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Research paper

Analysis of Young's modulus for Carboniferous sedimentary rocks and its relationship with uniaxial compressive strength using different methods of modulus determination

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ABSTRACT

Young's modulus (*E*) is one of the basic geomechanical parameters used in rock engineering in practice. It is determined based on uniaxial compressive test (*UCS*). However, according to International Society of Rock Mechanics it can be calculated by three different ways: as the tangent, secant and average modulus. The results from each method are significantly different. The *UCS* tests was carried out on 237 rock specimens with the slenderness ratio 2 of Carboniferous claystones, mudstones and sandstones. The axial deformation was always measured automatically by the displacement measurement device (LVDT) built into the testing machine and connected to the hydraulic piston. Then the Young's modulus was calculated for each test by all three methods. The analysis of the results is presented in this paper to show the difference between all the three moduli calculated for each specimen, and to recommend the best method of Young's modulus determination. First, the typical range of the elastic linear deformability for the chosen rock types was determined as 25–75% of the peak strength at confidence interval 95% for these sedimentary rocks. The modulus value distributions obtained from each calculation method were compared using statistical parameters: mean value, median, minimum, maximum, standard deviation, mean difference at confidence interval 95%, and non-uniformity coefficient. The proportions between average-secant modulus (E_{av}/E_{sec}) and average-tangent modulus (E_{av}/E_{tan}) for the rock samples were estimated. For the studied rocks the obtained values were: 1.10–1.32% for E_{av}/E_{sec}, 1.08–1.25% for E_{tan}/E_{sec} and 1.01–1.06 for E_{av}/E_{tan} (for E_{tan} with the range of 20–80% of peak strength). These values show low coherence between secant and average modulus (ca. 23% difference) and good consistency of average and tangent modulus. Based on the analysis, tangent Young's modulus is recommended as the guiding one at the constant range of 30–70% of the ultimate stress. Secant Young's modulus, as it comprises not only elastic strain but the pore compaction as well, should be named as modulus of deformability. This conclusion was further confirmed by the regression analysis between *UCS* and *E*. The highest regression coefficients and the lowest standard error of the regression was obtained for tangent Young's modulus determination method. In addition, modulus ratio *MR* for claystones, mudstones and sandstones was studied and determined as 274, 232 and 223 respectively.

1. Introduction

Engineering rock mechanics is applied in mining engineering practice to describe rock mass behaviour due to mining activity. The response of rocks and rock masses to changes in the stress fields affected by mining is a key issue for the evaluation of roadway stability and support design. The stress and strain distribution around a tunnel or mining openings strictly depends on the rock mass properties. The parameters that describe the elastic properties prior to rock failure are: Young's modulus, shear modulus, bulk modulus and Poisson's ratio. Since the shear modulus and bulk modulus are functions of Young's modulus and Poisson's ratio the last two parameters play the most important role in solving geomechanical problems. As rock is elastic-brittle material, some authors stress that the elastic modulus has become a critical parameter for describing rocks behaviour under loading (Bieniawski, 1989; Brady & Brown, 2006; Hoek & Brown, 1980; Santi, Holschen, & Stephenson, 2000). Young's modulus of intact rock is used by all numerical models for stress and deformation analyses such as FLAC, PHASE or UDEC for rock engineering problems (Ulusay, 2016). Young's modulus is also a basis to derive the deformation modulus of

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blocky and jointed rock masse which is non-elastic (Hoek & Diederichs, 2006; Karaman, Cihangir, & Kesimal, 2015).

The International Society of Rock Mechanics suggests three methods for determining deformation rock properties in uniaxial compression (Ulusay & Hudson, 2007). Three different values of the same parameters are then obtained based on the interpretation methods. This can be confusing for the designers and lead to wrong conclusions and wrong decisions in mining and rock engineering.

It is worth noting that Palmström and Singh (2001) also described three methods of *in situ* deformation modulus (E_m) measurements of rock masses. However, they clearly underlined the uncertainties of deformation measurements in the field and provided some empirical factors for the best rock mass characterization. This approach does not apply to laboratory research on small samples, where the results are, as expected, unequivocal.

In this research, 237 uniaxial compression tests were carried out to examine the methods of Young's modulus determination and to analyse the values obtained. The tests were carried out on Carboniferous claystones, mudstones and sandstones. The range of elasticity and modulus ratio (*MR*) for each rock type have been investigated and the relationship between Young's modulus and compressive strength was determined using regression analysis. The best method for interpreting Young's modulus from laboratory *UCS* tests has been recommended.

2. The elastic behaviour of rocks and suggested methods for determining their deformability

When dealing with the mechanical behaviour of solids the assumption is that they are homogeneous, continuous and isotropic, but rocks are much more complex and their mechanical properties vary according to scale, mineral composition or matrix type (Bell & Lindsay, 1999; Malik & Rashid, 1997; Roshan et al., 2018; Sabatakakis, Koukis, Tsiambaos, & Papanakli, 2008; Ulusay, 2016). However, for practical applications, engineers need definite values of rock property parameters for each rock type. The only way to obtain these values is through broad laboratory and field investigation.

The laboratory uniaxial compression test, triaxial compression test as well as ultrasonic or acoustic emission investigations can be used to evaluate the strength parameters of rocks (Madhubabu et al., 2016; Yu, Ji, & Li, 2016). In the test process, elastic rock behaviour can be well observed and measured up to the point when cracks start propagating and change the way of deformation into quasi-plastic and reaching. Generally, four phases of rock deformation before it fails (prefailure) and one phase beyond the peak strength (post-failure – Fig. 1) can be identified. The phases are defined as follows:

- *Compaction phase (I)* – the pre-existing cracks and joints, and inter-grain pores close under rising load. The stress-strain curve can be linear or non-linear because of the primary micro cracks' density and their geometry;

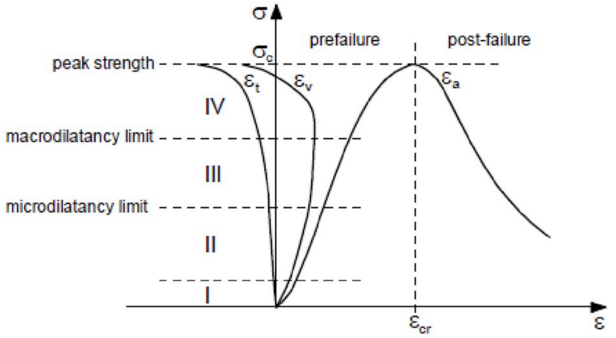

Fig. 1. Typical stress-strain chart for rocks, ε_a – axial deformation, ε_t – transverse deformation, ε_v – volumetric deformation.

- *Linear elastic deformation phase (II)* – elastic deformation predominates in this phase, but some non-linear behaviour is possible. The stress-strain curve is linear;
- *Stable fracturing phase (III)* – the start of this phase is the microdilatancy limit when the separation of cracks and their propagation in the directions parallel to the main compressive stress direction starts. The stress-strain curves for volumetric and transversal deformations stop being linear. Acoustic emission grows;
- *Unstable fracturing phase (IV)* – crossing the macro-dilatancy limit the crack opening mode starts, then the crack sliding mode initiates and their unstable propagation. Through the increase and joining of the cracks the shear surface forms. The opening of cracks causes rock volume to increase quickly. All the stress-strain curves are non-linear and a sharp rise in acoustic emission is observed. The phase ends when the stress reaches peak strength;
- *Rock degradation phase (V)* – macro shear surfaces form and then slip failure occurs.

The range and the extent of all the above phases depend on the type of rock. Rock type is determined by mineral composition and rock structure and texture (Bell & Lindsay, 1999; Roshan, Masoumi, & Hagan, 2016; Rybacki, Reinicke, Meier, Makasi, & Dresen, 2015; Sabatakakis et al., 2008). For example, igneous rocks with low porosity, formed in high pressure and temperature, show no compaction phase (I) with a long linear stress-strain curve. Sedimentary rocks formed in different conditions, usually by the deposition of the weathered remains of other rocks or by the accumulation and the consolidation of sediments, can reveal very short elastic behaviour during their loading. Taking into consideration that sedimentary rocks, such as sandstone, claystone or shale, cover a substantial part of the Earth's surface, knowledge of their behaviour during loading is very important.

The stress-strain curve for axial deformations is the best way to test rock elasticity and to determine the elastic modulus (Young's modulus). The International Society of Rock Mechanics suggests three standard methods for its determination. They are as follows (Brady & Brown, 2006; Ulusay & Hudson, 2007):

- Tangent Young's modulus E_{tan} – at fixed percentage of ultimate stress. This is defined as the slope of a line tangent to the stress-strain curve at a fixed percentage of the ultimate strength (Fig. 2a);
- Average Young's modulus E_{av} – of the straight-line part of a curve. The elastic modulus is defined as the slope of the straight-line part of the stress-strain curve for the given test (Fig. 2b);
- Secant Young's modulus E_{sec} – at a fixed percentage of ultimate stress. It is defined as the slope of the line from the origin (usually point (0; 0)) to some fixed percentage of ultimate strength, usually 50% (Fig. 2c).

There are other methods which can be used for Young's modulus determination, but they are not widely applied (Santi et al., 2000).

The range of investigation for tangent and secant modulus is chosen by a researcher. The first limit is the end of phase (I) – pores compressibility. Micro- and macro-dilatancy limits can be used for deciding on the second limit. They correspond to the limits of the elastic state of the rock (Pinińska, 2004). They are also called crack damage stresses (Brady & Brown, 2006; Palchik & Hatzor, 2002) and defined as the stress at the onset of dilation. When the crack damage stress is reached, the volume begins to increase. This information, together with deformation, strength and acoustic parameters can be essential, e.g. in gas and oil research drilling, shale gas research projects or in mining project decisions and should be updated and modified as research progresses (Pinińska, 2004).

