Mariusz BUCIAKOWSKI, Marcin WITCZAK, Marcel LUZAR
UNIWERSYTET ZIELONOGÓRSKI, INSTYTUT STEROWANIA I SYSTEMÓW INFORMATYCZNYCH,
Licealna 9, 65-417 Zielona Góra

Robust fault-tolerant control for a multi-tank system

M.Sc. Mariusz BUCIAKOWSKI

Received his M.Sc degree in automatic and robotics in 2013 from the Faculty of Electrical Engineering, Computer Science and Telecommunications at the University of Zielona Góra. Currently a Ph.D. student in this faculty. His current research interests are Robust Control, Fault-Tolerant Control (FTC) and Modern Control.

e-mail: mariusz.bu@wp.pl

Ph.D. D.Sc. Marcin WITCZAK

Received the M.Sc. degree in electrical engineering from the University of Zielona Góra (Poland), the Ph.D. degree in automatic control and robotics from the Wrocław University of Technology (Poland) and the D.Sc. degree in electrical engineering from the University of Zielona Góra in 1998, 2002 and 2007, respectively. His current research interests include computational intelligence, Fault Detection and Isolation (FDI), Fault-Tolerant Control (FTC), and control theory.

e-mail: M.Witczak@issi.uz.zgora.pl

M.Sc. Marcel LUZAR

Received the M.Sc. degree in computer science from the University of Zielona Góra (Poland) in 2009. Currently an Assistant Lecturer at Faculty of Electrical Engineering, Computer Science and Telecommunications at the University of Zielona Góra. His current research interests include Fault Diagnosis, Robust Control and Neural Networks. Marcel Luzar has published 11 papers in conference proceedings, 1 in international journal and 1 book chapter.

e-mail: M.Luzar@issi.uz.zgora.pl

Abstract

The paper deals with the problem of robust fault-tolerant control for non-linear discrete-time systems. The main part of this paper describes sensor a fault diagnosis scheme using *virtual sensors*, which recover the measurement of the fault sensor based on the fault-free ones. The virtual sensor is designed in such a way that a prescribed attenuation level is achieved with respect to the fault estimation error while guaranteeing the convergence of the robust observer underlying the virtual sensor. The subsequent part of the paper deals with the design of robust controller as well as the proposed fault-tolerant control scheme. The final part of the paper shows the experimental results regarding the multi-tank system, which confirm the effectiveness of proposed approach.

Keywords: fault diagnosis, fault identification, robust estimation, nonlinear systems, virtual sensor.

Odporne sterowanie tolerujące uszkodzenia układem wielu zbiorników

Streszczenie

Artykuł przedstawia problem odpornego sterowania tolerującego uszkodzenia układem nieliniowym w czasie dyskretnym. Główna część artykułu opisuję diagnostykę czujników pomiarowych z wykorzystaniem wirtualnych czujników. Wirtualny czujnik został zaprojektowany w taki sposób, aby dla określonego poziomu tłumienia zakłóceń uzyskać możliwie mały błąd estymacji stanu oraz zapewnić dobra zbieżność obserwatora, na którym bazuje wirtualny czujnik. Kolejna część artykułu opisuję metodę projektowania odpornego regulatora oraz proponowaną metodę detekcji uszkodzeń. W końcowej części artykułu przedstawiono wyniki eksperymentalne dotyczące układu wielu zbiorników. Zastosowany układ wielu zbiorników charakteryzuję się nieliniowością, przez co może zostać wykorzystany do weryfikacji przedstawionych w artykule techniki sterowania tolerującego uszkodzenia. Przedstawione wyniki potwierdzają skuteczność proponowanego podejścia bazującego na detekcji uszkodzeń czujników pomiarowych z wykorzystaniem wirtualnych czujników pomiarowych. Wyniki obrazują działanie układu w przypadku wystąpienia uszkodzenia przy braku sterowania tolerującego uszkodzenia oraz działanie układu w przypadku zastosowania sterowania tolerującego uszkodzenia.

Słowa kluczowe: Diagnostyka błędu, identyfikacja uszkodzenia, estymacja odporna, systemy nieliniowe, czujnik wirtualny.

1. Introducton

The problem of fault diagnosis (FD) of non-linear industrial systems [5, 12, 7, 8] has received considerable attention during the last three decades. Indeed, it developed from the art of designing satisfactory performing systems into the modern theory and practice that it is today. Within the usual framework, the system being diagnosed is divided into three main components, i.e. plant (or system dynamics [5, 12]), actuators and sensors. The paper deals with the problem of full fault diagnosis of sensors, i.e. apart from the usual two steps consisting of fault detection and isolation (FDI), the fault identification is also performed. This last step is especially important from the viewpoint of Fault-Tolerant Control (FTC) [10, 11, 1], which is possible if and only if there is information about the size of the fault being a result of fault identification (or fault estimation).

