Vol. 10, No 2/2021, 127-136

DOI: 10.17512/bozpe.2021.2.16

Construction of optimized energy potential
Budownictwo o zoptymalizowanym potencjale energetycznym

ISSN 2299-8535 e-ISSN 2544-963X

2D and 3D preliminary numerical analysis of a masonry arch – case study

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Abstract: Masonry arches are regularly used in construction. The main purpose of this work was to perform numerical calculations using the Finite Element Method to estimate the load capacity of the designed masonry arch and to identify optimal locations for sensors on a real structure to aid advanced structural analysis. The model was implemented in two computer programs – Autodesk Robot and DIANA, and the results were compared. The arch was analysed statically and dynamically in the elastic range. A preliminary 2D static analysis was performed in Autodesk Robot, while in DIANA both flat and spatial models were analysed using both static and dynamic analysis. Numerical estimation of the load capacity of the masonry structure gave the opportunity to show the basic failure mechanisms together with the character of the stress distribution as well as the eigenfrequencies and eigenmodes. Satisfactory results were obtained, after a review of the literature.

Keywords: masonry structures, masonry constructions, FEM (Finite Element Method), failure mechanisms

Please, quote this article as follows:

Jurczak M.A., 2D and 3D preliminary numerical analysis of a masonry arch – case study, Construction of Optimized Energy Potential (CoOEP), Vol. 10, No 2/2021, 127-136, DOI: 10.17512/bozpe.2021.2.16

Introduction

Masonry arches have been used for a very long time and remains a great interest of scientists. It is worth noting that in the past they were constructed intuitively based on experience passed down from generation to generation, without performing calculations, and yet many of them have survived to this day. However, new theories, studies and computational processes began to be introduced over the centuries. The theory of circular arches was developed by Jacques Antoine Charles Bresse, who in 1848 systematized the work around arches and drew up a table of individual cases (Bresse, 1848). In the 1860s, Spanish engineer and architect Eduardo Saavedra analysed elastic arches (Saavedra, 1860). Soon afterwards, the Italian engineer Castigliano applied his spring system theorem to masonry bridges (Castigliano, 1879). Consequently, as engineers at the end of the 19th century were looking for a real solution that best reflected the behaviour of masonry arches, the theory of elasticity turned out to be the best solution. Nevertheless, arches cracked under the influence of an non-centrally applied load, and although the material was taken as anisotropic and discontinuous, this theory was displaced by Huert's "new arch theory" (Huerta, 2001). These days, most practical problems in engineering are solved by numerical methods, due to the anisotropic properties of the wall.

Masonry structures, including arches, are still subjected to numerical and experimental analyses, which are confirmed by publications (Ramos et al., 2010; Giordano et al., 2020). In this work, a single-curved wall vault in the shape of a semicircle was designed on the basis of a review of the literature, and then numerical calculations using the Finite Element Method were carried out, giving accurate enough results to be acceptable to the technical environment (Sanches, 2007). The main purpose of the work was to identify suitable locations of sensors on a real structure, i.e. vibration sensors, strain gauges, for more advanced structural analysis, based on the behaviour of the structure under load until destruction. As a first step, numerical static and dynamic analysis was performed and is presented in this work. The static analysis concerned the presentation of the most strained areas. Dynamic analysis allowed the representation of the places with the largest displacements, and vibration forms would allow the location of dynamic sensors (e.g. accelerometers). It should be remembered that places where the displacement of the arch site is equal to 0 be avoided when locating these sensors. The detailed purpose of the analysis was to determine the strain of the most sensitive areas of the arch model – areas of potential damage in 2D and 3D analysis. A comparison of these two analyses would answer the question whether a 2D analysis of this type of structure would be sufficient. In addition, calculations were made with two programs in order to authenticate the results. The Autodesk Robot program is widely used in the construction industry (more commercial), and DIANA is more extensive in terms of the adaptability of its own models (more scientific).

Identification of damage at an early stage is very important, as it avoids the destruction of the structure during the construction phase and at the later stage of operation in order to protect human life and preserve cultural heritage.