Young's modulus of selected sedimentary rock types was tested in this study. From the perspective of mechanical properties, they are the most unpredictable rocks in rock engineering practice.

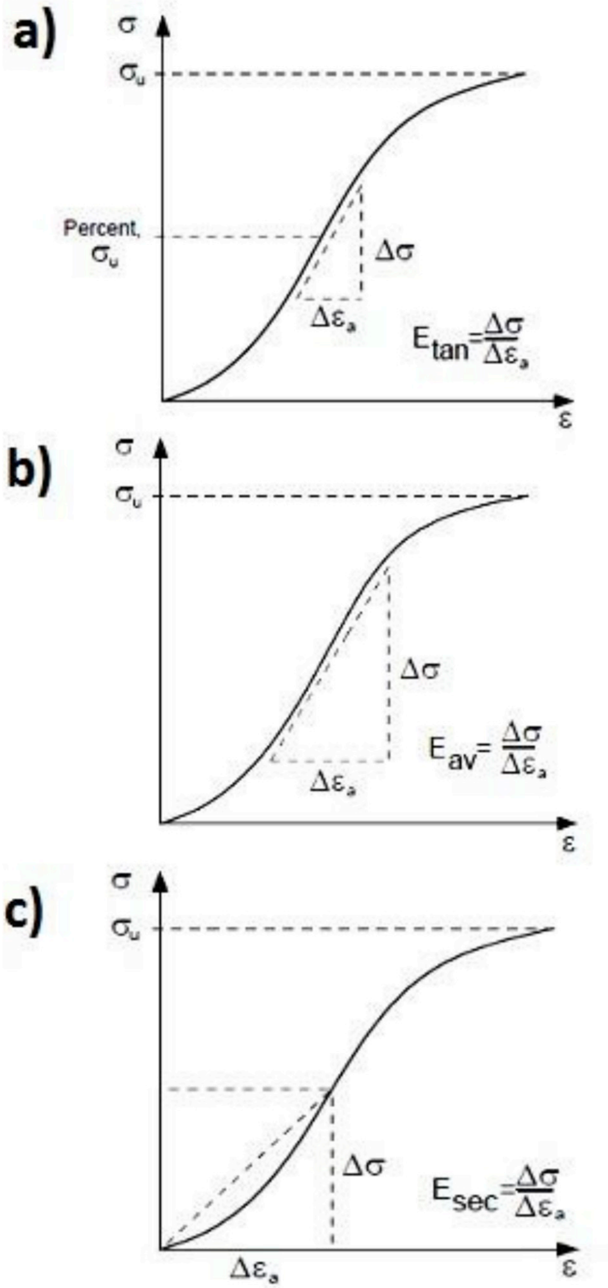

Fig. 2. a) Tangent Young's modulus E_{tan}, **b**) Average Young's modulus E_{av}, **c**) Secant Young's modulus E_{sec}.

3. Laboratory tests on rocks

Three modulus determination methods were used on three rock types to evaluate the coherence of the results. All together 237 compression tests were run: 86 on sandstones, 70 on mudstones and 81 on claystones. All three types of rock were Carboniferous from the Upper Silesia Basin in Poland, lying next to the hard coal beds of the 400, 500 and 600 group, at a depth of 700–1000 m. These rocks are typical geological formations extending from Ukraine, Poland, Czechia to Germany. The sandstone studied had a fine-grained structure, occasionally medium-grained, sometimes with silt or coal lamina (Fig. 3). The mudstone was strong and contained variable amounts of quartz minerals. The grey or light-grey claystone often contained some quartz, and infrequently contained carbonized plant material. Some of the samples showed shale structures (Fig. 3).

Fig. 3. The sandstone specimen with coal lamina (cylindrical) and the claystone specimen with quartz laminae (cuboid).

Fig. 4. The compressive strength test on claystone on Walter + Bai servo-operated press.

The tests were carried out on a servo-operated press made by Walter-Bai (Fig. 4) with a load rate of 0.3–1.5 kN/s depending on the sample strength (it was adjusted after the first test). The above rate related to a strain rate of 10^{-3}–10^{-4}/s. Load on the specimen was applied continuously up to its failure. Axial strain was recorded continuously. The samples were tested in air-dry state.

Since the method of displacement measurement influences the results (Korinets & Alehossein, 2002; Masoumi, Bahaanddini, Kim, & Hagan, 2014), during all the tests axial deformation was measured in the same manner – automatically using the displacement measurement device (LVDT) built into the machine and connected to the hydraulic piston.

The samples were cylindrical or cubical (cuboid) with the height to diameter (width) ratio of H/D = 2.0, which is slightly below the ratio range 2.5–3.0 recommended by ISRM (Ulusay & Hudson, 2007). This effect was due to the available borehole cores being heavily fractured along natural planes (Fig. 5). The different height to diameter ratio may be a cause of doubt as to whether test standards were upheld. Meng, Zhang, Han, Pu, and Li (2016) underlines the different behaviour of samples if the H/D ratio changes from 1.0 to 2.0. According to his investigation, when the H/D ratio was less than 2.0, the rock samples exhibited an increased number of split surfaces along the axial direction rather than conical damage (Masoumi, Bahaanddini, Kim, & Hagan, 2014; Meng et al., 2016), and the ultimate stress of the rock specimens decreases as the H/D ratio increases. A conical shape, or less often a shear plane, of the specimens after damage was observed in the tests (Fig. 6).

Fig. 5. Typical view of a core log from the borehole.

Fig. 6. Typical conical shape of rock samples after their damage.

Many pieces of research show that rock type (Liang, Zhang, Li, & Xin, 2016; Malik & Rashid, 1997; Meng et al., 2016; Roshan et al., 2018; Rybacki et al., 2015; Tuncay & Hasancebi, 2009; Yoshinaka, Osada, Park, Sasaki, & Sasaki, 2008) and porosity (Griffiths, Heap, Xu, Chen, & Baud, 2017; Hasselman, 1963; Madhubabu et al., 2016; Roshan et al., 2018; Rybacki et al., 2015; Sabatakakis et al., 2008; Yu et al., 2016) are the key factors affecting the results of laboratory tests on rocks. Experience shows that the porosity of the selected rock types is rather invariable and within a 1–4% range for claystones and mudstones, and 2–6% for sandstones. These values were confirmed in several samples used for this project.

The samples were 35–65 mm in diameter. Their diameter depended on the core diameter – the size of the borehole drilled on site. Cube side length (specimens cut from the rock blocks) was always 50 mm. The change in specimen diameter is reported as the key factor affecting laboratory strength-strain test results (Hoek & Brown, 1980; Kidybiński, 1982; Yoshinaka et al., 2008 – large samples,; Darlington, Ranjith, & Chol, 2011; Roshan et al., 2016; Quiñones et al., 2017). Nonetheless, scholars have proven that compressive strength and Young's modulus of rock does not change significantly in such a small range of specimen diameter, as the diameters used in this research.

The *UCS* test results were analyzed using seven statistical parameters: mean value, median, minimum, maximum, standard deviation, mean difference at confidence interval 95% and non-uniformity coefficient. The last parameter shows how much extreme values from the average (equation (1)).

$$\nu = \frac{S_x}{x_m} \tag{1}$$

where:

S_x – standard deviation of x,
x_m – arithmetic mean of x.

3.1. Range of elasticity

The range of testing rock elasticity is not set by standards and should be adjusted for common rock. The limits for this range are individual for each rock sample. Although the micro-dilatancy limit is the first threshold for a rock of losing its elasticity, it is not until the macro-dilatancy limit is reached a non-linear deformation is shown, if one considers the longitudinal deformation – the easiest and the most common way of rock sample deformation measurement. Hence, the elastic range for this investigation was established between the stress at the end of phase (I) – the end of pore compaction – and the stress at the end of phase (III) – macro-dilatancy limit (Fig. 1).

The results of lower elastic limit investigation for the selected rock types are shown in Table 1 and in Fig. 7 and the results of the upper elastic limit investigation for these types are also shown in Table 1 and in Fig. 8. The distribution of both limits for all the rock types studied appears to be asymmetrical and normal. Claystone shows the most definite lower elastic limit which is in the range of 20–25% of ultimate stress σ_u – uniaxial compressive strength *UCS* (nearly 35% cases – Fig. 7), but it does not have a clear upper elastic limit (Fig. 8). Elastic limit distribution for the tested rocks are rather flat and close to the Poisson distribution, but the histograms (line courses) are usually irregular. The lower elastic limit varies mainly from 15 up to 35% of the ultimate stress for claystone, from 15 to 30% for mudstone and from 12 to 37% for sandstone, while the upper elastic limit varies from 60 up to 88% of ultimate stress for claystone and mudstone and from 75 up to 90% of ultimate stress for sandstone. It is worth noting that the full range of determined lower elastic limit for sandstone amounted to 9.4–46.5% σ_u, and the determined upper elastic limit amounted to 45.7–95.2%, which is higher than for the other tested rocks (Table 1). Generally, it can be concluded that the range of linear elasticity may vary considerably. It is interesting that all three rocks reveal similar mean values of lower elastic limits – ca. 25% of ultimate stress and quite similar mean upper elastic limits, as well as minimum and maximum values of these limits.