In this paper there is proposed a robust fault estimation approach can be efficiently applied to realise the above-mentioned three-step procedure. The proposed approach is based on the general idea of an Unknown Input Observer (UIO) [12, 13], which was initially designed to tolerate a degree of model uncertainty and hence increased the reliability of fault diagnosis. The proposed approach can be perceived as a combination of the linear-system strategies [3] and [9] for a class of non-linear systems [15]. The proposed approach is designed in such a way that a prescribed disturbance attenuation level is achieved with respect to the fault estimation error while guaranteeing the convergence of the observer. The fault-tolerant control scheme is based on replacing the faulty sensor measurements and feeding them into the robust controller. The paper is organized as follows. Section 2 describes the proposed virtual sensor. Section 3 describes the robust controller and an integration procedure with the virtual sensor strategy. The final part of the paper presents a comprehensive case study regarding the multi-tank system, which clearly indicates the performance of the proposed approach.

2. Virtual sensor design

The main objective of this section is to provide a detailed design procedure of the virtual sensor, which can be used for sensor fault diagnosis. In other words, the main role of this sensor is to provide the information about the sensor fault. Indeed, apart from serving as a usual residual generator, the virtual sensor should be designed in such a way that a prescribed disturbance attenuation level is achieved with respect to the sensor fault estimation error while guaranteeing the convergence of the observer.

Let us consider the following non-linear system:

$$x_{k+1} = Ax_k + Bu_k + g(x_k) + W_1 w_k, \tag{1}$$

$$y_{k+1} = Cx_{k+1} + L_s f_{s,k} + W_2 w_{k+1}, \qquad (2)$$

where $x_k \in \mathbb{R}^n$ is the state vector , $u_k \in \mathbb{R}^r$ stands for the input, $y_k \in \mathbb{R}^m$ denotes the output, $f_{s,k} \in \mathbb{R}^m$ stands for the sensor fault, $w_k \in l_2$ is a en exogenous disturbance vector with $W_1 \in \mathbb{R}^{nxn}$, $W_2 \in \mathbb{R}^{mxn}$ being its distribution matrices while

$$l_2 = \{ \; w \in \Re^n | \; \|w\|_{l_2} < +\infty \; \}, \qquad \|w\|_{l_2} = \left(\sum_{k=0}^{\infty} \|w\|^2 \right)^{\frac{1}{2}}. \qquad (3)$$

Moreover, let us define the matrix X be partitioned in such a way that

$$X = \begin{bmatrix} x_1^T \\ \vdots \\ x_{n_x}^T \end{bmatrix} \qquad (4)$$

where x_j stands for the jth row of X. Let us also denote X^j as the matrix X without the jth row and y^j as a vector y without the jth element.

The sensor fault diagnosis will be realised by a set of m observers of the form:

$$\hat{x}_{k+1} = A\hat{x}_k + g(\hat{x}_k) + K_o^j \left(y_k^j - C^j \hat{x}_k \right), \qquad j = 1, \cdots, m, \qquad (5)$$

while the jth output (for $L_{s,k} = I$) is described by

$$y_{f,j,k} = c_j^T x_{f,k} + w_{2,j}^T w_k + f_{s,j,k}. \qquad (6)$$

Thus:

$$f_{s,j,k} = y_{f,j,k} - c_j^T x_{f,k} - w_{2,j}^T w_k, \qquad (7)$$

and an jth fault estimate is

$$\hat{f}_{s,j,k} = y_{f,j,k} - c_j^T \hat{x}_k. \qquad (8)$$

The fault estimation error $\varepsilon_{fj,k}$ of the jth sensor is

$$\varepsilon_{f,j,k} = f_{s,j,k} - \hat{f}_{s,j,k} = -c_j^T x_{f,k} + c_j^T \hat{x}_k + w_{2,j}^T w_k = -c_j^T e_k - w_{2,j}^T w_k \cdot \qquad (9)$$

while the state estimation error is:

$$e_{k+1} = Ae_k - s_k - K_o^j C^j e_k - K_o^j W_2 w_k + W_1 w_k, \qquad (10)$$

$$e_{k+1} = \left(A - K_o^j C^j \right) e_k + s_k - \overline{W} w_k, \qquad (11)$$

