

# Using Queue Theory to Analysis and Evaluation of the Logistics Centre Workload

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## Abstract

The article presents an application of queuing theory (mass service) to describe transportation process. It presents the essential concepts of the queuing theory. The process of moving a load unit through the logistics centre was presented. To illustrate the issues an example of five-level, two-input queuing networks was examined.

## 1. Introduction

Logistics is defined as the area of scientific research about organizing processes such as: raw materials and finished goods movement and storage in terms of the system, which aims to optimize the supply chain (from obtain raw material to the consumer), as well as dealing with the management of waste products [1]. The consequence of the operation of logistics is to introduce innovative solutions to improve the quality of the supply chain functioning<sup>1</sup>. This involves finding ways to meet the expectations of customers, acquiring new customers, providing long-term relationships with clients, but also to identify potential competitors and their current positions.

In engineering terms logistics is defined as a field of knowledge and skills necessary to designing the processes of cargo and information movement in logistics systems to meet the needs in the area, with a minimum investment and cost [7].

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<sup>1</sup> The supply chain is cooperating in various functional areas of mining companies, manufacturers, retailers, service providers and their customers, between which are flowing streams of products, information and funds.

In the literature can be found a variety and sometimes contradictory definitions of the logistics system. Starting from systems theory, the system is a set of components with different characteristics, functions, tasks, and a set of relations between them resulting from the realization of the objective 915. However, logistic system, in general, is defined as structured and integrated - within given economic system - the flow of materials and products, and the information accompanying that allow optimization of the materials flow 79151718.

Due to the scale of performed tasks and functions performed different logistics systems can be identified. Taking into account the aspect of scale (or the institutional aspect) we distinguish system: micro-logistic (e.g. company), meta-logistics (e.g. supply chain), mezzo-logistic (vertically integrated meta-logistics systems), macro-logistic (domestic economy scale). The other hand taking into account the functional aspect we distinguish logistics systems in supply, production, distribution, return of goods and packages and logistics systems for material management and marketing 14.

Analysis of logistics systems usually is to determine the best for them or the evaluation of the current structure and characteristics<sup>2</sup>. These objectives can be achieved, inter alia, through simulation research carried out using models of logistics systems 1116. These research, allows to observe the effect on the efficiency of these systems, approved equipment and applied work organization. They also allow to get answers to many questions about the expected behaviour of the logistics system in some more or less typical situations 3.

Therefore dynamics model of the actual transport process should include the occurrence of traffic congestion and bottlenecks. One of the best methods to include these issues in the model is using queuing theory to describe the transport process. Queuing Theory is the study of mathematical models of real processes where there are outages, waiting, queue and loss. Is an area that uses the application of probability theory as research tools in complex analysis, the theory of differential and integral equations and other areas of mathematics 2.

Considering the above, in order to carry out simulation studies for both designing and existing logistics systems, such system must be described as a queuing network. In general, a network for the logistics system can be represented as in Figure 1. As follows from this figure streams of goods arrived in the logistic system from environment are directed to operate inside that system in the next locations supported by internal roads connection.

Every place of cargo stream service ( $w, w'$ ), is treated as an elementary queuing system in which can be identified place of waiting for service (queue) and place of service (service channels) (Fig. 1).

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<sup>2</sup> In the case of point logistical infrastructure, the problem is to determine the best size for them and their equipment.

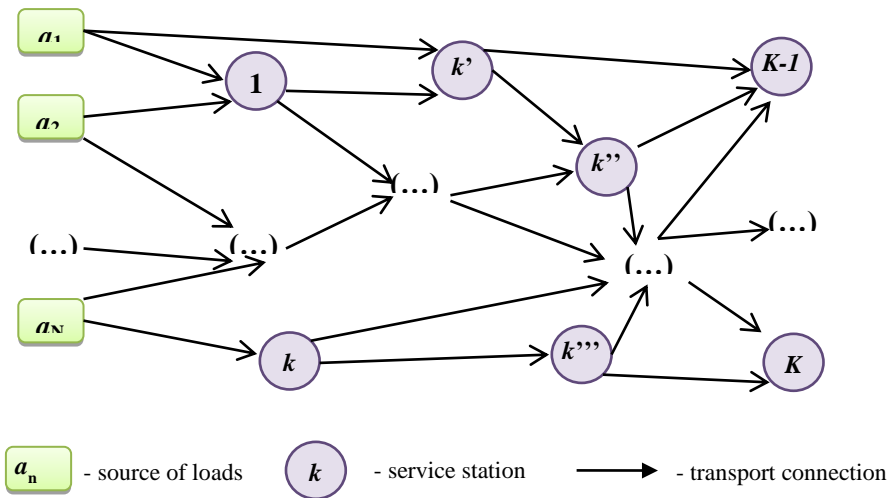


Fig. 1. The structure of the logistics system treated as a queuing network

Source: developed based on 19

Introducing a set of waiting and a set of work stations, anywhere in cargo streams service allows for the expression of situation in which one piece of equipment logistics system (e.g. crane) supports various forms of goods with varying intensity, and before that component loads expected in different places (railway wagon waiting on the truck, and the semi trailer with the truck unit on the way).

In the article queuing method used to analyze and evaluate the burden of cargo stream in logistics centre. Research is conducted in order to determine the optimal size and equipment of the newly designed logistics facilities. For analysis a computer package WinQsb was used.

## 2. Queue Theory - Assumptions

Constructing the model of the dynamics of cargo handling process in logistics centre to describe relations that exist between the states of the system and sometimes as an independent variable. Configuring the process of cargo moving it is need to take into account the relations between the states and the states of the elements of road transport means forming a stream of traffic. These relations reduce the number of possible states of the system states phase space. These restrictions allow for representation of the movement vector in two scenes 1:

- as the state changes in the elements of the roads system,
- as the state changes in the means of transport making cargo movement in the system, such as vehicles, trains, wagons, containers.

