Modelling and Control of Flexible Manufacturing Systems by Means of Interpreted Petri Nets

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Abstract:

Because flexible manufacturing systems (FMS) are discrete event systems (DES), their modelling and control by means of Petri nets (PN) is widely used. While PN transitions are observable and controllable and PN places are measurable, place/transition PN (P/T PN) are sufficient for this aim. However, when some PN transitions are unobservable and/or uncontrollable and some places are non-measurable/unobservable, P/T PN are insufficient for modelling and especially for control. In such a case interpreted Petri nets (IPN) seem to be an appropriate replacement for P/T PN. In this paper a possibility of usage of IPN for FMS modelling and control is pointed out. Illustrative examples as well as the case study on a robotized assembly cell are introduced. By means of using timed PN (TPN) also the performance evaluation of the IPN model of controlled plant is accomplished whereby the simulation in Matlab.

Keywords: *control, discrete event systems, flexible manufacturing systems, interpreted Petri nets, modelling, performance evaluation, place/transition Petri nets, timed Petri nets*

1. Introduction

Discrete event systems (DES) are frequently modelled by Petri Nets (PN). As to their structure PN are bipartite directed graphs with two kinds of nodes - places $p_i \in P, i = 1, \ldots, n$, and transitions $t_j \in T, j = 1, \ldots, m$, and two kinds of edges - first ones directed from places to transitions, being expressed by means of the incidence matrix $\mathbf{F} \in \mathbb{Z}^{(n \times m)}$, and second ones directed from transitions to places, being expressed by means of the incidence matrix $\mathbf{G} \in \mathbb{Z}^{(m \times n)}$, where $\mathbb{Z}$ represents integers. Places model particular operations of DES, states of which are expressed by the so called marking - i.e. by the number of tokens $n_t \in \{0, \ldots, \infty\}$ put into them. Transitions model the discrete events in DES. A transition can be disabled (it cannot be fired) or enabled (it can be fired). The occurrence of a discrete event is modelled by means of firing the corresponding transition. As to dynamics (the marking evolution) PN are expressed (see e.g. [10]) by the linear discrete state equation as follows

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{B}.\mathbf{u}_k, \quad k = 0, \ldots N \quad (1)$$

restricted in any step k by means of the inequality

$$\mathbf{F}.\mathbf{u}_k \leq \mathbf{x}_k \quad (2)$$

Here,

$\mathbf{x}_k = (\sigma_{p_1}, \ldots, \sigma_{p_n})^T$ is the state vector of places in the step k with $\sigma_{p_i}^k \in \{0, \ldots, \infty\}, i = 1, \ldots, n$;

$\mathbf{u}_k = (\gamma_{t_1}, \ldots, \gamma_{t_m})^T$ is the state vector of transitions in the step k (named as the control vector) with $\gamma_{t_j} \in \{0, 1\}$, where 0 denotes the disabled transitions and 1 denotes the enabled ones;

$\mathbf{B} = \mathbf{G}^T - \mathbf{F}$ is the structural matrix of PN.

Hence, the formulae (1) and (2) represent the PN-based model of a system of the type DES. More details about PN can be found e.g. in [3, 8, 9] which are basic (*historical*) sources and/or on many other papers. In [3] the name P/T PN was introduced for such a kind of PN instead of PN.

1.1. Timed Petri Nets

However, P/T PN do not contain explicitly time. The steps of their evolution depends only on the occurrence of discrete events. Of course, events occur implicitly in real time but time is not incorporated into the P/T PN model. To see time relations explicitly, timed Petri nets (TPN) [10, 12, 13] can be used. Consequently, TPN are suitable also for finding the performance evaluation and throughput of DES. Namely, TPN directly yield the marking evolution with respect to (wrt.) time. In this paper, time specifications are assigned exclusively to the P/T PN transitions as their duration function $D : T \to \mathbb{Q}_0^+$, where $\mathbb{Q}_0^+$ symbolizes non-negative rational numbers. In such a way P/T PN turn to TPN. The time specifications are represented by certain time delays of the transitions (in deterministic cases), or by the probability density of timing the transitions (in non-deterministic cases) - e.g. uniform, exponential, Poisson's, etc. Most often the uniform probability density

$$^u f_x = \begin{cases} 1/(b-a) & \text{if } x \in (a, b) \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

is used in DES models, especially in FMS ones.