$$e_{k+1} = A_1 e_k + s_k - \overline{W} w_k, \qquad (12)$$

where

$$s_k = g(x_k) - g(\hat{x}_k). \qquad (13)$$

Note that both e_{k+1} and $\varepsilon_{fs,k}$ are non-linear with respect to e_k. To settle this problem within the framework of this paper, the following solution is proposed. Using the Differential Mean Value Theorem (DMVT) [14], it can be shown that

$$g(a) - g(b) = M_x (a - b), \qquad (14)$$

with

$$M_x = \begin{bmatrix} \frac{\partial g_1}{\partial x}(c_1) \\ \vdots \\ \frac{\partial g_{n1}}{\partial x}(c_n) \end{bmatrix}, \qquad (15)$$

where $c_1, \ldots, c_n \in Co(a,b)$, $c_i \neq a$, $c_i \neq b$, $i = 1, \ldots, n$. Assuming that

$$\bar{a}_{i,j} \geq \frac{\partial g_i}{\partial x_j} \geq \underline{a}_{i,j}, \qquad i = 1, \cdots, n, \qquad j = 1, \ldots, n, \qquad (16)$$

it is clear that:

$$\mathrm{M}_x = \{ \; \mathrm{M} \in \Re^{n \times n} | \bar{a}_{i,j} \geq m_{x,i,j} \geq \underline{a}_{x,i,j}, i, j = 1, \ldots, n, \; \} \qquad (17)$$

Thus, using (14), the term $A_1 e_k - s_k$ in (12) can be written as

$$A_1 e_k + s_k = (\bar{A} + M_{x,k} - K_o^j C^j) e_k \qquad (18)$$

where $M_{x,k} \in \mathbb{M}_x$.

From (18), it can be deduced that the state estimation error can be converted into an equivalent form

$$e_{k+1} = A_2(\alpha) e_k + s_k - \overline{W} w_k, \qquad (19)$$

$$A_2(\alpha) = \tilde{A}(\alpha) - K_o C,$$

which defines an LPV polytopic system [2] with

$$\tilde{\mathrm{A}} = \{ \; \tilde{A}(\alpha): \qquad \tilde{A}(\alpha) = \sum_{i=1}^{N} \alpha_i \tilde{A}_i, \sum_{i=1}^{N} \alpha_i = 1, \alpha_i \geq 0 \; \}, \qquad (20)$$

where $N = 2^{n2}$. Note that this is a general description, which does not take into account that some elements of $M_{x,k}$ may be constant. In such cases, N is given by $N = 2^{(n-c)2}$ where c stands for the number of constant elements of $M_{x,k}$.

Thus, the state estimation error (12) can be described as

$$e_{k+1} = A_2(\alpha) e_k - \overline{W} w_k, \qquad (21)$$

$$A_2(\alpha) = A_2(\alpha) e_k - K_o^j C^j. \qquad (22)$$

The problem of H_∞ observer design [6] is to determine the gain matrix K_o such that

$$\lim_{k \to \infty} e_k = 0 \qquad for \qquad w_k = 0, \qquad (23)$$

$$\|\varepsilon\|_{l_2} \leq \omega \|w\|_{l_2} \qquad for \qquad w_k \neq 0, e_0 = 0. \qquad (24)$$

The general framework for designing a robust observer is:

$$\Delta V_k + \varepsilon_{f_{j,k}}^T \varepsilon_{f_{j,k}} - \mu^2 w_k^T w_k < 0, k = \ldots, \infty, \qquad (25)$$

with

$$\Delta V_k = V_{k+1} - V_k, \qquad V_k = e_k^T P(\alpha) e_k, \qquad \omega = \sqrt{\mu}. \qquad (26)$$

Consequently, it can be shown that:

$$\Delta V + \varepsilon_{f_{j,k}}^T \varepsilon_{f_{j,k}} - \mu^2 w_k^t w_k =$$
$$e_k^T \left(A_2(\alpha)^T P(\alpha) A_2(\alpha)\right) e_k^T +$$
$$e_k^T \left(A_2(\alpha)^T P(\alpha)\overline{W}\right) w_k + \tag{27}$$
$$w_k^T \left(\overline{\overline{W}}^T P(\alpha) A_2(\alpha)\right) e_k +$$
$$w_k^T \left(\overline{W} P(\alpha) \overline{W}\right) w_k < 0.$$