The mapping scope of logistics centre infrastructure and the range of goods stream mapping using the characteristics of the elements of the logistics centre due to the purpose and scope of the research for which the model is constructed. We assume that the state of the system is defined as a phase space defined by the Cartesian components of the logistics

centre states and states of vehicles forming freight flow in the system. This means that the description of the system state vector can be used with components defining a point in the state space of the system.

A mathematical model of the functioning of queues based on a theory of stochastic processes. The essence of the theory of mass service are the following terms:

1. **Arrival** – the queuing theory assumes random character creation service requests, ie the interval between adjacent moments of arriving requests to the system is a positive random variable. The same is with other examinees quantities.
2. **Input arrival stream** – sequence of consecutive intervals between adjacent moments of arrival notifications to the system. Most often it is recurrent, i.e. intervals are independent of each other and have the same distribution.
3. **Device service** – person or device that performs services. Service time is also a positive random variable and just like the input stream usually assume that the service is recurrent, i.e. the device service work independently of each other, and the processing time of each device has the same distribution.
4. **Queue** – queue is creating in service system when it is not possible (at a given moment of time) to service all requests, since all units are currently occupied.

The basic condition for the application of analytical methods is the assumption that the arrival stream is:

- **stationary process** – the probability of the entries depends only on the length of the respective periods of time, but does not depend on their position on the timeline. The appearance of  $k$  entries in the interval  $(t, t + \tau)$  is a function of the variables  $k$  and  $\tau$
- **memoryless** – the probability of the  $k$  entries in a given time interval does not depend on the number entries and how it is occurred up to this point
- **single** – lack of appearance of two or more entries at the same time.

In formulating the model of queue there should be defined:

- type of probability distribution of random variables;
- dependence or independence of random variables of waiting time for the arrivals and service time;
- limited or unlimited the number of service waiting stations, queue length;
- the queue's discipline.

The time elapsed between the subsequent reports entry to the system, the service time of one of the service station, the number of service stations, the number of places in the waiting queue waiting for service is defined as random variables in the model of analyzed type.

Figure 2 shows a simple queuing system, where  $\lambda_{WE}$  -is the intensity of the arrivals to the system, while the  $\mu$ -is the intensity of the service time.

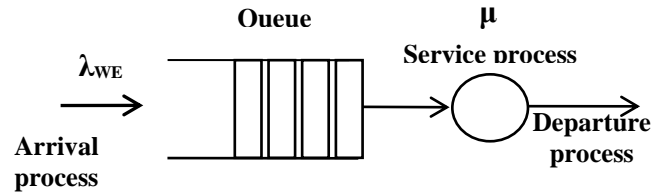


Fig. 2. Diagram of a simple queuing system

Source: own development.

Describing the queuing system can take over the classification of queuing systems developed by A. M. Lee, which is an extension of the Kendall notation. Code describing the queuing system has the form:

$$X/Y/m/d/l \quad (1)$$

where:

$X$  –type of distribution of the arrival stream in to the system,

$Y$  –type of service time distribution,

$m$  –number of service channels in the system,

$d$  –queue's discipline,

$l$  – size of the system, i.e., the maximum number of entries that can fit in the system (service channels and queue).

The most common distributions of arrival stream and service time are:

$M$  – exponential distribution of service time or intervals between subsequent arrival, i.e. arrive Poisson distribution,

$E_k$  – Erlang distribution (type  $k$ ), service times or time intervals between subsequent arrival,

$D$  – determined or regular stream,

$N$  – normal distribution,

$G$  – stream having any type distribution of service time,

$GI$ – stream of the general type, general and independent,

$H_r$ – hyper-exponential distribution with  $r$  parameter,

$C_k$ – Cox distribution (type  $k$ ),

$K_n$ –  $\chi^2$  distribution of intervals between subsequent arrival (with  $n$  degrees of freedom) or  $\chi^2$  distribution of service times.

For each of the queue system must be defined the rule of queue (the discipline), specifying the order of selecting entries from the queue. The basic discipline are:

- FIFO – (*first in first out*) customer who stayed longest in the queue is serviced first.;
- LIFO – (*last in first out*) client which is last in the queue is served as the first.;
- RSS. – (random service selection) customers are selected to serve from the queue randomly;
- RR – (*round robin*)

- customers are managed according to the FIFO discipline described above, but the service is terminated at the end of the time interval that is called the quantum  $T$ . If the service is not completed, the application takes place in the queue with probability  $p$  or in a situation where the service has been completed with a probability of  $(1 - p)$  customer is leaving the system;
- PS – (processor sharing) RR special case in which the time interval  $T$  approaches 0 and the probability  $p$  to 1. Quotient  $E(C) = T/(1-p)$  is constant.  $C$  – service time. Queue discipline based on RR and PS is used in computer science.

Set of queuing systems, connected with each other, between which customers with the need for service are moving, is creating a network of queuing. It can be seen that in the queue network elements are analogous to elements of the transport network, which allows the use of technical terms used in the claims and transport networks to describe queuing network properties (Table 1).

Table 1

**Comparison of the transport network with a queue network**

Elements of the network	Queue network	Transport network
Input	the arrival (source)	vertex with only outgoing edges
Output	the departure (sink)	vertex with only incoming edges
Flowrate	arrival stream	volume of the flow
Verticles	queue system	transport node
Edges	possible transition	arc between nodes
Direction of the edge	direction of the transition	flow direction
Value describing the edge	transition probability $0 < p \leq 1$	capacity

Source: developed based on 5.

When defining logistics process as a series of phases, single phase of the process will be characterized by the duration, and the event will be made up of just a change of state. The resulting structure was thus defined as the structure of the phases of the logistics network. In such an arrangement, such defined phases of the network structure is a representation of the structure of the logistics system such as a distribution centre and logistics centre. This means that the structure of the phases of the logistics network can be written in the form of queuing networks.