Table 1
Lower elastic limit (compaction limit) – the end of phase (I) and upper elastic limit (macro-dilatancy limit) – end of phase (III) for tested rocks.

Elastic limit type	Rock type	Mean, e_L, $\%\sigma_u$	Estimation error (α = 0.05), %	Median, e_L, $\%\sigma_u$	Minimum, $e_{L\ \min}$, $\%\sigma_u$	Maximum, $e_{L\ \max}$, $\%\sigma_u$	Standard dev. S_{eL}, $\%\sigma_u$	Non-uniformity coefficient ν_{eL}, %
Lower	Claystone	26.4	± 1.53	24.3	13.0	42.3	7.0	26.7
	Mudstone	24.4	± 1.64	24.0	10.8	46.4	7.0	28.7
	Sandstone	25.1	± 1.77	25.8	9.4	46.5	8.4	33.4
Upper	Claystone	72.4	± 2.30	72.9	50.1	90.3	10.6	14.6
	Mudstone	74.6	± 2.17	74.9	53.9	93.3	9.3	12.4
	Sandstone	77.0	± 2.22	78.7	45.7	95.2	10.5	13.7

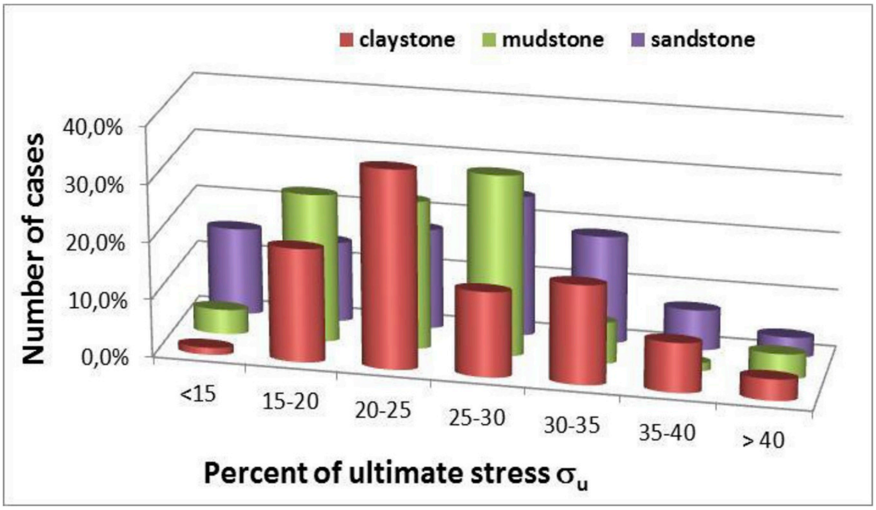

Fig. 7. Lower elastic limit distribution for the selected rock types.

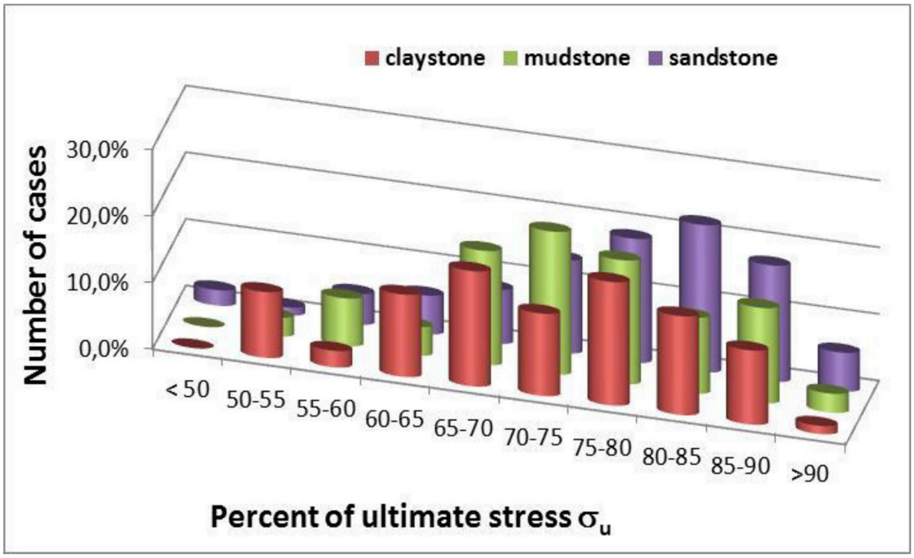

Fig. 8. Lower elastic limit distribution for the selected rock types.

The average limit values, mean and median, are closest to each other for mudstone and amount to 24.4% and 24.0% of σ_u and 74.6% and 74.9% of σ_u respectively. For claystone and sandstone these values are also not far from each other and vary by 0.5–2.1%.

It is important that the standard deviation of limits of elasticity for all tested sedimentary rocks is equal to ca. 7–8% of σ_u for the lower limit and 10% of σ_u for the upper limit. The non-uniformity coefficient also does not change considerably and amounts to 27–33% for the lower limit and 12–15% for the upper limit. The estimation error was calculated to be at a significance level of 0.05 for the calculated standard deviations for each rock type. The determined lower elastic limit for all rock types can change by ca. 1% of ultimate stress and the upper elastic limit by 2%. Hence, the results show that both determined limits can be used in rock engineering practice. However, the non-conformity coefficient proves that for sedimentary rocks the uncertainty of the lower elastic limit is nearly three times higher than the upper elastic limit. This characteristic has been observed for coal (sedimentary, but anisotropic rock), for which the lower and upper elastic limits are very similar, 22.4% and 74.3% of ultimate stress respectively (Małkowski & Ostrowski, 2017).

The ranges of lower and upper elastic limit for the investigated rock types are shown in Fig. 9.

3.2. Deformation modulus of the selected rock types

Three moduli were determined for rocks based on ISRM standards and under the assumption that the fixed percentage values of ultimate stress for tangent Young's modulus calculation were 20% and 80% of σ_u respectively. This range was based on the experience of AGH UST research and has been used so far by authors. However, the presented analysis proves that the primarily assumed range should be changed. Review of the references concerning Young's modulus test reveals that only a few authors provide the methodology of its determination, such as Martínez-Martínez, Benavente, and Garcia-del-Cura (2012) who calculated it as the slope of the straight line which links the origin of the stress-strain curve with the corresponding point at 70% of ultimate strength, or Bell and Lindsay (1999) and Gholami and Rasouli (2014) who determined it at 50% of the load at failure. It is found that in all the above references the authors used secant Young's modulus only. It is

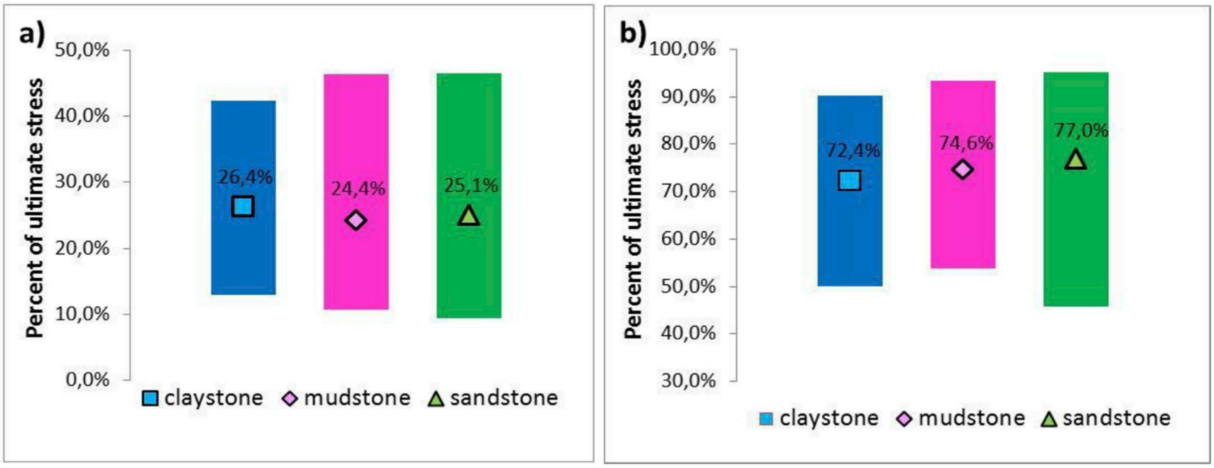

Fig. 9. The range of elastic limits for the selected rock types, a) Lower elastic limit, b) Upper elastic limit.

Table 2
Young's Modulus for chosen rocks.

Young's modulus type	Rock type	Mean μE_{tan}, GPa	Estimation error (α = 0.05), GPa	Median E_{tan}, GPa	Minimum E_{min}, GPa	Maximum E_{max}, GPa	Standard dev. S_E, GPa	Non-uniformity coefficient ν E_{av}, %
Tangent E_{tan}	Claystone	13.41	± 1.32 (± 9.8%)	13.90	1.87	33.21	6.067	45.2
	Mudstone	14.59	± 1.42 (± 9.7%)	15.11	2.72	32.45	6.074	41.6
	Sandstone	13.09	± 1.20 (± 9.2%)	13.21	2.39	29.56	5.687	43.4
Average E_{ave}	Claystone	13.44	± 1.31 (± 9.7%)	13.87	1.87	30.65	6.030	44.8
	Mudstone	15.06	± 1.44 (± 9.6%)	15.90	3.71	34.58	6.168	40.9
	Sandstone	13.75	± 1.30 (± 9.5%)	12.86	4.23	32.01	6.135	44.6
Secant E_{sec}	Claystone	13.35	± 1.40 (± 10.5%)	14.33	1.59	27.98	6.448	48.3
	Mudstone	13.14	± 1.49 (± 11.3%)	14.92	2.34	29.99	6.363	48.4
	Sandstone	10.91	± 1.11 (± 10.2%)	9.87	2.94	26.46	5.274	48.4

worth mentioning that Martinez-Martinez team research was performed on the basis of American Standards Testing Method – D 3148–96 from 1996 but originally published in 1972.