By defining

$$v_k = \left[e_k^T, w_k^T\right]^T, \tag{28}$$

the inequality (27) becomes

$$\Delta V_k + \varepsilon_{f_{j,k}}^T \varepsilon_{f_{j,k}} - \mu^2 w_k^T w_k = v_k^T M_{V_0} v_k < 0, \tag{29}$$

where

$$M_{V_0} = \begin{bmatrix} A_2(\alpha)^T P(\alpha) A_2(\alpha) - P(\alpha) + c_j c_j^T & A_2(\alpha)^T P(\alpha)\overline{W} + c_j w_{2,j}^T \\ \overline{W}^T P(\alpha) A_2(\alpha) + w_{2,j} c_j^T & \overline{W}^T P(\alpha)\overline{W} + w_{2,j} w_{2,j}^T - \mu^2 I \end{bmatrix} \tag{30}$$

The following two lemmas can be perceived as the generalization of those presented in [2].

Lemma 1. *The following statements are equivalent*
1. There exists $X \succ 0$ such that

$$V^T X V - W \prec 0, \tag{31}$$

2. There exists $X \succ 0$ such that

$$\begin{bmatrix} -W & V^T U^T \\ UV & X - U - U^T \end{bmatrix} \prec 0. \tag{32}$$

Proof. Applying the Schur complement to (*2*) gives

$$V^T U^T \left(U^T + U - X\right)^{-1} UV - W \prec 0. \tag{33}$$

Substituting $U = U^T = X$ yields

$$V^T X V - W \prec 0. \tag{34}$$

Thus, (*1*) implies (*2*).
Multiplying (32) by $T = [I\ V^T]$ on the left and by T^T on the left of (32), gives (31), which means that (*2*) implies (*1*) and hence the proof is completed.

Lemma 1. *The following statements are equivalent*
1. There exists $X \succ 0$ such that

$$V(\alpha)^T X(\alpha) V(\alpha) - W(\alpha) \prec 0, \tag{35}$$

2. There exists $X \succ 0$ such that

$$\begin{bmatrix} -W(\alpha) & V(\alpha)^T U^T \\ UV(\alpha) & X(\alpha) - U - U^T \end{bmatrix} \prec 0. \tag{36}$$

Proof. The proof can be realised by following the same line of reasoning as the one of Lemma 1. It is easy to show that (36) is satisfied if there exist matrices $X \succ 0$ such that

$$\begin{bmatrix} -W_i & V_i^T U^T \\ UV_i & X_i - U - U^T \end{bmatrix} \prec 0 \quad i = 1, \ldots, N. \tag{37}$$

Theorem 1. *For a prescribed disturbance attenuation level $\mu>0$ for the fault estimation error (9), the H_∞ observer design problem for the system (1)-(2) and the observer (5) is solvable if there exist matrices $P_i \succ 0$ ($i=1,\ldots,N$), U and N such that the following LMIs are satisfied:*

$$\begin{bmatrix} -P_i + c_j c_j^T & c_j w_{2,j}^T & A_{2,i}^T U^T \\ w_{2,j} c_j^T & w_{2,j} w_{2,j}^T - \mu^2 I & \overline{W}^T U^T \\ UA_{2,i} & UW & P_i - U - U^T \end{bmatrix} \prec 0. \tag{38}$$

where

$$UA_{2,i} = U(\hat{A}_i - K_o C) = U\hat{A}_i - NC. \tag{39}$$

Proof. Observing that the matrix (30) must be negative definite and writing it as

$$\begin{bmatrix} A_2(\alpha)^T \\ \overline{W}^T \end{bmatrix} P(\alpha) \left[A_2(\alpha)^T \overline{W}^T\right] + \begin{bmatrix} -P(\alpha) + c_j w_T^2 & c_j w_{2,j}^T \\ w_{2,j} c_j^T & w_2 w_{2,j}^T - \mu^2 I \end{bmatrix} \tag{40}$$

and then applying Lemma 2 and (37) leads to (38), which completes the proof. Finally, the gain matrix of the virtual sensor is given by

$$K_0 = U^{-1} N. \tag{41}$$

3. Controller design

The main objective of this section is to present the design procedure of the robust controller, for which a predefined disturbance attenuation level with respect to the state of the system is achieved. Following the same line of reasoning as in the preceding section, the state equation (1) can be written in an equivalent form:

$$x_{k+1} = A(\alpha) x_k + B u_k + W_1 w_{k.} \tag{42}$$

Substituting

$$u_k = -K_c x_k, \tag{43}$$

into (42) yields

$$x_{k+1} = A_3(\alpha) x_k + W_1 w_k, \tag{44}$$

where

$$A_3(\alpha) = \left(A(\alpha) - BK_c\right) x_k, \tag{45}$$

with

$$\mathrm{A} = \{\ A(\alpha): \quad A(\alpha) = \sum_{i=1}^{N} \alpha_i A_i, \sum_{i=1}^{N} \alpha_i = 1, \alpha_i \geq 0\}. \tag{46}$$