For each sequence of phases of the transport unit of the logistics process is described by vector phases. Due to phase out is tightly structured vector. Of course, the lack of space in the queue at the award phase of the service process will move the start or end service manual for the unit of time in the period preceding a given phase. However, the lack of free service channel in the highlighted phase shifts until the beginning of service at this stage. The transport unit, which is located at a further stage in the queue waiting for service. There is no space in the waiting phase following the highlighted phase shifts until the end of service for the unit of time. Presenting the logistics process in queue notation usually use the distribution of applications to the system. In the case where the intensity of a service channel will be compared to the actual need for a time before operating channels will form a queue. Because the most common:

- periods between subsequent appearances of vehicles in the nodes have identical and independent distributions, the probability that between  $t+\Delta t$  will be only one vehicle depends only on the length of the interval  $\Delta t$  and is independent of  $t$ .
  - in any interval of length  $\Delta t > 0$ , the probability of the vehicles is positive,
  - into consideration any sufficiently short period of time may appear at most the appearance of vehicles,
- should be used a queuing theory.

Assume that logistics system starts working at  $t=0$  and that the first vehicle occurs at the time  $t$ , ( $t>0$ ), then the probability  $F(t) = P(T < t)$ , that the vehicle will be on time  $t$  will be equal to:

$$F(t) = 1 - \int_0^t f(x) dx \quad (2)$$

where  $f(t)$  is the probability density function. Because the events are events that occur independent, therefore:

$$P(t + \Delta t) = P(t)P(\Delta t) \quad \text{for } t > 0, \Delta t > 0 \quad (3)$$

The solution to the tasks described by queuing theory are parameters describing the behaviour of the system such as queue length and waiting time in the queue. Since the purpose of logistics systems is the implementation of logistics tasks, it is important to adapt the infrastructure facilities of these systems to the size of the tasks. In the case of systems such as logistics centre, the essential problem is to determine the workload on each of its components. The workload is determined by the desired performance components for logistics systems.

### 3. Logistic Centre in Queue Theory Approach

One of the key elements of the supply chain network structure is a logistics centre. It is defined as an independent economic entity, which is located close to a major economic and industrial centres, as well as near roads, at least two different modes of transport featuring: a dedicated area of communication associated with the environment (mainly road network), infrastructure (roads, plazas, parking lots, construction and engineering buildings), equipment, personnel and organization, logistics service providers in the continuous emergency orders or agreements with external companies 1.

In terms of tasks performed, logistic centres may have a different range of cargo stream transformations, depending on the size of logistical tasks in a given area. This means that the functional structure of the centre should be expanded to different degrees. The more that the area of operation of the centre is a place of many operators, carriers, freight

forwarders, specialist companies are able to provide comprehensive logistics services. This imposes a requirement to dispose of the logistics centre include:

- high storage warehouse with reloading ramps for cars and rail;
- container storage yard with the gantry;
- warehouse for sorting and packing cargo delivered to the logistics centre (in containers and in the pallet load units);
- parking, for cars that do not use other services of the logistics centre;
- system of rail track (railway siding) enabling comprehensive reloading support;
- road system, including, among others.: roads system (access and communication within the centre) as well as manoeuvring areas and parking (for trucks waiting for service in the square storage containers and trucks waiting for service on the loading platforms high storage);
- administrative and social buildings;
- associated facilities, including: cargo machines, lighting and power containers points, equipment and facilities of communication, sewage, fire protection and drainage installations;
- additional equipment.

Importantly, equipment logistics centre infrastructure should allow to change the type of transport, especially in terms of road-rail intermodal technology. Logistics centre analyzed in the article offers support for palletized cargo and container units. The logistics centre has access to a network of roads and railways, which means that it is the contact point between transport modes involved in the cargo movement. For the study will be taken into account only road and rail transport. However, the form of cargo that can be carried out using a considered centre that having: intermodal transport of containers and trailers and on pallets, crates and bundles. Denoting by  $R$  set of numbers of cargo classes, while the index  $r$  class number of the unit load, in the analysis centre has following types of classes are supported:

- truck that use only the parking ( $r=ST$ );
- truck with containers to unloading ( $r=CS+KR$ );
- truck with containers to transshipment ( $r=CS+KP$ );
- truck with trailer to loading ( $r=CS+NZ$ );
- truck with trailer to transshipment ( $r=CS+NP$ );
- trains to containers load ( $r=PTKZ$ );
- trains with empty containers ( $r=PTCZ$ );
- container trains to transshipment ( $r=PTKP$ );
- trains to unload ( $r=PTZR$ );
- trains to transshipment ( $r=PTZP$ ).

Service types that will be implemented in the centre include:

1. arrive by road transport – document management – departure by road transport (without changing the type of transport);



2. arrive by road or rail transport – transshipment and selection according to the order – departure by road or rail transport (possible change of the transport type but not the mode of transport);
3. arrive by road or rail transport – transshipment and selection according to the order – departure by road or rail transport (change the mode of transport);
4. arrive by road or rail transport – transshipment with storage – departure by road or rail transport (change the mode of transport);
5. arrive by road or rail transport – transshipment with storage – departure by road or rail transport (possible change of the transport type but not the mode of transport).

In order to assess the size of the load on each component task, the symbol  $k$  indicates accumulations of tasks at the logistics centre. Set  $K$  of the location of tasks accumulations  $K$  in the reporting centre will be set as  $K=\{1, \dots, k, k', \dots, N\}$ . As the location of accumulations in the analyzed logistics centre is defined:

- Entrance gate for cars ( $k=WES$ );
- Waiting (parking) square ( $k=PP$ );
- Container storage square ( $k=PSK$ );
- The warehouse 1 ( $k=M1$ );
- The warehouse 2 ( $k=M2$ );
- Entrance gate for trains ( $k=WEEK$ );
- Arrival track ( $k=TP$ );
- Departure track ( $k=TW$ );
- Exit gate for cars ( $k=WYS$ );
- Exit gate for trains ( $k=WYK$ ).