Assigning time into a transition, the duration of operations modelled by the input places of the transition is set up in the DES model. Because the transition can be fired only in the case when all operations modelled by its input places are finished, assigned time represents the duration of the longest running operation. When in the case of simulation the duration of an operation is supposed to be fixed, we speak about deterministic case of timing and the corresponding time delay is assigned to the transition. Otherwise, we speak about non-deterministic timing. In such a case we are not able to guess exactly the du-

ration of running operations. Therefore, a probability density is assigned to the transition. It yields a probable time in which the longest running operation may be finished.

1.2. Interpreted Petri Nets

There exist also unobservable and uncontrollable transitions as well as the non-measurable/unobservable places in PN models of real DES. P/T PN are not able to deal with such transitions and places. One of the approaches, how to deal with such a non-determinism at DES modelling and control, is usage of interpreted Petri nets (IPN) [11]. IPN are an extension of P/T PN. They allow to represent the output signals of measurable/observable places (occurring when a marking is reached), and the input signals (connected with controllable transitions). IPN are also helpful at avoiding the state explosion problem occurring often in P/T PN models.

Formally, IPN can be represented by the 6-tuple

$$Q = \{(N, \mathbf{x}_0), \Sigma, \Phi, \lambda, \Psi, \varphi\} \qquad (4)$$

where

$PN = (N, \mathbf{x}_0)$ is the PN with the structure N and the initial state $\mathbf{x}_0$;

$\Sigma = \{\alpha_1, ..., \alpha_r\}$ is the input alphabet of the IPN with α_i, $i = 1, \ldots, r$, being input symbols;

$\Phi = \{\delta_1, ..., \delta_s\}$ is the output alphabet of the IPN with δ_j, $j = 1, \ldots, s$, being the output symbols;

$\lambda : T \to \Sigma \cup \{\varepsilon\}$ is a labelling function assigning an input symbol to each PN transition with the following constraints: $\forall t_j, t_k \in T, j \neq k$, if $\forall p_i, F(p_i, t_j) = F(p_i, t_k) \neq 0$ and $\lambda(t_j) \neq \varepsilon, \lambda(t_k) \neq \varepsilon$, then $\lambda(t_j) \neq \lambda(t_k)$. Here, ε represents a spontaneous system event which cannot be influenced from outside - i.e. internal system event. If for a transition t_i holds $\lambda(t_i) \neq \varepsilon$, then the transition is controllable. Otherwise the transition is uncontrollable. Denote the set of controllable transitions as T_c and the set of uncontrollable transitions as T_u;

$\Psi : P \to \Phi \cup \{\varepsilon\}$ is a labeling function of the places assigning an output symbol $\delta \in \Phi$ or the null symbol ε to each place - it means that $\Psi(p_i) = \delta_k$ when p_i models an output signal, otherwise $\Psi(p_i) = \varepsilon$. Thus, the set P of all places is divided into two subsets - the set of measurable places $P_m = \{p_i | \Psi(p_i) \neq \varepsilon\}$ and the set of non-measurable places $P_{nm} = \{p_i | \Psi(p_i) = \varepsilon\}$. Of course, it holds $P_{nm} = P \backslash P_m$. The number of the measurable places is $q = |P_m|$;

$\varphi : R(N, \mathbf{x}_0) \to \mathbb{Z}_{\geq 0}^q$ is an output function, where $R(N, \mathbf{x}_0)$ is a reachability set of $(N, \mathbf{x}_0)$ and $\mathbb{Z}_{\geq 0}^q$ represents non-negative integers including 0. It maps a reachable marking $\mathbf{x}_k$ to a $(q \times 1)$ observation vector $\mathbf{y}_k$ of non-negative integers. The output function is a $(q \times n)$-dimensional matrix φ. Each its row is an elementary $(1, n)$-dimensional vector $\varphi(i, \bullet)$, $i = 1, \ldots, q$, having only one nonzero entry equal to 1, namely $\varphi(i, j) = 1$, if the place p_j is the i-th measured place. When the i-th place is non-measured, $\varphi(i, j) = 0$.