Similarly as in Section 2, the general framework for designing robust controller is:

$$\Delta V_k + x_k^T x_k - \mu^2 w_k^T w_k < 0, k = \ldots, \infty, \quad (47)$$

with

$$V_k = x_k^T P(\alpha) x_k. \quad (48)$$

Consequently, it can be shown that:

$$\begin{aligned} &\Delta V + x_k^T x_k - \mu^2 w_k^t w_k = \\ &x_k^T \left(A_3(\alpha)^T P(\alpha) A_3(\alpha)\right) x_k^T + \\ &x_k^T \left(A_3(\alpha)^T P(\alpha) W_1\right) w_k + \\ &w_k^T \left(W_1^T P(\alpha) A_3(\alpha)\right) x_k + \\ &w_k^T \left(W_1^T P(\alpha) W\right) w_k < 0. \end{aligned} \quad (49)$$

By defining

$$v_k = \left[x_k^T, w_k^T\right]^T, \quad (50)$$

the inequality (49) becomes

$$\Delta V_k + x_k^T x_k - \mu^2 w_k^T w_k = v_k^T M_{V_c} v_k < 0, \quad (51)$$

where

$$M_{V_0} = \begin{bmatrix} A_3(\alpha)^T P(\alpha) A_3(\alpha) - I & A_3(\alpha)^T P(\alpha) W_1 \\ W_1^T P(\alpha) A_3(\alpha) & W_1^T P(\alpha) W_1 - \mu^2 I \end{bmatrix}. \quad (52)$$

Theorem 2. *For a prescribed disturbance attenuation level $\mu>0$ for the state (42), the H_∞ controller design problem for the system (1)-(2) is solvable if there exist matrices $P_i \succ 0$ $(i=1,\ldots,N)$, U and V such that the following LMIs are satisfied:*

$$\begin{bmatrix} I & 0 & A_{3,i}U \\ 0 & -\mu^2 I & W_1 U \\ U^T A_{3,i}^T & UW & P_i - U - U^T \end{bmatrix} \prec 0, \quad (53)$$

witch

$$A_{3,i}U = \left(A_i - BK_c\right)U = A_i U - BV. \quad (54)$$

Proof. Observing that the matrix (49) must be negative definite and writing it as

$$\begin{bmatrix} A_3(\alpha)^T \\ W_1^T \end{bmatrix} P(\alpha) \left[A_3(\alpha)^T W_1^T\right] + \begin{bmatrix} I & 0 \\ 0 & -\mu^2 I \end{bmatrix} \prec 0 \quad (55)$$

and then applying transposed version of Lemma 2 leads to (53), which completes the proof.

Thus, the final design procedure is: given a prescribed disturbance attenuation level μ, obtain $P_i \succ 0$, U, V by solving (53). Finally, the gain matrix of the FTC controller is:

$$K_c = VU^{-1}. \quad (56)$$

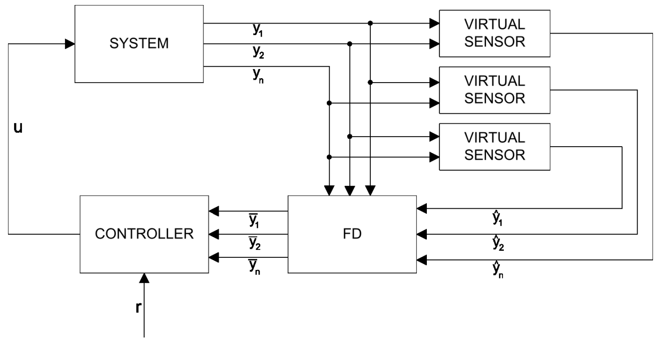

Fig. 1. FTC scheme
Rys. 1. Schemat FTC

Since the design procedure of the robust controller is provided, then the FTC scheme can be described in details. The FTC scheme is portrayed in Fig. 1. The main idea behind the proposed approach is that the faulty sensor measurement is replaced by the fault-free one. The decision is realised by the FD part, which aims at providing appropriate switching in case of a sensor fault. The decision are based on thresholding the fault estimates provided by the virtual sensors [12].