Schematically, the flow of cargo through the logistics centre is shown in Figure 3.

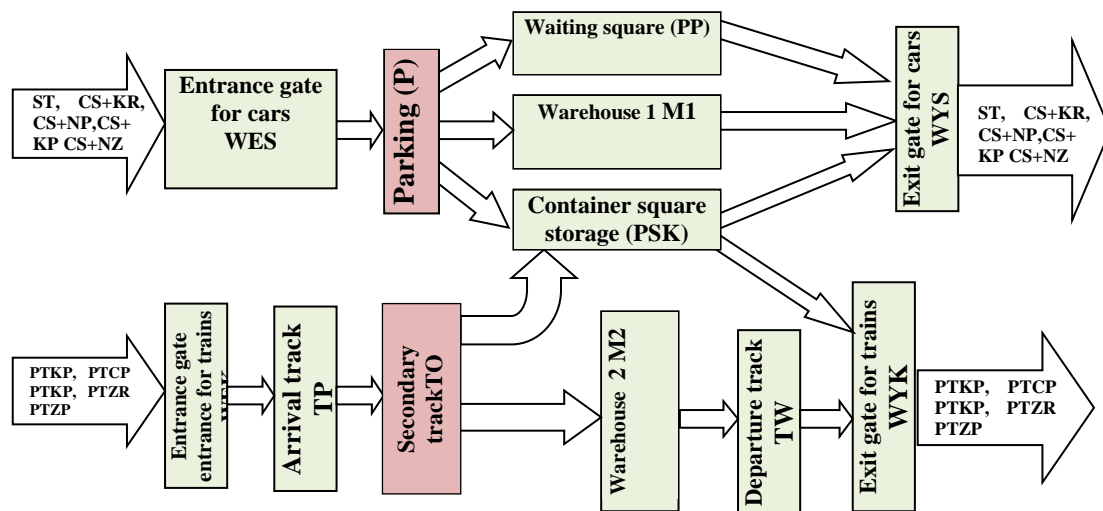


Fig. 3. Flow diagram of the cargo in logistics centre

Source: own development

For the study, it was assumed that the share of rail container handling is 20% higher than the freight carried road transport, while the unit load handling pallet rail share is 23% lower than the freight carried by road. The daily workload of cargo stream in logistics centre are shown in Table 2.

Table 2

Daily workload of cargo stream in logistics centre

Specification	TEU [pcs/24 h]	Pallet load units [pcs/24 h]
Arrival:	79	2139
– rail transport	43	959
– road transport	36	1180
Departure:	87	1746
– rail transport	48	783
– road transport	39	963
Arrival - sending	-8	393
Empty containers	8	-
Empty trucks	-	8
Cars in transit	28	

Source: own development.

For the purpose of assessing the workload analysis of individual elements of the logistics centre the size of tasks has been some assumptions. The first defines the set  $S(k)$  of the types of service station as a set of the form:

$$S(k) = \{s(k) : k \in K\}$$

However, the symbol  $P(k)$  established a set of service types at the logistics centre on the elements:

$$P(k) = \{p(k) : k \in K\}$$

Given the above, each of the location of queue  $k$  is described by an ordered pair:

$$\langle S(k), P(k) \rangle$$

At the same time, in order to place service was the queuing system is required to sets  $S(k)$  and  $P(k)$  was non-empty, i.e.  $S(k) \neq \emptyset$  i  $P(k) \neq \emptyset$ . Furthermore, assume that the symbol  $M((k, k'), t)$  will be marked state of the road between the positions of queues at the logistics centre,  $(k, k') \in L$  in moment  $t$  defined by a set of units of vehicles that are on it. Thus, the state of the road at any<sup>3</sup> given time interval can be defined by the following equation:

$$M((k, k'), t + \Delta t) = M((k, k'), t) \cup \bigcup_{k'' \in \Gamma_k^{-1}} M((k'', k), t) \setminus \bigcup_{k'' \in \Gamma_k} M((k', k''), t + \Delta t) \quad (4)$$

<sup>3</sup> In the interval  $t + \Delta t$

where  $\Gamma_k^{-1}$  - set of predecessors  $k$ ,  $\Gamma_k$  - set consequents  $k$ .

Through the network queuing, which is analyzed in the article is understood set of queuing systems and linked, between which movement of the demand for service was implemented. It was assumed that examined the logistics centre for the following types of queuing systems:

- M/M/m/FIFO/1;
- M/N/m/FIFO/1.

It is also assumed that the model type M/M/1/FIFO/ $\infty$  is a special case of model M/M/n/FIFO/ $\infty$ , in which it is assumed that the service channels of which is  $m$ , have the same intensity service  $\mu$ .

In Figure 4 is a diagram of a logistics centre in terms of a network of queues.

