$ , results in increasing of the mode I critical strain energy release rate in the multi-axial regime (see Fig. 1). The critical loading for  $[0/60]^\circ$  configurations are given in the Tab. 1.

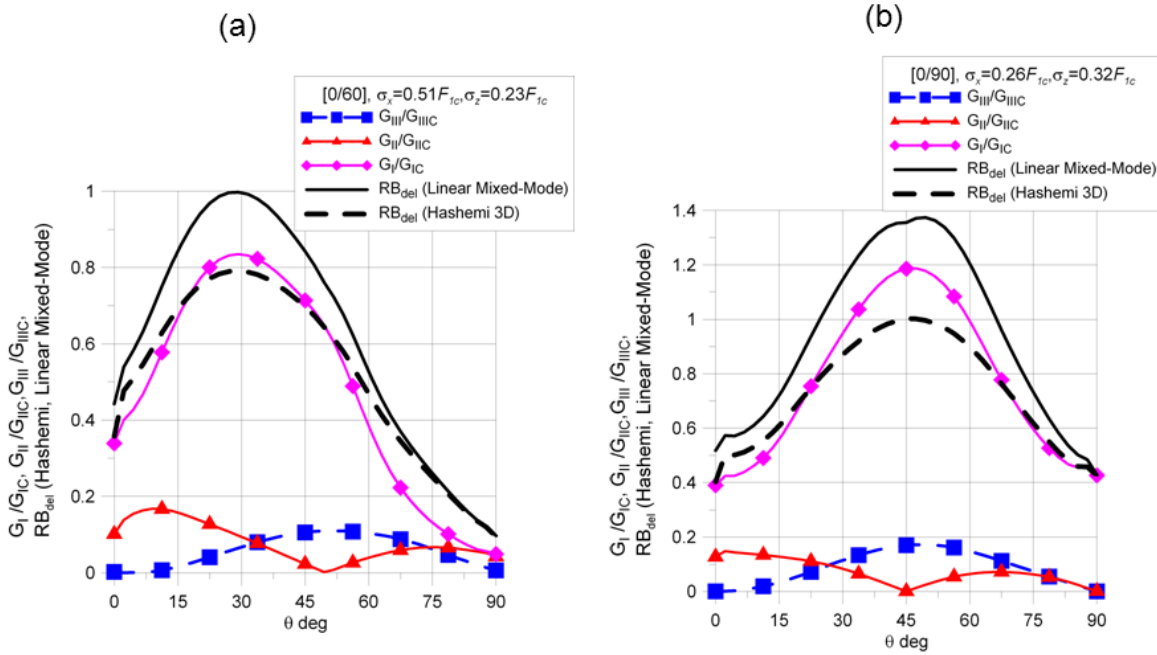


Fig. 3. Strain energy release rate and damage parameter  $RB_{del}$  for composite plate with circular delamination,  $h=1.5\text{mm}$ ,  $a/R = 4$ ,  $\sigma_x=0.51F_{1c}$ ,  $\sigma_z = 0.23F_{1c}$  layer orientation  $[0/60]^\circ$  (a),  $\sigma_x=0.26F_{1c}$ ,  $\sigma_z = 0.32F_{1c}$  layer orientation  $[0/90]^\circ$  (b)

Tab. 1. Critical loading for plate with circular delamination, AS4/3501-6, thickness  $h= 1.5\text{ mm}$ , delamination radius  $R = 15\text{ mm}$ , length  $a = 60\text{ mm}$

Orientation	0/90°		0/60°	
	$\sigma_x/F_{1c}$	$\sigma_z/F_{1c}$	$\sigma_x/F_{1c}$	$\sigma_z/F_{1c}$
Hashemi 3D interaction model	0.26	0.32	0.67	0.23
Linear mixed-mode criterion	0.26	0.23	0.51	0.23

## 6. Conclusions and future works

Following conclusions can be drawn:

- complex multi-axial form of the gap opening occurs in delaminated plate subjected to bi-axial compression,
- local or global buckling may occur in such structures and may be main form of the final failure,
- the proposed modification of the Hashemi formula, gives good fitting of the fracture curve to the experimental results for graphite/epoxy composite materials,
- using the proposed modification of Hashemi formula it is possible to estimate critical radius of delaminated zone or critical loadings which leads to the final failure,
- the mode I (tensile) is the most dominant form of the crack propagation in a composite plates with circular delamination subjected to the compressive load, however the shear modes have significant influence on damage parameter,
- linear mixed-mode criteria is not able to predict influence of interaction between particular tensile the mode I and shear the mode II and the mode III parameters on fracture strength for graphite/epoxy composite structures,
- the critical loading for investigated plate with circular delamination was designated,
- in order to determine the interaction between  $G_{III}$  and  $G_I$  the experimental multi-axial tests including the mode III should be performed.

In the future works the experimental tests of such structure for verification of the proposed model are planned.

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