4. Case study

To verify the proposed approach, it is implemented for the multi-tank system. The considered multi-tank system (Fig. 2) is designed for simulating the real industrial multi-tank system in the laboratory conditions [4]. It can be efficiently used to practically verify both linear and non-linear control, identification and diagnostics methods.

Fig. 2. Multi-tank system
Rys. 2. Układ wielu zbiorników

The considered system consists of three separate tanks placed each above other and equipped with drain valves and level sensors based on a hydraulic pressure measurement. Each of them has a different cross-section in order to reflect system nonlinearities. The lower bottom tank is a water reservoir for the system. A variable speed water pump is used to fill the upper tank. The water outflows the tanks due to gravity. The considered multi-tank system has been designed to operate with an external, PC-based digital controller. The control computer communicates with the level sensors, valves and a pump by a dedicated I/O board and the power interface. The I/O board is controlled by the real-time software, which operates in a Matlab/Simulink environment.

The system matrices and non-linearities are

$$A = I_n, \quad B = \begin{bmatrix} 0.014 \\ 0 \\ 0 \end{bmatrix}, \quad C = I_m, \tag{57}$$

$$L_a = I_m,$$

$$g(x_{f,k}) = \begin{bmatrix} \frac{1}{\beta_1(x_{1,k})} C_1 x_{1,k}^{\alpha_1} \\ \frac{1}{\beta_2(x_{2,k})} C_1 x_{1,k}^{\alpha_1} - \frac{1}{\beta_2(x_{2,k})} C_2 x_{2,k}^{\alpha_2} \\ \frac{1}{\beta_3(x_{3,k})} C_2 x_{2,k}^{\alpha_2} - \frac{1}{\beta_3(x_{3,k})} C_3 x_{3,k}^{\alpha_3} \end{bmatrix}$$

where $x_{i,k}$, $i \in 1,\ldots,3$ is water level in the i-th tank, $\beta(x_{i,k})$ stands for cross section area of the i-th tank at the level $x_{i,k}$ and is, respectively, defined as:

1. Constant cross-sectional area of the top tank

$$\beta_1(x_{1,k}) = aw$$

2. Variable cross-sectional area of the middle tank

$$\beta_2(x_{2,k}) = \frac{x_{2,k}}{x_{2\max}} bw$$

3. Variable cross-sectional area of the bottom tank

$$\beta_3(x_{3,k}) = w\sqrt{R^2 - (R - x_{3,k})^2}$$

The numerical values of the above parameters are as follows: $C_1 = 1.0057 \cdot 10^{-4}$, $C_2 = 1.1963 \cdot 10^{-4}$, $C_3 = 9.8008 \cdot 10^{-5}$, $b = 0.34$, $c = 0.1$, $w = 0.035$, $R = 0.364$, $x_{2max} = 0.35$, $\alpha_1 = 0.29$, $\alpha_2 = 0.2256$, $\alpha_3 = 0.2487$ and $h = 0.01$s. Since the pump is not equipped with flow sensor, it is impossible to explicitly identify the sensor fault in the upper tank. Thus, to present the results, the upper tank liquid level is measured based on the real sensor signal and the other levels in the tanks are measured with the estimated and real values. The assumed initial state and its estimate are equal and the system input is generated using a random signal $0.08 \le u_k \le 2.3$ while $w_k \sim N(0,0.01I)$.

The following fault scenario for the second sensor was introduced:

$$f_{s_2,k} = \begin{cases} y_{2,k} - 0.35, & \text{for} \quad 100 \le k \le 300. \\ 0, & \text{otherwise.} \end{cases}$$

Moreover, the strategy of switching between the virtual sensor output and the system output presented in Fig. 1 is based on the following rule:

$$\bar{y}_{2,k} = \begin{cases} \hat{y}_{2,k}, & \text{if} \quad \left|\hat{f}_{s,2,k}\right| > \varepsilon, \\ y_{2,k}, & \text{otherwise,} \end{cases}$$

where $\varepsilon > 0$ is a given threshold and the fault estimate is the result of (8). The system performance with (Fig. 5) and without (Fig. 4) proposed FTC strategy. Indeed, the reference signal (solid line) is the target that has to be achieved by the controller. The fault of the second sensor appears for $k = 100,\ldots,300$, which is clearly depicted on both figures. While in the right figure, it can be observed that the faulty measurement is replaced by its estimate provided by the virtual sensor.