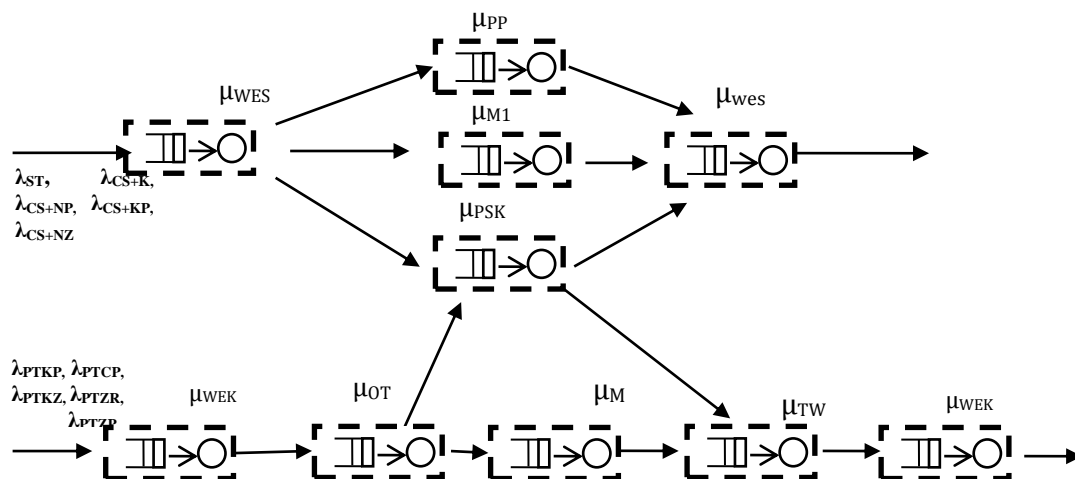


Fig. 4. The block diagram of a network of queues at the logistics centre

Source: own development.

## 4. Service Time Distribution on Service Station

Given the size of transformations implemented in the logistic centre to describe the size and character of distribution streams of arrival for different types of means of transport that are in the analyzed example as clients. At the same time, we assume that the name of the type of transport we mean not only the type (train, car), but also other information such as whether the vehicle is in transit. Therefore we assume that, at the logistics centre are used for different types of vehicles, including cars in transit, trucks and semi-trailers with containers and other types of intermodal units, pallet trucks and freight trains with intermodal units and units subject to the pallet unloaded or transhipped.

In the analyzed network (Fig. 4), we have two inputs with each input is characterized by the intensity of arrival for each customer  $\lambda$ . Analyzing the distribution: the input stream

of vehicles to the network and services times it was found that these distributions are distributions of the normal type and, in most, exponential distributions. Thus, in the remainder of this article is limited to presenting models of queuing systems based on this type of distribution.

Probability distributions to describe the interval between two arrivals and the size for each customer are described by exponential distribution and normal. Thus, according to the notation A. M. Lee [9] code describing the queuing systems has form:

$$M / M / m / FIFO / 1 \quad (5)$$

And

$$M / N / m / FIFO / 1 \quad (6)$$

Probability density for the exponential distribution is given by:

$$f(t) = \lambda e^{-\lambda t} \quad (7)$$

for each  $t$  greater than zero. However, for a normal distribution

$$f(t) = \frac{e^{-\frac{(t-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}} \quad (8)$$

In the present case  $t$ -is a time. The numbers characteristics of distribution are included in Table 3.

Table 3

**Time distribution of arrival implications and the single size of customer needs of the analyzed queuing system**

Customer symbol	Time distribution of arrival implications	The size distribution of a single arrival
ST	Exp(1.91; 0.19)	Exp(1; 0.5)
CS+KR	Exp(2.1; 0.15)	Exp(1; 1)
CS+KP	Exp(1.96; 0.25)	Exp(1; 1)
CS+NZ	Exp(2.78; 0.25)	Exp(1; 1.5)
CS+NP	Exp(1.4; 0.64)	Exp(1; 1.5)
PTKP	Exp(24; 1.2)	Normal(31; 0.3)
PTCP	Exp(48; 1.3)	Normal(28; 0.3)
PTKZ	Exp(24; 1)	Normal(29; 0.51)
PTZR	Exp(24; 2)	Normal(30; 0.1)
PTZP	Exp(24; 1.5)	Normal(27; 0.3)

Source: own development.

For proper evaluation of analyzed logistics centre was made identification of processes where transform a input stream (described in Table 2) in to output stream. The main

characteristics of the work stations is the average time it takes to service the customer. It was assumed that the customer service times at different positions are distributed exponentially. As mentioned above service time is one of the most important characteristics of each operating position and determines its capacity. Service time  $T_0$  should be considered as random variable with distribution:

$$F(t) = P(T_0 < t) \quad (9)$$

For the exponential distribution, which we have in analyzed situation, distribution function has the form:

$$F(t) = 1 - e^{-\lambda t} \quad t \geq 0 \quad (10)$$

Where  $\lambda = \frac{1}{T_0}$   $T_0$  is the mean time of service a single customer. The time interval between next arrivals is exponential and has the form:

$$E(t) = \int_0^{\infty} t f(t) dt = \lambda \int_0^{\infty} t e^{-\lambda t} dt = \frac{1}{\lambda} \quad (11)$$

For a normal distribution while distribution function is:

$$F(t) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^t e^{-\frac{(x-m)^2}{2\sigma^2}} dx \quad (12)$$

Service times at selected stations and their distribution as well as the operations performed on them and the number of place in queue are shown in Table 4.

Table 4

**Service time on a selected stations**

Service station	Type classes serviced on station	Mean service time	Probability distribution of the service (t, p)	Operations	Number of place in queue
Entrance gate for cars (PWES)	ST, CS+KR, CS+KP, CS+NP	5 min.	exp(0,083; 0,0069) hour	1) registration of cargo arrived into logistics centre <sup>4</sup> ; 2) visual inspection of the technical condition of the cargo;	15
	CS+NZ	2 min.	exp(0,0333; 0,0011) hour	3) documents inspection; 4) indication of the service place.	
Entrance gate for trains (PWEK)	PTKZ, PTCP, PTKP, PTZR, PTZP	3 min / 28 wag..	exp(0,0018; 0,000003) hour	1) cargo registration; 2) indication on the right arrival track;	40
Arrival track (TP)	PTKP, PTZR, PTZP	27 min / 28 wag..	exp(0,0161; 0,00026) hour	1) removing end of train markers, coming off the traction unit; 2) composition transfer and inspection of technical	40