Additionally, only a few papers present recommendations regarding the investigation. From studying secant modulus determined from 25%, 50% and 75% of maximum stress based on the standards of the American Institute For Standards (Pells, 1993) suggested that Young's modulus should be determined at 50% of maximum strength. Investigation carried out by Santi et al. (2000) who took the short length of stress-strain curve next to 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80% and 90% of ultimate stress led to the conclusion that the best repeatability of the results appears for the analysis at 50% of maximum load.

There is an interesting study by Hsieh, Dyskin, and Dight (2014) who proved the increase of Young's modulus of rock under uniaxial compression if it is loaded several times. He claimed that the change in the tangent modulus under different stress levels is attributable to the combination of crack closure, sliding and dilatancy. This can lead to significant differences in tangent and secant modulus under different stress levels in the range of 15–95% of *UCS*. This aspect is rarely addressed when carrying out research for the determination of Young's modulus, and the types of determination method were rarely varied.

These examples confirm that the manner of calculation of Young's modulus is entirely arbitrary. There are numerous studies concerning the influence of fissures or voids (Martínez-Martínez et al., 2012), porosity and mineral assemblage (Griffiths et al., 2017; Hasselman, 1963; Madhubabu et al., 2016; Rybacki et al., 2015; Sabatakakis et al., 2008; Yu et al., 2016), water content and permeability on the elastic properties of rock (Bell & Lindsay, 1999; Gholami & Rasouli, 2014; Rybacki et al., 2015), as well as studies on weakness planes in rocks and their transversal anisotropy (Gholami & Rasouli, 2014) or the study of relationships between Young's modulus and other physical properties (Madhubabu et al., 2016; Ocak, 2008; Sabatakakis et al., 2008). However, these works do not elaborate on the methodology of Young's modulus determination.

The results of the studies carried out by the authors are presented in Table 2 and in Figs. 10–12. The charts presented show that all the distributions are normal and asymmetric. It appears that the secant Young's modulus distribution (green colour – Figs. 10–12) is more irregular and disturbed than the others. It is apparent that the average and tangent Young's modulus values are close to each other while secant Young's modulus distinctly differs in some ranges. As foreseen, the highest range of data concerning average Young's modulus reveals the highest standard deviation (Table 2). Average and tangent modulus are more stable parameters, with lower percentage deviations from the mean value and lower non-uniformity coefficients (Table 2) than for secant modulus. Comparison of Young's modulus estimation error at 0.05 significance level shows the highest values for secant modulus – over 10%. Hence, the average and tangent modulus should be recommended as the parameters which describe the rock mechanical elastic properties.

As the average Young's modulus is based on the full linear part of the stress-strain curve to describe the rock elasticity, the analysis of the difference between the modulus for consecutive samples was carried out by comparing the average modulus with the tangent and secant modulus. Figs. 13–15 present the proportion average-secant modulus

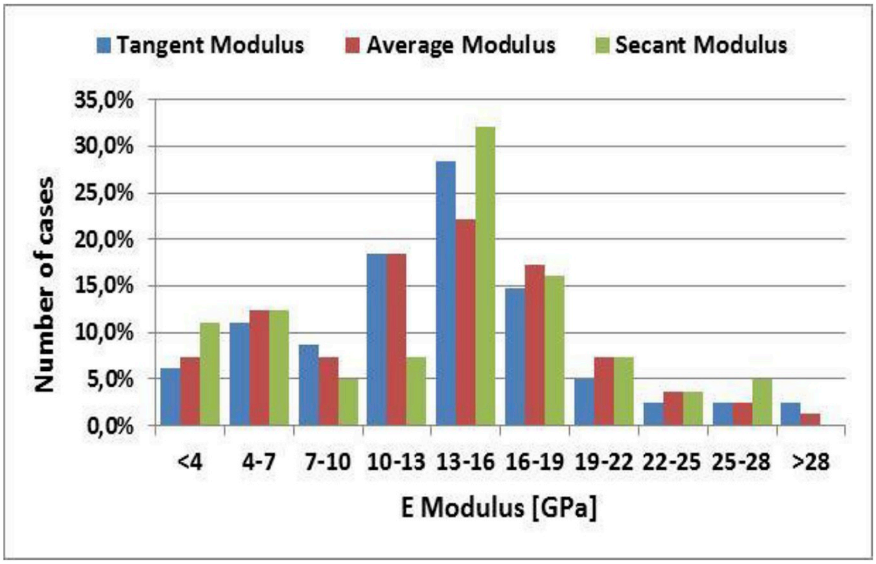

Fig. 10. Young's moduli for claystone.

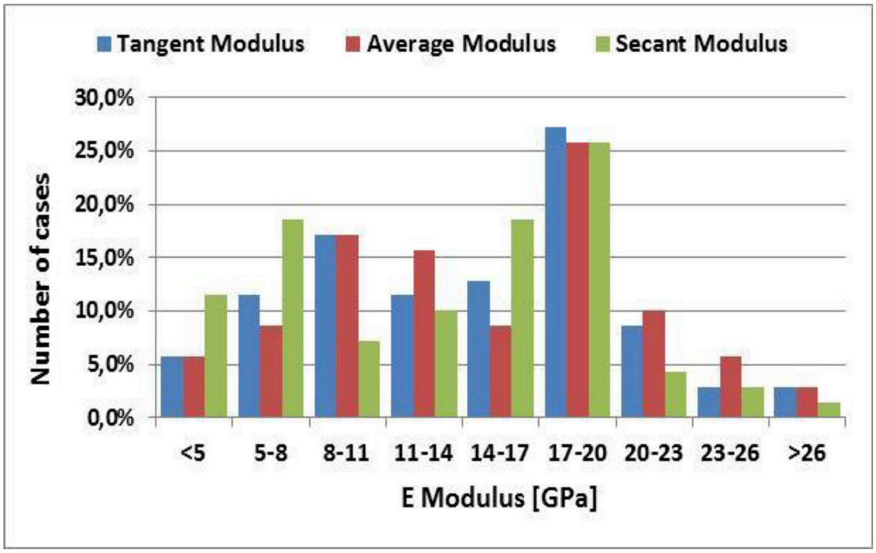

Fig. 11. Young's moduli for mudstone.

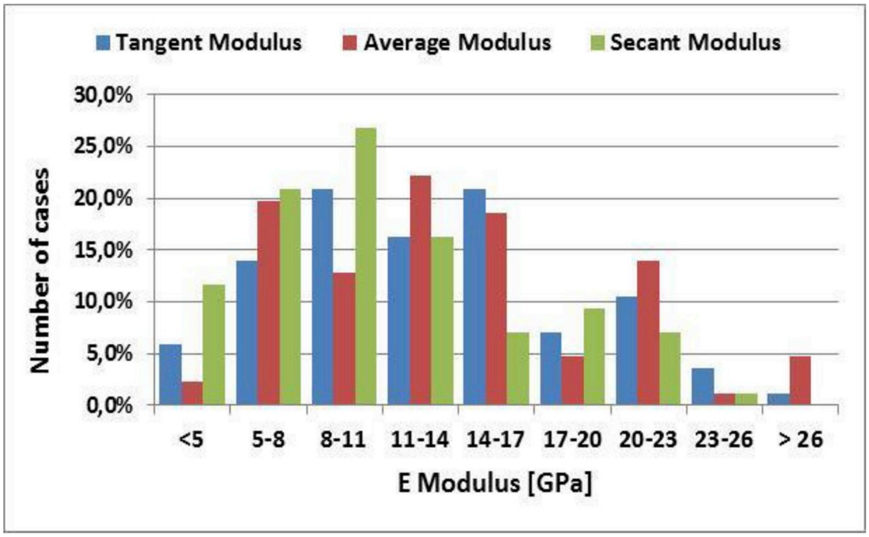

Fig. 12. Young's moduli for sandstone.

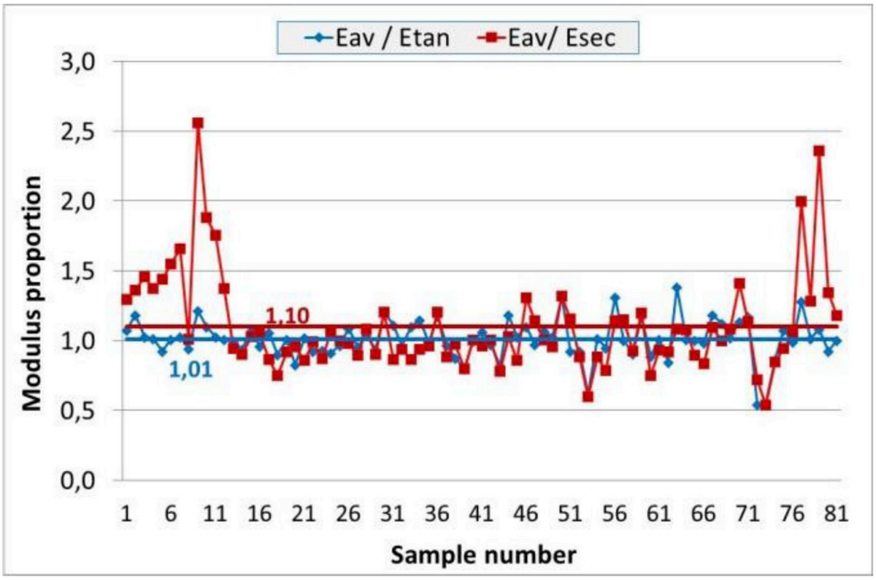

Fig. 13. Proportion average-tangent modulus and average-secant modulus for claystone samples.