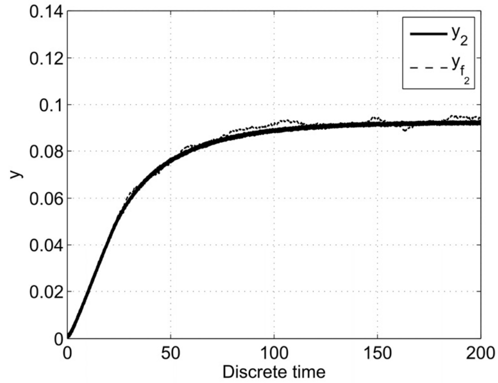

Fig. 3. Second output and its estimate
Rys. 3. Poziom w zbiorniku drugim oraz estymata poziomu

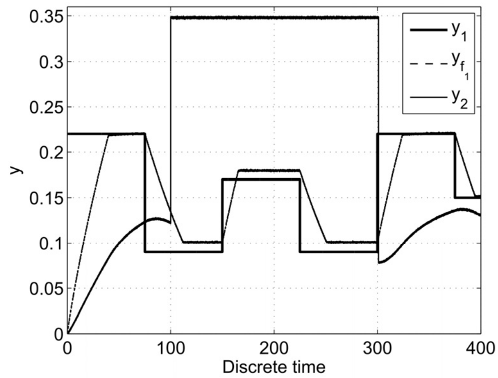

Fig. 4. Performance of the system without FTC.
Rys. 4. Wydajność systemu bez zastosowania FTC

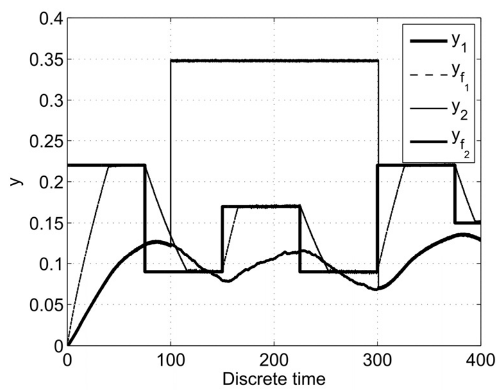

Fig. 5. Performance of the system with FTC.
Rys. 5. Wydajność systemu z zastosowania FTC

A straight comparison clearly indicates that the FTC controller outperforms the usual robust controller. Finally it should be mentioned that the similar results were obtained for the first and second sensor.

5. Conclusion

The paper deals with the problem of robust FTC for a class on non-linear systems. In particular, a combination of the celebrated generalised virtual sensor scheme with the robust H_∞ approach is proposed to settle the problem of robust fault diagnosis. The proposed approach is designed in such a way that a prescribed disturbance attenuation level is achieved with respect to the sensor fault estimation error while guaranteeing the convergence of the observer. Moreover, the controller design, which realises the switching strategy between the virtual sensor and the real sensor output, is carefully analysed. The final part of the paper concerns a comprehensive case study regarding the multi-tank system. The achieved results show the performance of the proposed approach, which confirm its practical usefulness.

The work was financed as a research project with the sciencfunds for years 2011-2014 with the kind support of the National Science Centre in Poland under the grant NN514678440 Predictive fault-tolerant control for non-linear systems.

6. References

[1] De Oca S. and Puig V. and Witczak M. and Dziekan Ł.: Fault-tolerant control strategy for actuator faults using LPV techniques: application to a two degree of freedom helicopter. International Journal of Applied Mathematics and Computer Science, 22(1):161–171, 2012.

[2] De Oliveira M.C. and Bernussou J. and Geromel J.C.: A new discrete-time robust stability condition. Systems and Control Letters, 37(4):261–265, 1999.

[3] Gillijns S. and De Moor B.: Unbiased minimum-variance input and state estimation for linear discrete-time systems. Automatica, 43:111–116, 2007.

[4] INTECO. Multitank System - User's manual. 2013.

[5] Korbicz J. and Kościelny J.M. and Kowalczuk Z.: Fault Diagnosis, Models, Artificial Intelligence, Applications. Springer-Verlag, Berlin, 2004.

[6] Li H. and Fu M. A linear matrix inequality approach to robust H∞ filtering. IEEE Trans. Signal Processing, 45(9):2338–2350, 1997.

[7] Luzar M. and Czajkowski A. and Witczak M. and Mrugalski M.: Actuators ans sensors fault diagnosis with dynamic, state-space neural networks. Methods and Models in Automation and Robotics - MMAR 2012: Proceedings of the 17th IEEE International Conference.