Above introduced description means that IPN distinguish controllable and uncontrollable transitions as

well as the measurable and non-measurable places. When we consider (in analogy with continuous systems) the equation (1), restricted by (2), to be the state equation of a PN-based model, then

$$\mathbf{y}_k = \varphi . \mathbf{x}_k, k = 0, \ldots, N \qquad (5)$$

is its output equation.

More details about theory of IPN can be found e.g. in [1, 2, 4–7].

1.3. Illustrative Example on IPN

To illustrate the previous definition of IPN let us introduce Figure 1. Suppose that the measured pla-
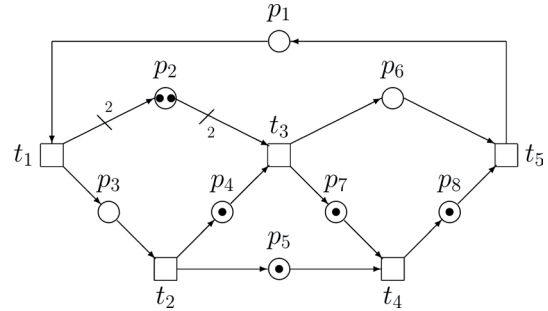
Figure 1. An example of IPN

ces are $P_m = \{p_1, p_5, p_6\}$ and the non-measurable places are $P_{nm} = P \backslash P_m = \{p_2, p_3, p_4, p_7, p_8\}$. Suppose that the controllable transitions are $T_c = \{t_1, t_5\}$ and the uncontrollable transitions are $T_u = T \backslash T_c = \{t_2, t_3, t_4\}$. Consider that the input and output alphabet are, respectively, $\Sigma = \{a, b\}$ and $\Phi = \{\delta_1, \delta_2, \delta_3\}$. Hence, $\lambda(t_k)_{k=1,\ldots,5} = \{a, \varepsilon, \varepsilon, \varepsilon, b\}, \Psi(p_i)_{i=1,\ldots,8} = \{\delta_1, \varepsilon, \varepsilon, \varepsilon, \delta_2, \delta_3, \varepsilon, \varepsilon\}$. Consequently, the IPN output vector in the step k is given by (5) where

$$\varphi = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \qquad (6)$$

It means, that for the state $\mathbf{x}_k = (0, 2, 0, 1, 1, 0, 1, 1)^T$ displayed in Figure 1, the output vector can be obtained in the following form $\mathbf{y}_k = \varphi . \mathbf{x}_k = (0, 1, 0)^T$. As we can see, the output vector represents a *crippled* state vector free of the non-measurable places. Hereinafter, the problem of control will be analyzed in Section 2.

1.4. Terminative Remarks and Paper Organization

In this paper the P/T PN are used for modelling FMS (i.e. the plant) to be controlled. In case when all transitions are controllable and all places are measurable, there are different methods for the model based control (e.g. the supervisory control). However, in case of the P/T PN model with uncontrollable transitions and non-measurable places, the specific IPN-based controller (different from a supervisor) has to be added. Thus, the IPN model of the controlled FMS rises. Applying TPN, i.e. assigning time to the transitions of the IPN model of controlled system, the performance evaluation can be find by means of simulation. Simulation was performed in Matlab by means of the

HYPENS tool. Definitions all of kinds of PN used here were introduced above in this section - Section 1.

It is necessary to emphasize that there is used a specific kind of controller, completely different from a supervisor, in this approach. The principle of its construction is explained in the Section 2 together with a simple illustrative example. The detailed case study on the model of real FMS is introduced in Section 3.