(E_{av}/E_{sec}) and average-tangent modulus (E_{av}/E_{tan}) for the rock samples. For the tested rocks the proportion varies within the range of 1.10–1.32 for E_{av}/E_{sec} and 1.01–1.06 for E_{av}/E_{tan} (for E_{tan} with a range of 20–80% of peak strength). These values reveal low coherence between secant and average modulus (ca. 23% difference) and good consistency between the average and tangent modulus. Since detailed strain-stress curve analysis for rock samples can be troublesome in some cases (e.g. no data record), setting the fixed percentage of the ultimate strength for Young's modulus calculation is the most convenient method of its determination.

It is worth noting that the spread in calculated modulus for individual samples can be high. The highest difference between average and secant modulus appears for claystone and mudstone – up to 2.56 and 2.28 respectively (Figs. 13 and 14), while up to for sandstone 2.09 (Fig. 15). Generally, the proportion between the average and secant modulus for individual samples scatters more than proportion between average and tangent modulus and sometimes it is much less than one: for claystone – 0.54 (Fig. 13), for mudstone 0.66 (Fig. 14) and for sandstone 0.82 (Fig. 15). What is most significant is the difference between the mean values of the tangent and the average modulus is the smallest and amounts to 1% in the case of claystone and 5–6% for mudstone and sandstone, while for the mean values of secant and average Young's modulus it amounts to: 10%, 24% and 32% respectively for the tested rock types. These values show that Young's moduli may differ by up to a third depending on the derivation methodology.

3.3. Relationship between uniaxial compressive strength and average, secant and tangent modulus

3.3.1. Regression analysis

Modulus ratio *MR* (or linear correlation) between uniaxial compressive strength and Young's modulus can also be a factor which

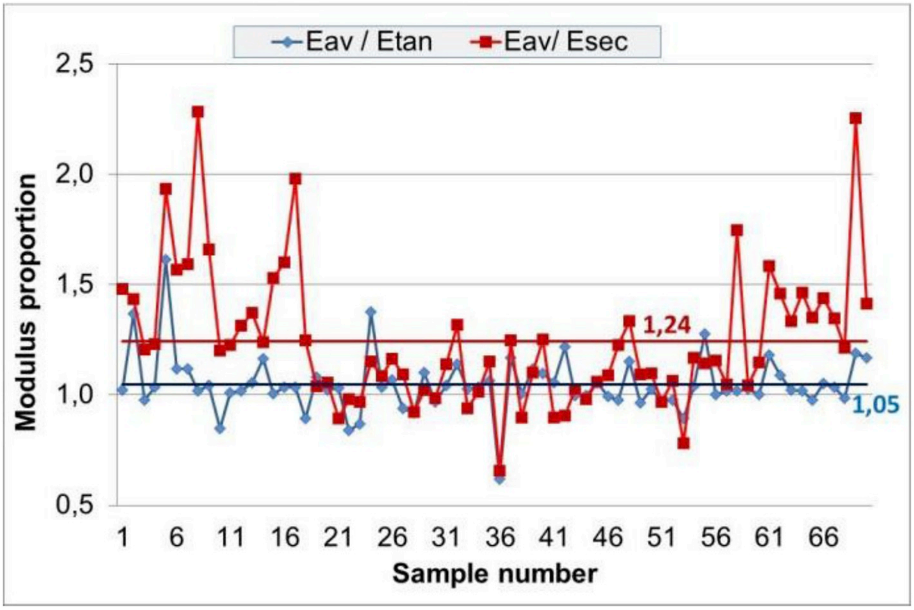

Fig. 14. Proportion average-tangent modulus and average-secant modulus for mudstone samples.

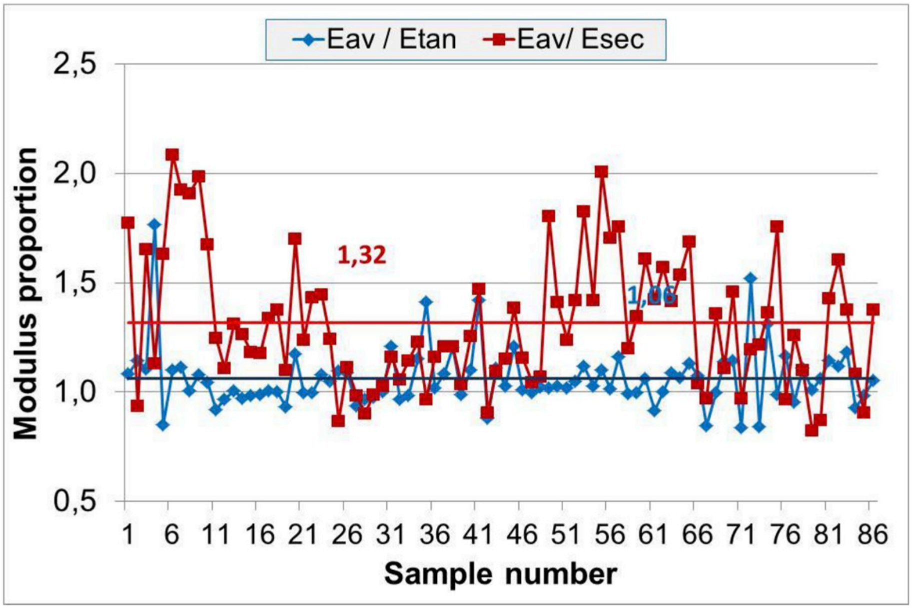

Fig. 15. Proportion average-tangent modulus and average-secant modulus for sandstone samples.

Table 3
Uniaxial compressive strength UCS.

Rock type	Average *UCS*, MPa	Difference at 95% *CI*, MPa	Median *UCS*, MPa	Minimum UCS_{min}, MPa	Maximum UCS_{max}, MPa	Standard dev. S_{UCS} MPa	Non-uniformity coefficient ν_{UCS}, %
Claystone	50.51	± 4.96	49.94	14.9	133.7	22.8	45.1
Mudstone	65.33	± 6.76	62.50	20.0	174.6	28.9	44.2
Sandstone	59.84	± 5.98	51.51	30.2	195.8	28.3	47.3

verifies Young's modulus determination methodology (Ocak, 2008). High regression coefficients and low standard error of the regression between *UCS* and *E* will point to the best Young's modulus determination method.

Analysis of uniaxial compressive strength *UCS* for the chosen rocks shows rather high standard deviation and non-uniformity coefficient (Table 3) and quite similar mean values which is surprising. The range of maximum stress that causes the collapse of the sample is very wide for all the rock types, which is typical for sedimentary rocks.

The regression analysis was performed for all three rock types for three variants of relationships between *UCS* and *E* for every rock, with an assumed significance level α of 0.05. Results of the investigation are presented in Table 4 and in Figs. 16–18. The UCS relationship with different Young's modulus gives correlation equations which differ significantly for the same type of rock. It is clearly visible that the coefficient of determination is always highest for the $UCS\text{-}E_{tan}$ relationship (Fig. 16) and the lowest for $UCS\text{-}E_{sec}$ analysis (Fig. 18). The best fit to the data described by the coefficient of determination suggests that tangent Young's modulus should be used for such a comparison (Table 4). It should be noted that 20–80% of the ultimate stress range was used in this case. Simultaneously, the standard error of the estimate that represents the average distance that the observed values fall from the regression line is also lowest for $UCS\text{-}E_{tan}$ analysis. It is worth underlining that the coefficient of correlation *R* amounting to ca. 0.7–0.8 for the all analyses, taking into account sedimentary rock characteristics, could be satisfactory. The highest values for sandstones suggest that this rock type is statistically the most homogenous among the tested rock types.

3.3.2. Modulus ratio MR

The most common parameter used in engineering practice for the fast and easy evaluation of Young's modulus is the modulus ratio *MR* (Hoek & Diederichs, 2006). This is the proportion between Young's modulus *E* and uniaxial compressive strength *UCS*. Unfortunately, researchers do not provide the procedure for the *E* calculation. The cited correlations are between compressive strength and Young's modulus, determined by one of the three methods.

The values of Young's modulus for claystone, mudstone and sandstone determined on the basis of three different calculations were juxtaposed with the results of other investigations on the same rock types (Table 5). Table 5 includes results from five references and the most well-known *MR* range gathered by Hoek and Diederichs (2006), who used results of Palmström and Singh (2001). Siltstone rock, which is mineralogically the most similar to mudstone, was also included in the table.

The review of modulus ratio values given in the papers shows that they vary a great deal and beyond the recommended range suggested by Hoek and Diederichs (2006). Moreover, the *MR* calculated in this research generally does not suit the recommended values as well. Again, the ratio *UCS* and secant modulus give other values than the

Table 4

Results of regression analysis for *UCS*-E_{tan}, *UCS*-E_{av} and *UCS*-E_{sec} (*UCS* in *MPa*, *E* in GPa).