[8] Mrugalski M.: An unscented Kalman filter in designing dynamic GMDH neural networks for robust fault detection. International Journal of Applied Mathematics and Computer Science, 23(1):157–169, 2013.

[9] Nobrega E.G. and Abdalla M.O. and Grigoriadis K.M.: Robust fault estimation of unceratain systems using an LMI-based approach. International Journal of Robust and Nonlinear Control, 18(7):1657–1680, 2008.

[10] Noura H. and Theilliol D. and Ponsart J. and Chamseddine A.: Fault-tolerant Control Systems: Design and Practical Applications, Springer-Verlag, Berlin, 2003.

[11] Puig, V.: Fault diagnosis and fault tolerant control using set-membership approaches: Application to real case studies. International Journal of Applied Mathematics and Computer Science, 20(4):619–635, 2010.

[12] Witczak M.: Modelling and Estimation Strategies for Fault Diagnosis of Non-linear Systems, Springer-Verlag, Berlin, 2007.

[13] Witczak M. and Pretki P.: Design of an extended unknown input observer with stochastic robustness techniques and evolutionary algorithms. International Journal of Control, 80(5):749–762, 2007.

[14] Zemouche A. and Boutayeb M.: Observer design for Lipschitz non-linear systems: the discrete time case. IEEE Trans. Circuits and Systems - II:Express Briefs, 53(8):777–781, 2006.

[15] Zemouche A. and Boutayeb M. and Iulia Bara G.: Observer for a class of Lipschitz systems with extension to H_∞ performance analysis. Systems and Control Letters, 57(1):18–27, 2008.

otrzymano / received: 19.11.2013
przyjęto do druku / accepted: 02.12.2013 *artykuł recenzowany / revised paper*

INFORMACJE

Informacje dla Autorów

Redakcja przyjmuje do publikacji tylko prace oryginalne, nie publikowane wcześniej w innych czasopismach. Redakcja nie zwraca materiałów nie zamówionych oraz zastrzega sobie prawo redagowania i skracania tekstów oraz streszczeń.

Artykuły naukowe publikowane w czasopiśmie PAK są formatowane jednolicie zgodnie z ustaloną formatką zamieszczoną na stronie redakcyjnej www.pak.info.pl. Dlatego artykuły przekazywane redakcji należy przygotowywać w edytorze Microsoft Word 2003 (w formacie DOC) z zachowaniem:
- wielkości czcionek,
- odstępów między wierszami tekstu,
- odstępów przed i po rysunkach, wzorach i tabelach,
- oznaczeń we wzorach, tabelach i na rysunkach zgodnych z oznaczeniami w tekście,
- układu poszczególnych elementów na stronie.

Osobno należy przygotować w pliku w formacie DOC notki biograficzne autorów o objętości nie przekraczającej 450 znaków, zawierające podstawowe dane charakteryzujące działalność naukową, tytuły naukowe i zawodowe, miejsce pracy i zajmowane stanowiska, informacje o uprawianej dziedzinie, adres e-mail oraz aktualne zdjęcie autora o rozmiarze 3,8 x 2,7 cm zapisane w skali odcieni szarości lub dołączone w osobnym pliku (w formacie TIF).

Wszystkie materiały:
- artykuł (w formacie DOC),
- notki biograficzne autorów (w formacie DOC),
- zdjęcia i rysunki (w formacie TIF lub CDR),

prosimy przesyłać w formie plików oraz dodatkowo jako wydruki na białym papierze (lub w formacie PDF) na adres e-mail: wydawnictwo@pak.info.pl lub pocztą zwykłą, na adres: Redakcja Czasopisma Pomiary Automatyka Kontrola, Asystent Redaktora Naczelnego mgr Agnieszka Skórkowska, ul. Akademicka 10, p.21A, 44-100 Gliwice.

Wszystkie artykuły naukowe są dopuszczane do publikacji w czasopiśmie PAK po otrzymaniu pozytywnej recenzji. Autorzy materiałów nadesłanych do publikacji są odpowiedzialni za przestrzeganie prawa autorskiego. Zarówno treść pracy, jak i wykorzystane w niej ilustracje oraz tabele powinny stanowić dorobek własny Autora lub muszą być opisane zgodnie z zasadami cytowania, z powołaniem się na źródło cytatu.

Przedrukowywanie materiałów lub ich fragmentów wymaga pisemnej zgody redakcji. Redakcja ma prawo do korzystania z utworu, rozporządzania nim i udostępniania dowolną techniką, w tym też elektroniczną oraz ma prawo do rozpowszechniania go dowolnymi kanałami dystrybucyjnymi.