<sup>4</sup> Cars with cargo only

Service station	Type classes serviced on station	Mean service time	Probability distribution of the service (t, p)	Operations	Number of place in queue
	PTKZ, PTKP	20 min / 28 wag..	exp(0,0119; 0,00015) hour	condition of railway wagons <sup>5</sup> ; 3) attachment shunting locomotive.	
Waiting (parking) square (PP)	ST	10 h / 50 place.	exp(0,2; 0,04) hour	1) parking; 2) uses a place on parking; 3) launch vehicle and leaving the square parking	25
Container storage square (PSK)	CS+KR	8 min.	exp(0,133; 0,0178) hour	1) set the crane in the correct position for the wagon (or car); 2) grabbing the container by handle of crane; 3) pick up and move the container to the square storage; 4) return movement of the crane.	truck (parking) – 25;
	CS+KP	16 min.	exp( 0,266; 0,0708) hour		
	PTKZ	7 min.	exp(0,1167; 0,0136) hour		wagon (TO) – 120
	PTCP, PTKP	14 min.	exp(0,2333; 0,05444) hour		
Warehouse1 (M1)	CS+NZ	30 min.	exp(0,05; 0,0025) hour	1) entry of car under the ramp; 2) removal and insertion of load units from semi-trailer trucks and preliminary quantitative control; 3) car departure from the ramp.	25
	CS+NP	60 min.	exp(0,1; 0,01) hour		
Warehouse2 (M2)	PTZR	24 min.	exp(0,04; 0,0016) hour	1) enter the group of wagons under the ramp; 2) removal and insertion of load units from wagons and preliminary quantitative control; 3) shift of wagons group to the departure track.	120
	PTZP	48 min.	exp(0,08; 0,0064) hour		
Departure track (TW)	PTKZ, PTKP, PTZP	27 min. / 28 wag..	exp(0,0161; 0,00026) hour	1) disengagement and departure shunting locomotive; 2) exit and merge locomotive train; 3) seals check and technical inspection <sup>6</sup> ; 4) set up end of train markers; 5) detailed test of the brakes; 6) deregistration of cargo; 7) train departure from the group of departure tracks.	40
	PTZR	20 min / 28 wag.	exp(0,0119; 0,00015) hour		
Exit gate for cars (WYS)	ST, CS+KP, CS+NZ, CS+NP	5 min.	exp(0,083; 0,0069) hour	1) deregistration of cargo outgoing from logistics centre <sup>7</sup> ; 2) visual inspection of the technical condition of the cargo; 3) car departure on the external road;	3
	CS+KR	2 min.	exp(0,0333; 0,0011) hour		
Exit gate for trains (WYK)	PTKP, PTZR, PTZP, PTKZ, PTKP	27 min / 28 wag..	exp (0,0018; 0,000003) hour	1) deregistration of cargo coming from the base <sup>8</sup> ; 2) visual inspection of the technical condition of the cargo; 3) train departure on the tracks of general purpose;	40

Source: own development.

## 5. Research Results Analysis

The simulation was performed using the WinQsb. The description of the operation in the logistics centre uses the parameters described in second and third section. The study included 20-day operation of the logistics centre.

<sup>5</sup> Applies to wagons with cargo.

<sup>6</sup> Applies to train with cargo..

<sup>7</sup> Cars with cargo only

<sup>8</sup> Cars with cargo only

The analysis of the simulations shows that logistics centre is working without interruption, but its workload of posts is very small. Gradually workload increasing studies were made, how can be increased the workload the logistics centre to ensure greater use of its capacity without creating undue distortions in the flow of goods by these centre. In addition, a operating simulation of the logistics centre includes times of individual vehicles moving between different operating positions (Table 5).

In Table 6 results of simulation were presented.

Table 5

### Times for connections between the positions of service for each customer

Connection	Transition time	
	Minutes	Hours
Entrance gate for cars (WES) – parking (P)	2	0,033
Waiting (parking) square (PP) – exit gate for cars (WYS)	1	0,0167
Container storage square (PSK) – exit gate for cars (WYS)	3	0,05
Warehouse 1 – exit gate for cars (WYS)	1/28 wagons	0,0167
Entrance gate for trains (WEK) – arrival track (TP)	2/28 wagons	0,0012
Arrival track (TP) – secondary track(PTO)	4/28 wagons	0,0024
Departure track (TW) – exit gate for trains (WYK)	1/28 wagons	0,0006

Source: own development.

Table 6

### Simulation results

L.p.	Result	CS+N Z	CS+ KR	CS+ KP	CS+ NP	ST	PTK Z	PTC P	PTZ R	PTK P	PTZ P	Over all
1	Total Number of Arrival	328	434	421	348	560	557	250	523	540	488	4449
2	Total Number of Balking	0	0	0	0	0	0	0	0	0	0	0
3	Average Number in the System (L)	0.1647	0.292 1	0.276 2	0.290 0	0.453 1	0.878 8	0.210 1	0.983 4	0.307 9	1.162 6	5.018 8
4	Maximum Number in the System	5	8	5	8	9	31	28	30	30	28	182
5	Current Number in the System	0	0	0	0	0	0	0	0	29	0	29
6	Number Finished	341	446	432	368	587	556	250	492	510	438	4420
7	Average Process Time	0.1489	0.189 0	0.195 7	0.232 6	0.236 9	0.032 0	0.032 0	0.073 6	0.036 3	0.122 9	0.130 9
8	Std. Dev. of Process Time	0.0297	0.035 7	0.070 1	0.056 1	0.104 1	0.000 3	0.000 3	0.001 6	0.000 4	0.006 2	0.093 3
9	Average Waiting Time (Wq)	0.1476	0.177 1	0.179 7	0.254 6	0.255 4	0.724 3	0.371 3	0.826 3	0.250 9	1.035 8	0.437 6
10	Std. Dev. of Waiting Time	0.2660	0.266 0	0.318 1	0.333 9	0.440 7	0.627 6	0.448 5	0.638 5	0.152 2	0.619 9	0.546 4
11	Average Transfer Time	0	0	0	0	0	0	0	0	0	0	0
12	Std. Dev. of Transfer Time	0	0	0	0	0	0	0	0	0	0	0
13	Average Flow Time (W)	0.2889	0.354 4	0.368 0	0.460 1	0.470 5	0.756 7	0.403 3	0.900 5	0.287 2	1.158 8	0.561 0