In comparison with the author's conference paper [11], the aim of this paper is to deeply analyze and describe a situation in a robotized system (often occurring in practice), where uncontrollable transitions and unobservable places occur, by means of IPN-based model of a controlled system. Namely, there are explained and describe in more detail: (i) the principle of control of P/T PN model of FMS containing uncontrollable transitions and non-measurable places by means of the model of the IPN-based controller; (ii) the difference in comparison with deterministic model with controllable transitions and observable places is emphasized with introducing the reachability tree (RT); and (iii) the interconnection of both models - the model of the controlled plant and the controller - into the IPN based model of the controlled plant. Moreover, the performance evaluation of the IPN-based model of the controlled system was accomplished by means of simulation in Matlab using the HYPENS tool applying TPN - i.e. by means of assigning time to the transitions of the IPN model of controlled system. The description of the performance evaluation as well as its results are introduced in Section 4.

2. A View on Control of IPN

The principled idea of this control is to create a controller in such a way that the output of the controlled system always be equal to the prescribed control specification output. The specification describes some relevant sequences of states that the system must pass.

Let us introduce the principle how to control DES with P/T PN model containing uncontrollable transitions and non-measurable places by means of adding the IPN model of control specifications. Consider a segment of the IPN model of a controlled system in the form given in Figure 2. The upper line (containing
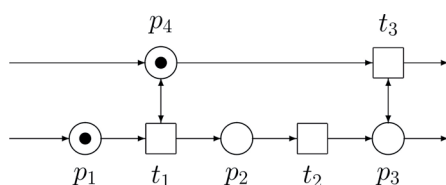
Figure 2. The segment of the IPN model of controlled FMS

the place p_4 and transition t_3) represents the fragment of the IPN model of the control system, while the lower line (containing p_1, p_2, p_3, t_1, t_2) represents the fragment of the P/T PN model of the controlled object/plant.

At the structure displayed in Figure 2 the transition t_1 is controllable. Moreover, it is enabled because p_1 and p_4 (being the state of a sensor) are active (they have the token). The self-loop between p_4 and t_1 represents the relation between the place of the control specification and the controllable discrete event of the plant. The transition t_2 is uncontrollable. The place p_2 is non-measurable. The transition t_3 is enabled because of the active p_4. It models the event which expresses the situation when the plant and control specification have the same output. The place p_3 models the state of another sensor. The self-loop between p_3 and t_3 expresses the relation between the measured place of the controlled plant and the event representing a control specification. In such a way the uncontrollable transition t_2 and non-measurable place p_2 are bypassed. In spite of this, after firing t_1 the place p_2 can be active and consecutively t_2 can be fired, only the activity of p_2 cannot be observed and firing of t_2 cannot be affected from outside.

The corresponding reachability graph (RG) is in Figure 3. Here, $\mathbf{x}_0 = (1,0,0,1)^T$, $\mathbf{x}_1 = (0,1,0,1)^T$,
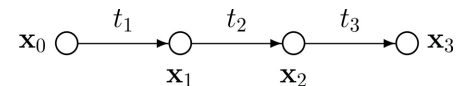
Figure 3. The corresponding reachability graph

$\mathbf{x}_2 = (0,0,1,1)^T$, and $\mathbf{x}_3 = (0,0,1,0)^T$. Because RG has no branching, it shows that the control process is unambiguous.

3. Case Study on FMS

Let us apply the IPN-based approach to modelling and control of a simple FMS. The scheme of the system is displayed in Figure 4. FMS represents a robotized assembly cell consisting of two input conveyers
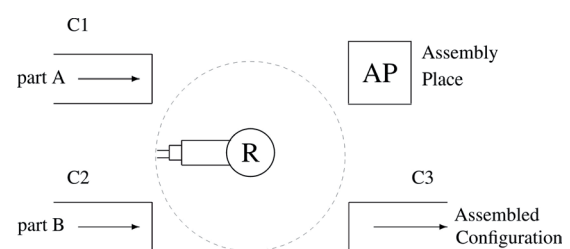
Figure 4. The scheme of the FMS

C1 (feeding parts of a kind A) and C2 (feeding parts of a kind B), the robot R, the assembly place AP, and the output conveyer C3 (carrying the assembled parts away). R takes subsequently the parts A, B from the conveyers C1, C2 and inserts them into the AP, where they are assembled (i.e. the assembly A + B is performed). After finishing the assembly process, R picks the assembled configuration from AP and put it on C3.