Regression analysis	Rock type	Equation	Coefficient of correlation *R*	Coefficient of determination *R*-squared	Standard *error* of the estimate *SSE*
UCS-E_{tan}	Claystone	$E = 0.198\ UCS + 3.392$	0.744	0.555	15.29
	Mudstone	$E = 0.165\ USC + 3.829$	0.783	0.613	19.09
	Sandstone	$E = 0.170\ UCS + 2.907$	0.847	0.718	15.12
UCS-E_{ave}	Claystone	$E = 0.188\ UCS + 3.938$	0.711	0.505	16.12
	Mudstone	$E = 0.162\ UCS + 4.476$	0.758	0.575	18.95
	Sandstone	$E = 0.175\ UCS + 3.308$	0.805	0.648	16.89
UCS-E_{sec}	Claystone	$E = 0.199\ UCS + 3.289$	0.704	0.495	16.29
	Mudstone	$E = 0.158\ UCS + 2.815$	0.717	0.514	20.28
	Sandstone	$E = 0.149\ UCS$ -1.959	0.802	0.644	16.99

quotient of compressive strength and tangent and average modulus. The range of *MR* for clastic rocks is wider than the range presented in other papers; notably the same conclusion is reached by Palchik (2011) in relation to carbonate rocks. It is significant that the investigations were carried out on different lithological rock types of considerably different mechanical properties (Malik & Rashid, 1997; Palchik & Hatzor, 2002; Sabatakakis et al., 2008), hence the results are barely comparable. This analysis cannot confirm unequivocally by which calculus the Young's modulus *E* should be determined from *UCS*. However, assuming that Young's modulus should present the elastic behaviour of rocks, and in the view of the obtained results (Table 5), the average and tangent Young's modulus should be considered only (Fig. 19). Therefore, the following modulus ratios for the selected rock types can be suggested (their variability is given in Table 5):

- Carboniferous claystone – *MR* = 274,
- Carboniferous mudstone – *MR* = 232,
- Carboniferous sandstone – *MR* = 222.

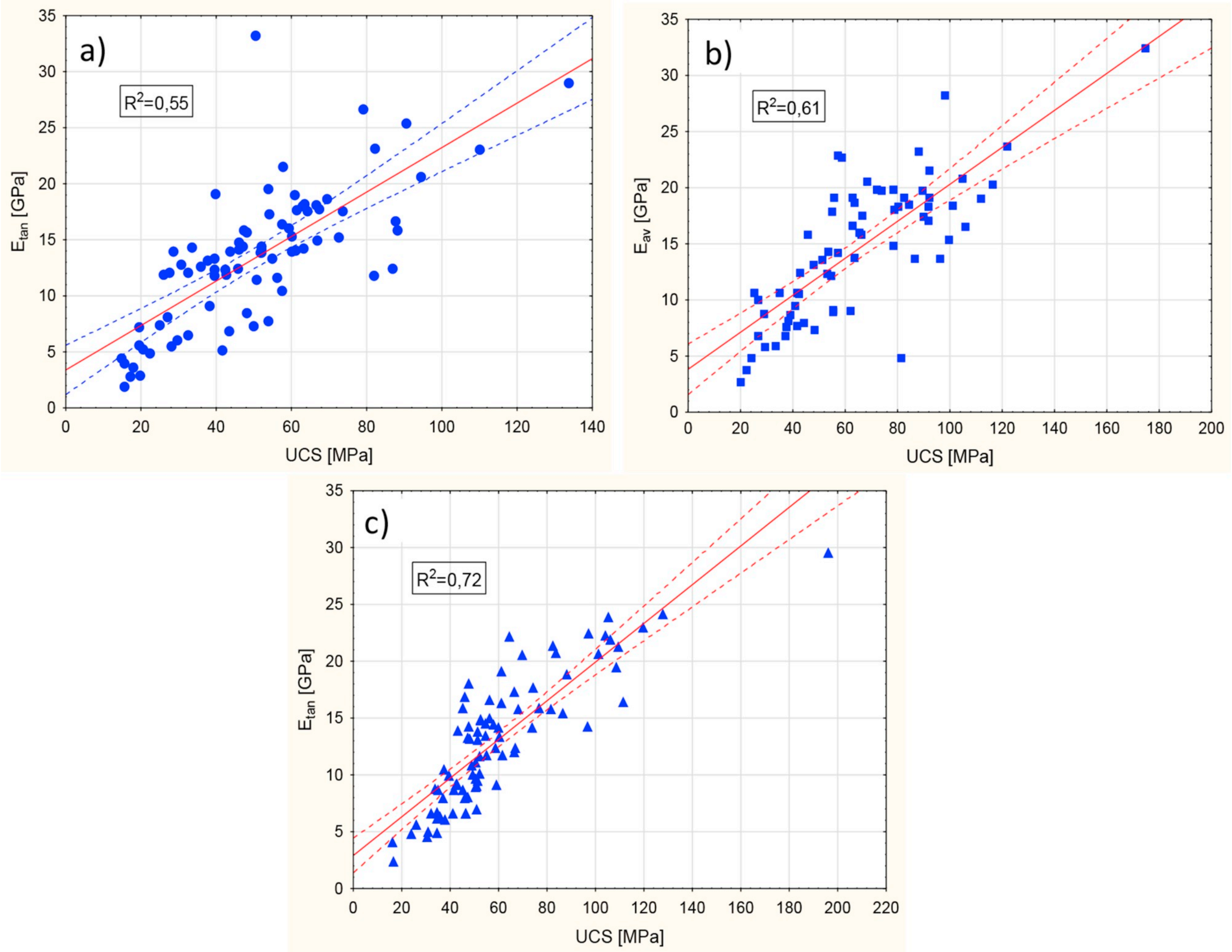

Fig. 16. Correlation (α = 0.05): Tangent Young's modulus – uniaxial compressive strength for: a) claystone, b) mudstone, c) sandstone.

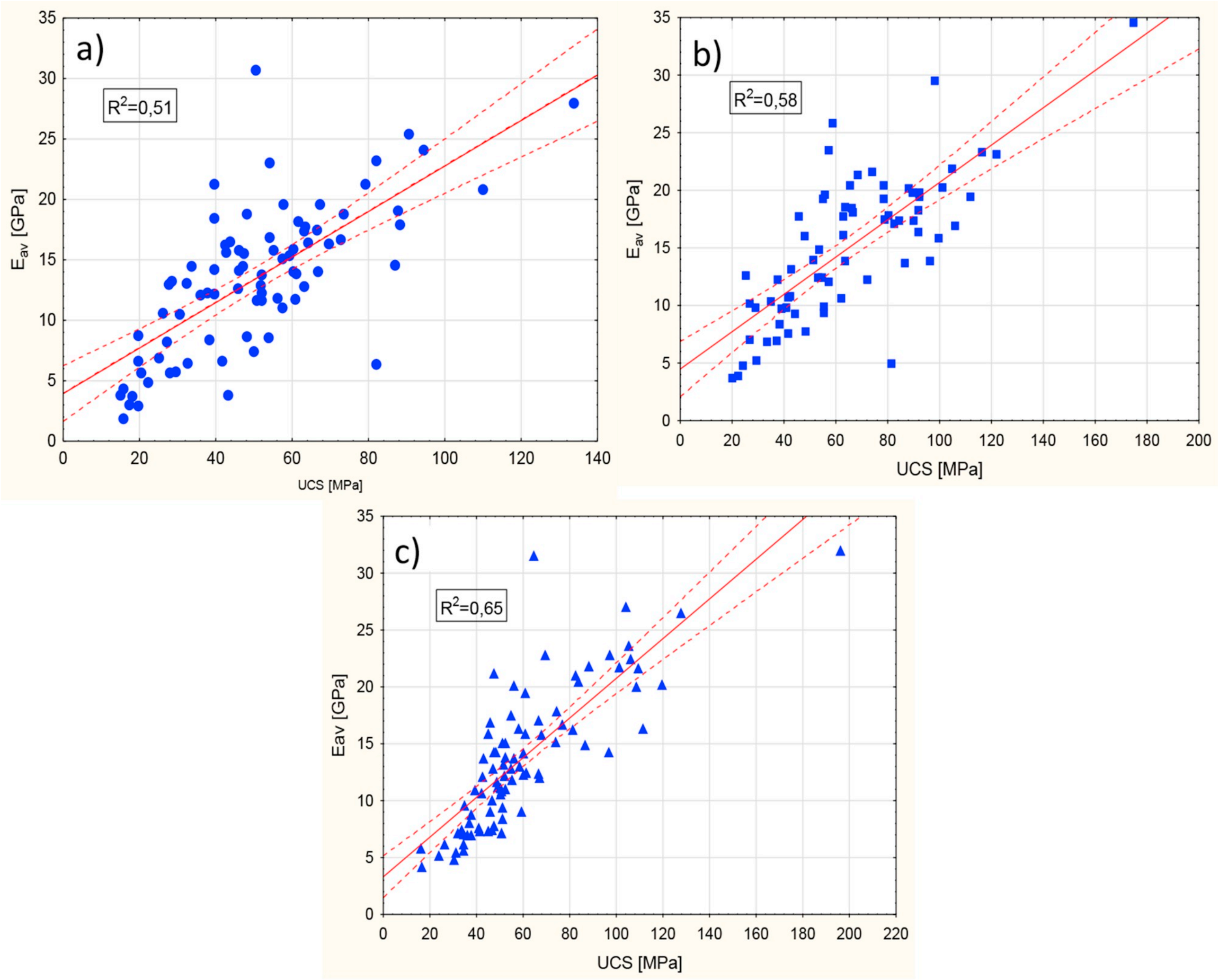

Fig. 17. Correlation (α = 0.05): Average Young's modulus – uniaxial compressive strength for: a) claystone, b) mudstone, c) sandstone.