L.p.	Result	CS+N Z	CS+ KR	CS+ KP	CS+ NP	ST	PTK Z	PTC P	PTZ R	PTK P	PTZ P	Over all
14	Std. Dev. of Flow Time	0.2399	0.255 8	0.346 2	0.266 4	0.384 1	0.628 1	0.448 4	0.640 7	0.152 2	0.620 4	0.523 9
15	Maximum Flow Time	2.1171	2.257 5	2.742 0	2.147 5	3.389 3	2.524 2	1.831 1	3.412 5	0.742 8	2.449 8	3.412 5

Source: own development based on WinQsb

Table 7 shows the results of the service stations in centre.

Table 7

### The results of service stations in logistic centre

Lp.	Server Name	Server Utilization	Average Process Time	Std. Dev. Process Time	Maximum Process Time	Blocked Percentage	#Customers Processed
1	WES	35.43%	0.0813	0.0212	0.1297	0.00%	2091
2	WEK	0.88%	0.0018	0.0000	0.0018	0.13%	2353
3	TP	7.22%	0.0149	0.0021	0.0176	0.00%	2330
4	PP	7.08%	0.0470	0.0959	0.3394	0.00%	723
5	PSK	1.07%	0.0619	0.1114	0.4449	0.00%	83
6	M1	5.74%	0.0214	0.0389	0.1431	0.00%	1285
7	M2	12.15%	0.0260	0.0342	0.1270	0.02%	2247
8	TW	7.22%	0.0149	0.0034	0.0181	0.00%	2329
9	WYS	39.27%	0.0901	0.0069	0.1328	0.00%	2091
10	WYK	0.84%	0.0017	0.0003	0.0018	0.00%	2329
	Overall	11.69%	0.0314	0.0421	0.4449	0.01%	17861

Source: own development based on WinQsb

Table 8 shows the results of the analysis of queues occurring at the logistics centre.

Table 8

### Results of the analysis of queues occurring at the logistics centre

Lp.	Queue Name	Average Q. Length (Lq)	Current Q. Length	Maximum Q. Length	Average Waiting (Wq)	Std. Dev. of Wq	Maximum of Wq
1	PWES	0.4726	0	12	0.1085	0.1340	1.0102
2	PWEK	0.1406	4	40	0.0286	0.0320	0.4585
3	P	0.0001	0	1	0.0000	0.0008	0.0274
4	PTP	1.0290	22	40	0.2117	0.1383	0.6560
5	TO	1.8217	0	63	0.3753	0.4970	2.2124
6	PWYS	0.1152	0	3	0.0265	0.0438	0.2744
7	PTW	0.2694	0	40	0.0555	0.1176	0.6555
8	PWYK	0.0001	0	1	0.0000	0.0002	0.0018
	Overall	3.8488	26	63	0.1029	0.2330	2.2124

Source: own development based on WinQsb



In Figures 4-7 are shown histograms of logistics centre parts performance obtained from the WINQSB.