3.1. P/T PN Based Model of the Plant

The P/T PN model of the robotized assembly workcell is given in Figure 5. The places represent there the following activities:

Figure 5. The P/T PN model of the uncontrolled FMS

Figure 6. The RT of the P/T PN model of the uncontrolled FMS when t_1, t_4, and t_{11} are fired only once

p_1 - means that C1 conveys the part A;

p_2 - means that C1 is available;

p_3 - means that R takes the part A from C1 and transfers it to AP;

p_4 - expresses that R inserts A into AP;

p_5 - models that C2 conveys the part B;

p_6 - models the availability of C2;

p_7 - represents the situation when R takes B from C2 and transfers it to AP;

p_8 - means that R inserts B into AP;

p_9 - ensures the mutual exclusion, because R cannot take A from C1 and B from C2 simultaneously;

p_{10} - models the situation that the parts A, B are assembled in AP;

p_{11} - models that R unloads the finished configuration from AP;

p_{12} - expresses that R transfers the finished configuration from AP to C3;

p_{13} - means that R put the finished configuration on C3;

p_{14} - represents that a free place on C3 is available.

The RT of this model corresponds to the initial state $\mathbf{x}_0$ depicted in Figure 5. Unlimited inputs ensured by t_1 and t_4 and unlimited output ensured by t_{11} in the model displayed in Figure 5 cause that the RT is too large and loops occur in some nodes. Owing to these reasons it can be introduced here neither in a graphical form nor in the form of matrix $\mathbf{X}^{PN}_{reach}$ of reachable states.

However, when t_1, t_4, and t_{11} are fired only once, the RT of the P/T PN model of the uncontrolled plant has the form displayed in Figure 6 with the nodes represented by the columns of the matrix (7).

$$\mathbf{X}^{PN}_{reach} = \begin{pmatrix} 0\,1\,0\,0\,1\,0\,0\,0\,1\,0\,0\,1\,0\,0\,0\,0\,0\,0\,0\,0 \\ 1\,0\,1\,1\,0\,1\,1\,1\,0\,1\,1\,0\,1\,1\,1\,1\,1\,1\,1\,1 \\ 0\,0\,0\,1\,0\,0\,0\,1\,0\,0\,0\,0\,0\,1\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,1\,0\,0\,0\,1\,0\,1\,0\,1\,0\,0\,0\,0\,0 \\ 0\,0\,1\,0\,1\,0\,0\,1\,0\,0\,1\,0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 1\,1\,0\,1\,0\,1\,1\,0\,1\,1\,0\,1\,1\,1\,1\,1\,1\,1\,1\,1 \\ 0\,0\,0\,0\,0\,1\,0\,0\,1\,0\,0\,0\,1\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,1\,0\,1\,0\,1\,1\,0\,0\,0\,0\,0\,0 \\ 1\,1\,1\,0\,1\,0\,1\,0\,0\,1\,1\,1\,0\,0\,1\,1\,1\,1\,1\,1 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,1\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,1\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,1\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,1\,0 \\ 1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,0\,1 \end{pmatrix} \quad (7)$$

Note that the first column and the last column are the same - i.e. the system comes back into the initial state. It is necessary to say that the RT corresponds to this model only when all transitions are controllable and all places are measurable.

Till now we have considered that all places are observable/measurable and all transitions are controllable and observable. However, in fact it is not true. In the model of a real plant several transitions could be considered to be uncontrollable and several pla-

ces could be non-measurable. Consider, for example, that only the assembly process cannot be influenced from outside. Essentially, it is true, of course, because the process run autonomously and cannot be affected from outside.

Moreover, faults can occur in real systems - e.g. a part can fall down from the robot gripper, etc. Here, in this paper, we will not study any non-determinism concerning faults. The simple application of IPN will be presented only to illustrate how to avoid problems with unobservable/non-measurable places and un-controllable and/or unobservable transitions at control synthesis of real FMS.