Attention should also be paid to the fact that there are not so many tests results on the *MR-UCS* relationship available in the literature. One of the best analysis of Young's modulus and uniaxial compressive strength was carried out by Wang and Aladejare (2016), who proved the high variation of *UCS-E* correlation, yet on the other hand they did not explain the method of Young's modulus determination.

4. Discussion

The IRSM recommends three methods of determination of Young's modulus from the *UCS* test: tangent, average and secant, and the results significantly differ. Therefore, the selection of a particular method needs to be substantiated to suit the individual engineering problem. An investigation carried out on three sedimentary rocks: claystones, mudstones and sandstones shows that Young's modulus value in the case of average and secant methods can differ by 1.10–1.32%, and in the case of tangent and secant methods by 1.08–1.25%. The results obtained from the average and tangent methods are the closest to each other (1–6% of difference), yet it depends on the range of stress-strain curve used for the analysis. A range corresponding to 20–80% of maximum stress was applied in this research based on the AGH UST Department of Geomechanics, Civil Engineering and Geotechnics earlier recommendations. However, the study of the elastic behaviour of the selected rock types rocks proves that the range 25–75% of σ_u (*UCS*), or even narrowed to 30–70% of σ_u is more appropriate. The *UCS* tests were carried out on rocks samples collected from various sites and they were therefore structurally different.

The regression analysis shows that by applying a linear equation for the *UCS* variable, Young's modulus can be calculated with a very high coefficient of correlation 0.7–0.8. However it is further noted that the highest value of this coefficient is achieved for tangent Young's modulus, so this modulus gives the best relationship with the uniaxial compressive strength. This conclusion is also confirmed by the modulus ratio analysis where the range of *MR* variability is the lowest for the relationship between *UCS* and tangent Young's modulus.

It is important that the axial deformation was always measured automatically by the displacement measurement device (LVDT) built into the testing machine and connected to the hydraulic piston.

By definition, Young's modulus describes the elastic properties of rock, therefore it should be inferred from the straight-line part of the stress-strain curve of the *UCS* test (*E* average). As this aspect usually causes a problem, the authors suggest the part of the curve corresponding to the fixed range 30–70% of the peak strength for the calculation of the tangent Young's modulus. The conducted analysis

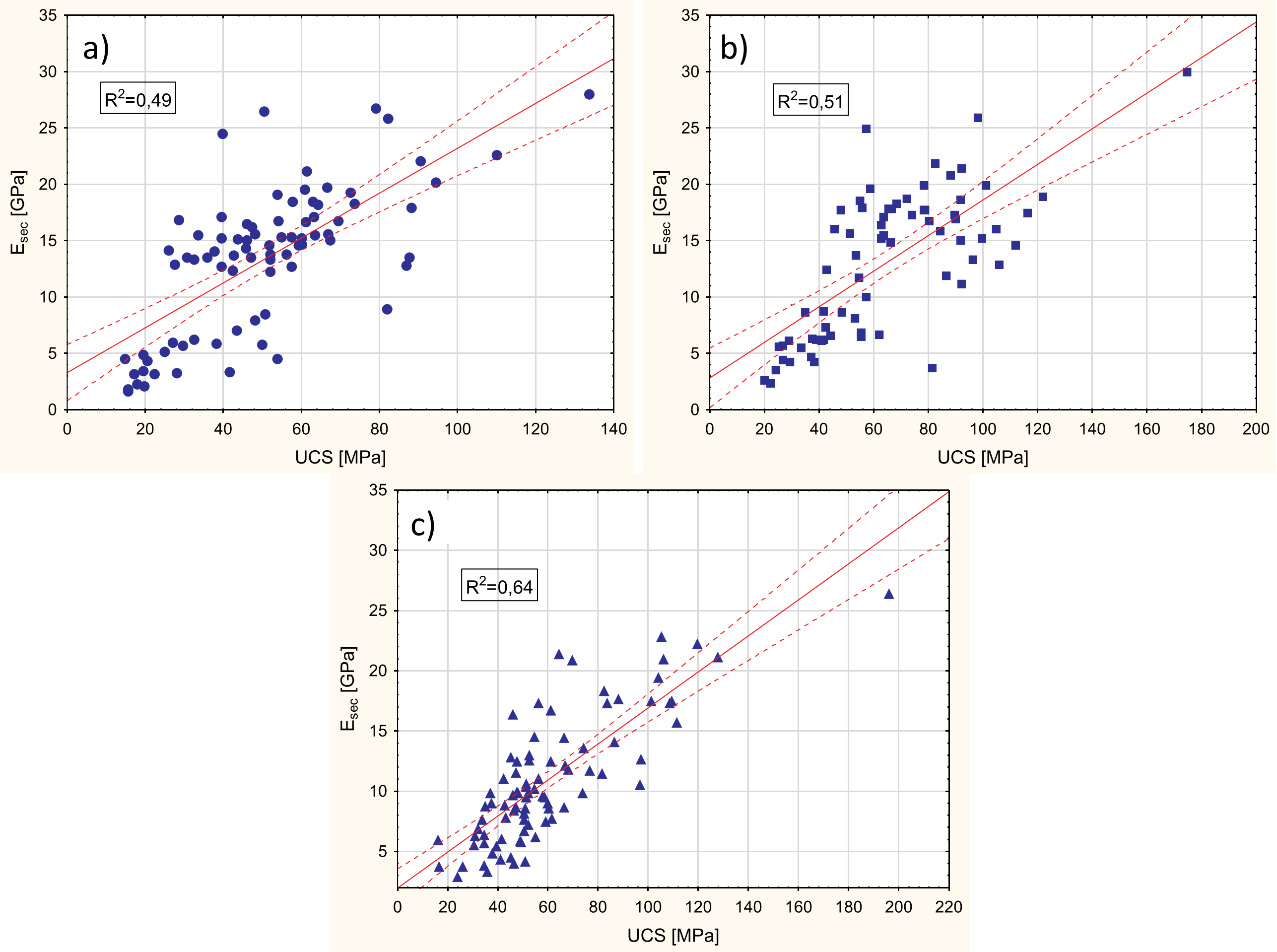

Fig. 18. Correlation (α = 0.05): Secant Young's modulus – uniaxial compressive strength for: a) claystone, b) mudstone, c) sandstone.

Table 5
Modulus ratio *MR* (*E* and *UCS* in MPa).

Rock type	Reference	Number of samples	*MR* (range)
Claystone	Hoek & Diederichs, 2006	nda	(200–300)
	Malik & Rashid, 1997	30	141 (87–228)
	Małkowski & Ostrowski, 2017 (Carboniferous)	**81**	**E_{tan} 274 (118–657)**
			E_{av} 276 (77–606)
			E_{sec} 269 (79–616)
Siltstone	Hoek & Diederichs, 2006	nda	(350–400)
	Malik & Rashid, 1997	30	137 (79–190)
Mudstone	Małkowski & Ostrowski, 2017 (Carboniferous)	**70**	**E_{tan} 232 (59–421)**
			E_{av} 242 (61–500)
			E_{sec} 203 (45–436)
Sandstone	Hoek & Diederichs, 2006	nda	(200–350)
	Bell & Lindsay, 1999	27	372 (141–680)
	Malik & Rashid, 1997	30	119 (76–157)
	Sabatakakis et al., 2008	36	303 (120–727)
	Małkowski & Ostrowski, 2017 (Carboniferous)	**86**	**E_{tan} 223 (139–381)**
			E_{av} 236 (141–491)
			E_{sec} 187 (82–379)

proves, with a confidence level of 95%, that the sedimentary Carboniferous rocks deform in an elastic manner within the suggested range. Additionally, this way of modulus determination is simpler and does not require the full record of stress-strain characteristic.

Secant Young's modulus, as it comprises not only elastic but also the first phase of rock deformation i.e. pore compaction, should rather be named a modulus of deformability instead of Young's modulus.

The risk analysis of the engineering case, which takes into

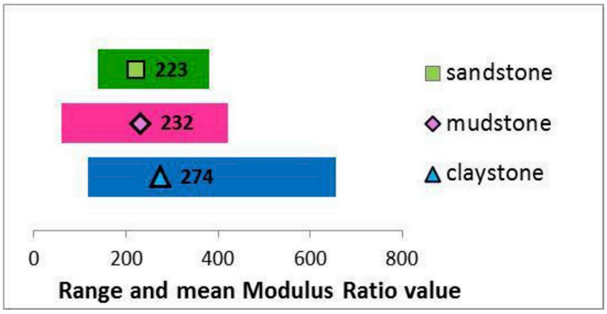

Fig. 19. Range and mean modulus ratio *MR* for the tested rock types.

consideration the variability of rock types as well as, for example, the availability of samples and tests, may suggest the more conservative use of secant Young's modulus. However, the Young's modulus defined to describe the elastic properties of materials needs to be determined on the elastic part of the rock characteristic.

Conflict of interest

None declared.

Ethical statement

Authors state that the research was conducted according to ethical standards

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References

Bell, F. G., & Lindsay, P. (1999). The petrographic and geomechanical properties of some sandstones from the Newspaper Member of the Natal Group near Durban, South Africa. *Engineering Geology, 53*(1), 57–81. https://doi.org/10.1016/S0013-7952(98)00081-7.