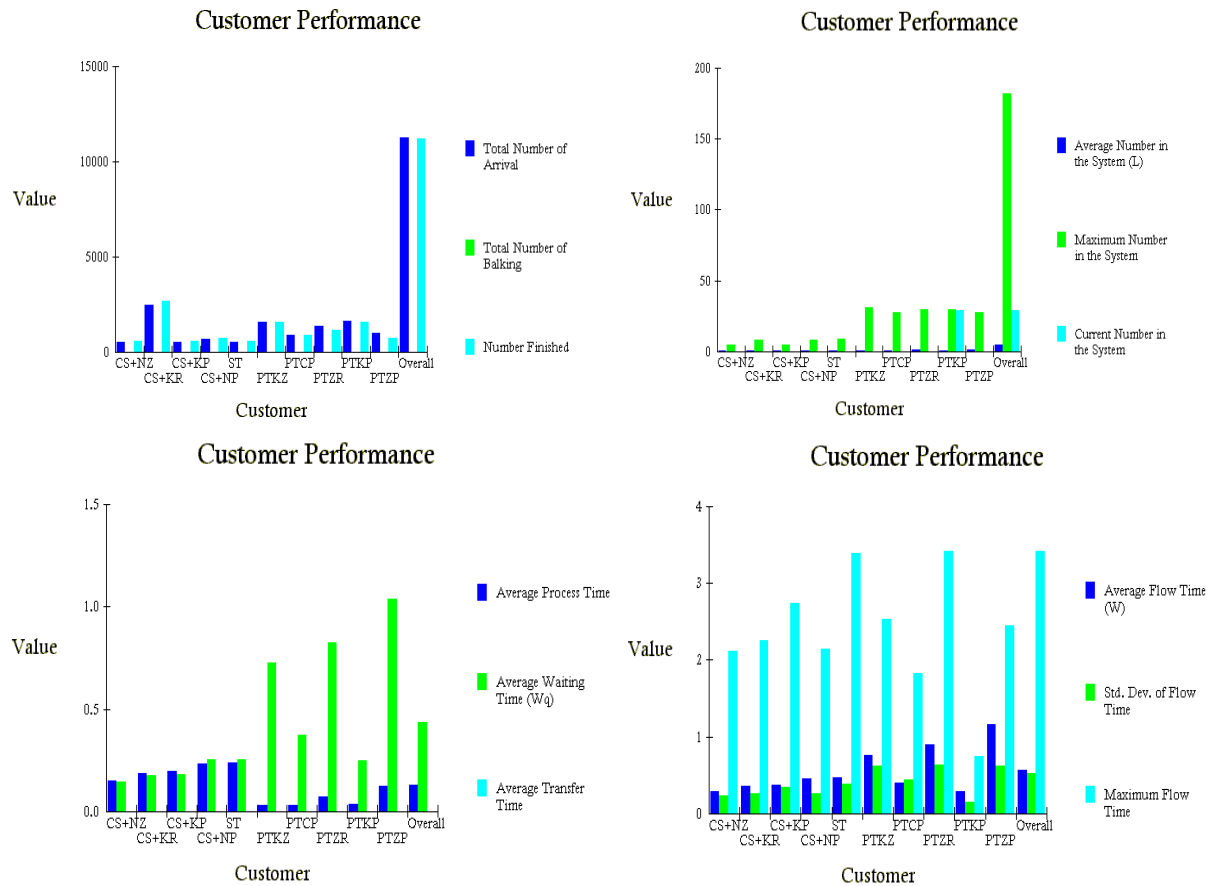


Fig. 5. Histogram of the logistics centre parts performance due to the flow of the stream of goods

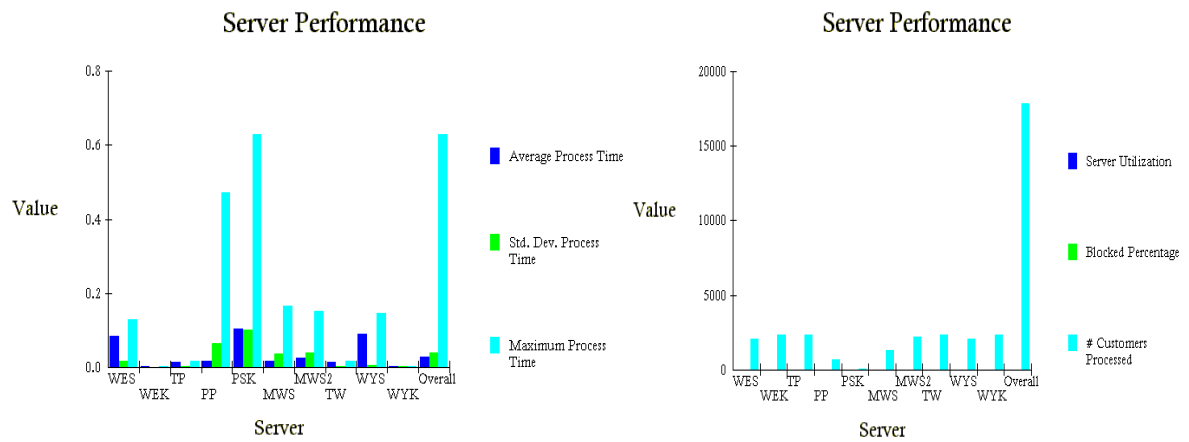


Fig. 6. Histogram of the logistics centre parts performance due to the size of the service channels

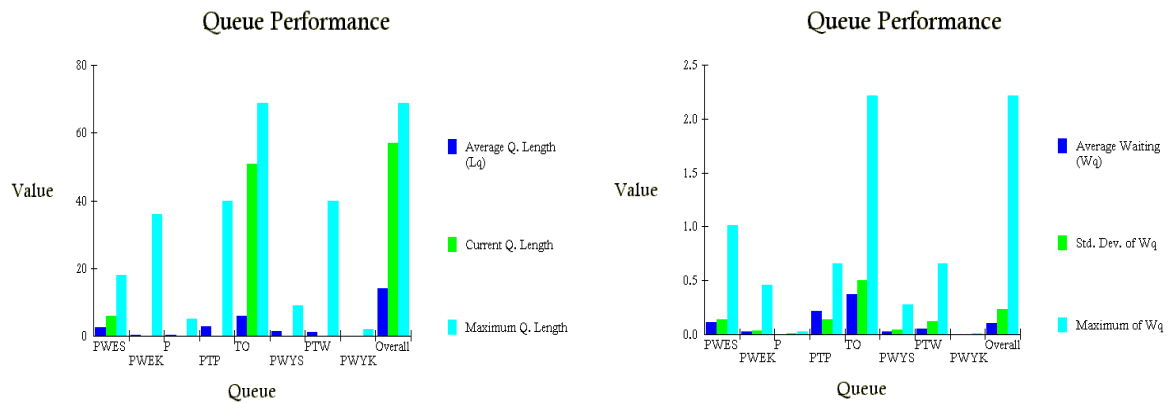


Fig. 7. Histogram of the logistics centre parts performance due to the size of the queues  
Source: own development

## Conclusion

The existing theoretical studies and practical applications are preferred deterministic models of workload degree assessment of the logistics centre.

According to the authors of the article the reasons for their use result from the convenience and the fact that probabilistic models require more data and constraints. Deterministic models are used to analyze and assess the degree of workload of logistics centres are prognostic models. Number of unit loads at the entrance to the logistics centre has a probabilistic character, therefore the authors suggest that to the analysis and evaluation probabilistic models should be used. Compared with deterministic models they reproduce in a higher degree of actual workload logistics centre.

Using probabilistic models can determine the change in the degree of workload of the logistics centre, depending on the time and the occurrence of random events. They allow ongoing analysis and evaluation of the degree of workload the centre, thus giving rise to changes in the organization of the Centre's operations depending on internal and external conditions, so above all changing the flow rates of charges.

Based on these results it can be concluded that the theory of queues is the right tool for the analysis and evaluation of the degree of workload of the logistics centre. Furthermore, when assessing the operating of the logistics centre can be seen that both the service time particular units in positions of cargo handling and transit times may impact on the equipment and the size of the logistics centre.

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