3.2. IPN Based Control of the Plant

Building the controller in the sense of the procedure described in Section 2, the IPN model of the controlled FMS is given as it is displayed in Figure 7. Here, in this model, it is supposed that the transition t_8 is uncontrollable and the place p_{10} is non-measurable. It corresponds to reality. Namely, the automatically running assembly process inside AP (being an automatic workstation) represented by p_{10} cannot be influenced from outside during its activity. It is fully autonomous. Thus, the current state of the assembly process cannot be measured in any way. Only two states of the assembly process - the start and the end - are observable.

While meaning of the plant places is the same as in the P/T PN model, meaning of the places in the control specification module is clear from the analogy with Figure 2. Namely, the control specification place p_{17} makes possible to fire t_7 representing the controllable discrete event. Thus, the assembly process can be started when the parts A and B are inserted into AP (see meaning of p_4 and p_8). When uncontrollable event represented by t_8 occurs (i.e. when the assembly process in AP was finished) the measurable/observable place p_{11} becomes active. Because of active p_{17} and p_{11} the transition t_{14} is enabled and can be fired. Consequently, the control process can continue.

Starting from the P/T PN model in Figure 5, parameters of the IPN model are as follows

$$\mathbf{F} = \begin{pmatrix} \mathbf{F}_p & \mathbf{F}_{pc} \\ \mathbf{F}_{cp} & \mathbf{F}_c \end{pmatrix} ; \ \mathbf{G}^T = \begin{pmatrix} \mathbf{G}_p^T & \mathbf{G}_{pc}^T \\ \mathbf{G}_{cp}^T & \mathbf{G}_c^T \end{pmatrix} \quad (8)$$

Here, the parameters of the PN model of the plant to be controlled are

$$\mathbf{F}_p = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (9)$$

$$\mathbf{G}_p^T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (10)$$

being, respectively, the incidence matrices of directed arcs from the plant places to its transitions and from the plant transitions to its places.

The cross parameters between the plant and control system are

$$\mathbf{F}_{pc} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} ; \mathbf{G}_{pc}^T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (11)$$

being, respectively, the incidence matrices of directed arcs from the plant places to the control system transitions and from the control system transitions to the plant places.

The cross parameters between the control system and plant are

$$\mathbf{F}_{cp} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad (12)$$

$$\mathbf{G}_{cp}^T = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (13)$$

being, respectively, the incidence matrices of directed arcs from the control system places to the plant transitions and from the plant transitions to the control system places.
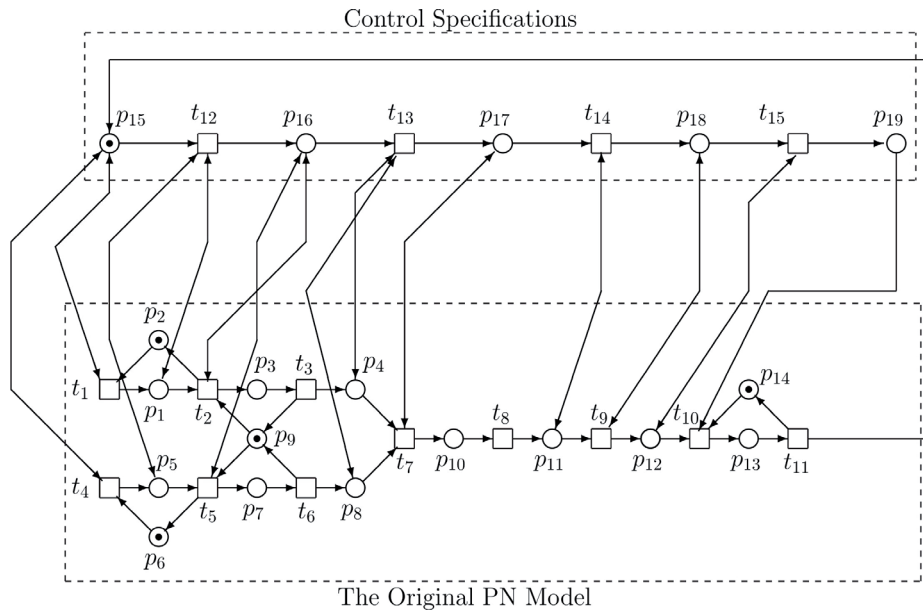
Finally, the parameters of the control system are

$$\mathbf{F}_c = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} ; \mathbf{G}_c^T = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (14)$$

being, respectively, the incidence matrices of directed arcs from the control system places to its transitions and from the control system transitions to its places.