Bieniawski, Z. T. (1989). *Engineering rock mass classifications: A complete manual for engineers and geologists in mining, Civil and petroleum engineering.* Canada: John Wiley & Sons, Inc.

Brady, B. H. G., & Brown, E. T. (2006). *Rock mechanics. For underground mining* (3rd ed.). Dordrecht: Springer Netherlands. https://doi.org/10.1007/978-1-4020-2116-9.

Darlington, W. J., Ranjith, P. G., & Choi, S. K. (2011). The effect of specimen size on strength and other properties in laboratory testing of rock and rock-like cementitious brittle materials. *Rock Mechanics and Rock Engineering, 44*, 513–529. https://doi.org/10.1007/s00603-011-0161-6.

Gholami, R., & Rasouli, V. (2014). Mechanical and elastic properties of transversely isotropic slate. *Rock Mechanics and Rock Engineering, 47*(5), 1763–1773. https://doi.org/10.1007/s00603-013-0488-2.

Griffiths, L., Heap, M. J., Xu, T., Chen, Ch-f., & Baud, P. (2017). The influence of pore geometry and orientation on the strength and stiffness of porous rock. *Journal of Structural Geology, 96*, 149–160. https://doi.org/10.1016/j.jsg.2017.02.006.

Hasselman, D. P. H. (1963). Relation Young's between effects of porosity on strength and on modulus of elasticity of polycrystalline materials. *Journal of the American Ceramic Society, 46*(11), 564–565. https://doi.org/10.1111/j.1151-2916.1963.tb14615.x.

Hoek, E., & Brown, E. T. (1980). *Underground excavations in rock.* London: CRC Press.

Hoek, E., & Diederichs, M. S. (2006). Empirical estimation of rock mass modulus. *International Journal of Rock Mechanics and Mining Sciences, 43*(2), 203–215. https://doi.org/10.1016/j.ijrmms.2005.06.005.

Hsieh, A., Dyskin, A. V., & Dight, P. (2014). The increase in Young's modulus of rocks under uniaxial compression. *International Journal of Rock Mechanics and Mining Sciences, 70*, 425–434. https://doi.org/10.1016/j.ijrmms.2014.05.009.

Karaman, K., Cihangir, F., & Kesimal, A. (2015). A comparative assessment of rock mass deformation modulus. *International Journal of Mining Science and Technology, 25*(5), 735–740. https://doi.org/10.1016/j.ijmst.2015.07.006.

Kidybiński, A. (1982). *Podstawy geotechniki kopalnianej (The base of mining geotechnics).* Katowice: Wydawnictwo Śląsk.

Korinets, A., & Alehossein, H. (2002). On the initial non-linearity of compressive stress-strain curves for intact rock. *Rock Mechanics and Rock Engineering, 35*(4), 319–328. https://doi.org/10.1007/s00603-002-0030-4.

Liang, C. Y., Zhang, Q. B., Li, X., & Xin, P. (2016). The effect of specimen shape and strain rate on uniaxial compressive behavior of rock material. *Bulletin of Engineering Geology and the Environment, 75*(4), 1669–1681. https://doi.org/10.1007/s10064-015-0811-0.

Madhubabu, N., Singh, P. K., Kainthola, A., Mahanta, B., Tripathy, A., & Singh, T. N. (2016). Prediction of compressive strength and elastic modulus of carbonate rocks. *Measurement, 88*, 202–213. https://doi.org/10.1016/j.measurement.2016.03.050.

Malik, M. H., & Rashid, S. (1997). Correlation of some engineering geological properties of the Murree formation at lower Topa (Murree district), Pakistan. *Geological Bulletin of University Peshawar, 30*, 69–81.

Martínez-Martínez, J., Benavente, D., & García-del-Cura, M. A. (2012). Comparison of the static and dynamic elastic modulus in carbonate rocks. *Bulletin of Engineering Geology and the Environment, 71*(2), 263–268. https://doi.org/10.1007/s10064-011-0399-y.

Masoumi, H., Bahaaddini, M., Kim, G., & Hagan, P. (2014). Experimental investigation into the mechanical behavior of Gosford sandstone at different sizes. In *Proceedings of 48th U.S. Rock Mechanics/Geomechanics Symposium, 1–4 June, Minneapolis, Minnesota: Vol. 2*, (pp. 1210–1215). American Rock Mechanics Association.

Małkowski, P., & Ostrowski, Ł. (2017). The methodology for the Young modulus derivation for rocks and its value. *Proceedia Engineering, 191*, 134–141. https://doi.org/10.1016/j.proeng.2017.05.164.

Meng, Q., Zhang, M., Han, L., Pu, H., & Li, H. (2016). Effects of size and strain rate on the mechanical behaviors of rock specimens under uniaxial compression. *Arabian Journal of Geosciences, 9*, 527. https://doi.org/10.1007/s12517-016-2559-7.

Ocak, I. (2008). Estimating the modulus of elasticity of the rock material from compressive strength and unit weight. *The Journal of The Southern African Institute of Mining and Metallurgy, 108*, 621–626.

Palchik, V. (2011). On the ratios between elastic modulus and uniaxial compressive strength of heterogeneous carbonate rocks. *Rock Mechanics and Rock Engineering, 44*(1), 303–309. https://doi.org/10.1007/s00603-010-0112-7.

Palchik, V., & Hatzor, Y. H. (2002). Crack damage stress as a composite function of porosity and elastic matrix stiffness in dolomites and limestones. *Engineering Geology, 63*(3–4), 233–245. https://doi.org/10.1016/S0013-7952(01)00084-9.

Palmström, A., & Singh, R. (2001). The deformation modulus of rock masses — comparisons between in situ tests and indirect estimates. *Tunnelling and Underground Space Technology, 16*(2), 115–131. https://doi.org/10.1016/S0886-7798(01)00038-4.

Pells, P. J. N. (1993). Unixial strength testing. In J. A. Hudson (Vol. Ed.), *Comprehensive rock engineering: Principles, practice and projects: Vol. 3*, (pp. 67–85). Oxford: Pergamon Press.

Pinińska, J. (2004). Geospatial data integration in rock engineering. *Przegląd Geologiczny, 52*(8/2), 806–812.

Quiñones, J., Arzúa, J., Alejano, L. R., García-Bastante, F., Mas Ivars, D., & Walton, G. (2017). Analysis of size effects on the geomechanical parameters of intact granite samples under unconfined conditions. *Acta Geotechnica, 12*(6), 1229–1242. https://doi.org/10.1007/s11440-017-0531-7.

Roshan, H., Masoumi, H., & Hagan, P. C. (2016). On size-dependent uniaxial compressive strength of sedimentary rocks in reservoir geomechanics. *Proceedings of 50th U.S. Rock Mechanics/Geomechanics Symposium: Vol. 3*, (pp. 2322–2327). American Rock Mechanics Association.

Roshan, H., Masoumi, H., Zhang, Y., Al-Yaseri, A. Z., Iglauer, S., Lebedev, M., et al. (2018). Microstructural effects on mechanical properties of shaly sandstone. *Journal of Geotechnical and Geoenvironmental Engineering, 144*(2), https://doi.org/10.1061/(ASCE)GT.1943-5606.0001831 06017019.

Rybacki, E., Reinicke, A., Meier, T., Makasi, M., & Dresen, G. (2015). What controls the mechanical properties of shale rocks? – Part I: Strength and Young's modulus. *Journal of Petroleum Science and Engineering, 135*, 702–722. https://doi.org/10.1016/j.petrol.2015.10.028.

Sabatakakis, N., Koukis, G., Tsiambaos, G., & Papanakli, S. (2008). Index properties and strength variation controlled by microstructure for sedimentary rocks. *Engineering Geology, 97*(1–2), 80–90. https://doi.org/10.1016/j.enggeo.2007.12.004.

Santi, P. M., Holschen, J. E., & Stephenson, R. W. (2000). Improving elastic modulus measurements for rock based on geology. *Environmental and Engineering Geoscience, 6*(4), 33–346. https://doi.org/10.2113/gseegeosci.6.4.333.

Tuncay, E., & Hasancebi, N. (2009). The effect of length to diameter ratio of test specimens on the uniaxial compressive strength of rock. *Bulletin of Engineering Geology and the Environment, 68*, 491–497. https://doi.org/10.1007/s10064-009-0227-9.

Ulusay, R. (2016). *Rock properties and their role in rock characterization, modelling and Design.* Retrieved 27 June 2018, from www.isrm.net/fotos/gca/1245695019 use_of_rock_properties.pdf.

Ulusay, R., & Hudson, J. A. (2007). *The complete ISRM suggested methods for rock characterization, Testing and Monitoring.* Ankara: ISRM Turkish National Group.

Wang, Yu, & Aladejare, A. E. (2016). Bayesian characterization of correlation between uniaxial compressive strength and Young's modulus of rock. *International Journal of Rock Mechanics and Mining Sciences, 85*, 10–19. https://doi.org/10.1016/j.ijrmms.2016.02.010.

Yoshinaka, R., Osada, M., Park, H., Sasaki, T., & Sasaki, K. (2008). Practical determination of mechanical design parameters of intact rock considering scale effect. *Engineering Geology, 96*(3–4), 173–186. https://doi.org/10.1016/j.enggeo.2007.10.008.

Yu, Ch, Ji, S., & Li, Q. (2016). Effects of porosity on seismic velocities, elastic moduli and Poisson's ratios of solid materials and rocks. *Journal of Rock Mechanics and Geotechnical Engineering, 8*(1), 35–49. https://doi.org/10.1016/j.jrmge.2015.07.004.