The RG corresponding to the model in Figure 7 is given in Figure 8. Its nodes are state vectors (the initial state $\mathbf{x}_0$ and all states reachable from it) represented by the columns of the following reachability matrix (15)

$$\mathbf{X}_r = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$(15)$$

The matrix φ in the output equation (5) is (18×19)-dimensional, because p_{10} is not observable. It has the following form

$$\varphi = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$(16)$$

The 10-th column is the zero vector. Thus, the output vector $\mathbf{y}_k$ is the (18×1)-dimensional vector.

4. Performance Evaluation

Let us view on the operation of the controlled plant in time. Consider the non-deterministic timing of the uncontrollable transition t_8. Treat other transitions as deterministic. In deterministic timing the duration of technological operations can be guessed and the transition delays represent the fixed duration of technological operations. In the uncontrollable transition t_8 the duration of the operation modelled by p_{10} cannot be guessed exactly. Therefore, the uniform probability density (3) will be applied in order to obtain probable time in which the running operation may be finished.

To accomplish the performance evaluation of the controlled plant by means of simulation, let us apply TPN-based approach on the controlled IPN model of the controlled plant. Consider uniform probability density for t_8 with parameters $a = 5.5, b = 8.5$. For other transitions consider the following time delays in a time unit. Namely, for t_1, t_2, t_4, t_5, t_{10}, t_{11} the delay $\Delta_1 = 1$, for t_3, t_6, t_9 the delay $\Delta_2 = 2$, for t_7 $\Delta_3 = 5$ and for t_{12}, t_{13}, t_{14}, t_{15} $\Delta_4 = 0.1$. All of the numerical values concerning the parameters are still multiplied by the constant 50. Simulation was performed on the time interval $\langle 0, 4000 \rangle$ of time units using the simulation tool HYPENS in Matlab.

4.1. Simulation Results

During the simulation process the graphical results expressing the performance evaluation of the controlled plant were found. Although the course of marking wrt. time can be displayed for any place of Figure 8, only courses of marking wrt. time of some places are introduced here. It has two reasons. Namely, on the one hand these places are most important as to understanding the system behaviour, and on the other hand the courses of marking all of 18 places wrt. time occupy much space. While the courses of markings of the places p_1 - p_8 ($M(p_1)$ - $M(p_8)$) wrt. time are not so interesting (they correspond with those being standard like in P/T PN model), the courses of markings of the places p_9 - p_{12} ($M(p_9)$ - $M(p_{12})$) wrt. time are given in Figure 9. The courses of markings of the places p_{13} - p_{21} ($M(p_{13})$ - $M(p_{21})$) wrt. time are also not so important like the previous ones because of reasons mentioned at p_1 - p_8 ($M(p_1)$ - $M(p_8)$). From the point of view of the IPN model application the most interesting is the course of marking in the places p_{10} and p_{11}. Namely, the length of the assembly process represented by p_{10} cannot be exactly measurable. Consequently, the robot which activity is modelled by p_{11}, *does not know* exactly when it can unload the assembly place. Just on that account timing the transition t_8, situated among these places, was modelled as non-deterministic one.

5. Conclusion

This paper presents a possibility how to model and control FMS by means of PN in case when some PN transitions are unobservable and/or uncontrollable and some places are non-measurable/unobservable.
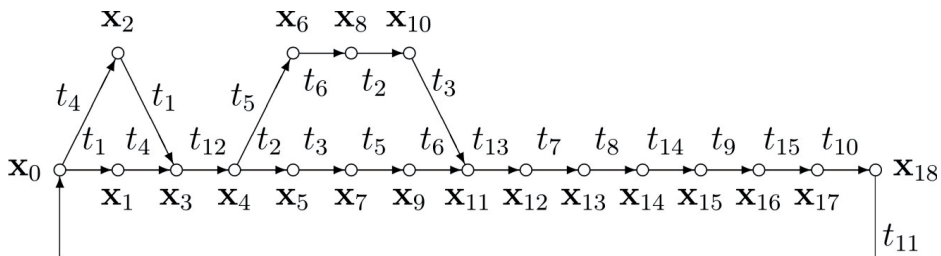
Figure 7. The IPN model of the controlled FMS

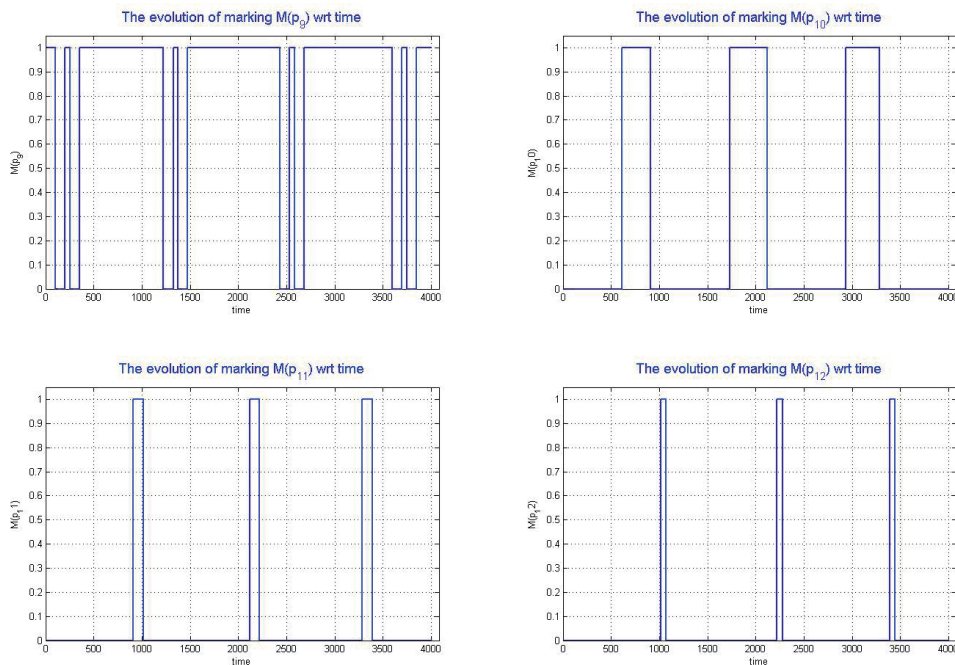*Figure 8. The corresponding RG of the IPN model of the controlled FMS*

Figure 9. The simulation results - performance evaluation of the places p_9 *-* p_{12}

Namely, in such a case P/T PN are not able to describe such a non-determinism. Therefore, IPN were applied here. They yield the appropriate replacement for P/T PN as well as the effective tool how to deal

with the non-determinism. IPN are an extension of P/T PN. Main difference between P/T PN and IPN consists in the fact that IPN allow to represent both (i) the output signals which are generated when a marking is reached; (ii) and the input signals being associated with the controllable transitions. Moreover, IPN make possible to express the symbiosis of both the model of controlled plant and the controller expressing directly the control specifications. These facts make possible to deal with the non-determinism caused by uncontrollable/unobservable transitions and non-measurable/unobservable places.

The mathematical description of IPN and their usage for FMS modelling and control were introduced. For illustration the explanation example was introduced in Subsection 1.3. The principle of the IPN-based control was explained and illustrated by example in Section 2. The main part of the paper - Section 3 - presents the simple case study on a robotized assembly cell. The non-determinism arises when an operation of FMS represented by a PN place cannot be observable/measurable - like the automatically performed assembly process cannot be affected from outside. The operation of assembly does not take (because of different reasons) the same time in each working cycle of the plant. This fact causes that the final assembled part cannot be unloaded from the assembly device before finishing the assembly process. A suitable bypass of the uncontrollable transitions and unobservable places of the plant by means of the controller leads to the successful control of the non-deterministic plant.

The simulation results concerning the performance evaluation of the controlled plant in the case study introduced in Section 4 corroborate the applicability of IPN-based models of DES and show that the usage of IPN for modelling and control of FMS can be effective and applicable in practice.

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