1. The mechanics of masonry structures

Masonry structures are heterogeneous materials with directional mechanical properties due to its masonry elements and mortar. Their interconnection creates a complex stress state even in the case of single-axis compression throughout the component. Masonry structures best carry compressive loads perpendicular to the system of supportive welds. This property has been used since the beginning of the construction of walls, especially arches and vaults, laying bricks in such a way that the pressure line remains in the middle of the cross-section. Scratches and cracks are most often caused by stretching, as the wall has almost ten times less tensile strength than compression.

In numerical calculations using the MES method, masonry structures can be thought of as a homogeneous or heterogeneous model – a division into masonry elements and mortar. Regardless of the calculation model selected, scratches occur when the limit load capacity is reached in the wall. A feature of these structures is that one scratch does not mean complete destruction, but at the point of its occurrence there is a redistribution of forces and stresses to adjacent finite elements. In order to further explore the mechanism behind the destruction of masonry structures, one can refer to the literature (Jemioło & Małyszko, 2013; Małyszko & Orłowicz, 2000).

2. Materials and methods

2.1. Geometry and material of the arch

A single-curvature vault in the shape of a semicircle with a radius of 0.90 m from autoclaved aerated concrete bricks connected by mortar was designed. The outer arrow of the arch is 1.02 m and its internal span is 1.80 m, a width equal to 0.50 m and a thickness equal to 0.12 m. The arch on both sides lies on 12x12x50 cm blocks. Details about the arch geometry are presented in Figure 1.

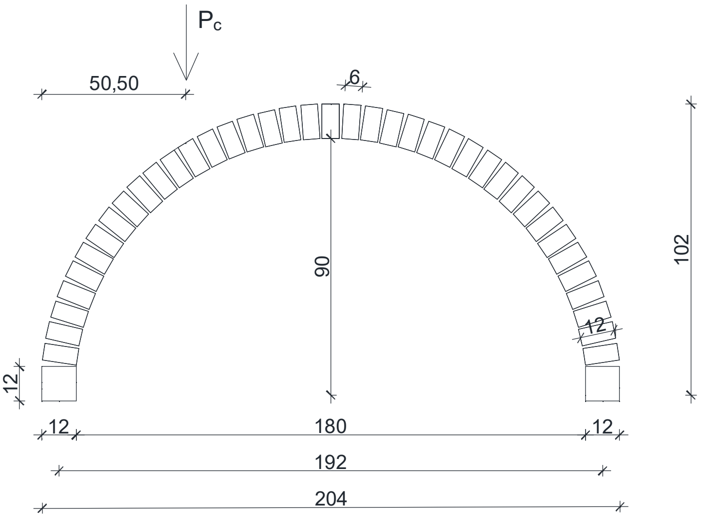


Fig. 1. The geometry of the masonry arch (*own study*)

The material parameters for concrete blocks and mortars are adopted in accordance with Table 1.

Table 1. Summary of the material parameters of concrete blocks and mortar (Małyszko et al., 2017)

	Autoclaved aerated concrete	Mortar
Young's modulus E [MPa]	1225	3411
Poisson ratio v [–]	0.30	0.30
Mass density ρ [kg/m^3]	6.86	15.00
Kirchhoff modulus G [MPa]	542	1312
Average compressive strength f [MPa]	4.0	2.0

2.2. Software

Numerical calculations were performed using the MES finite element method in two computer programs Autodesk Robot (User) and DIANA (DIANA). After reviewing the literature, it was decided to place the load in 1/4 of the arch span (Fig. 1), as this allows the determination of the smallest destructive load for masonry arches (Heyman, 1982). Additionally, Diana performed calculations in a 3D model to find out if the 2D model was accurate enough.

2.3. Models

For the modelled arch, static analysis was performed in Autodesk Robot, assuming a concentrated force load of P = 5 kN in 1/4 of the wall span. The arch was divided into masonry elements and mortar and assigned corresponding properties. Normal stresses, principal stresses and displacements were considered the most important calculation parameters.

For the 2D arch modeled in DIANA, the static effects of external interactions in the linear-elastic range were determined. The designed MES mesh consisted of 600 four-node, regular finite elements (Q8MEM), with 726 nodes and 1452 degrees of freedom. In this case, the material was homogenized, giving the whole properties as for autoclaved aerated concrete brick. Preliminary dynamic calculations were also carried out, obtaining eigenfrequencies and eigenmodes.

In order to compare the results and their possible correction, as well as to improve the accuracy of the calculations, an analysis was also performed in 3D space in DIANA 9.3. In this case an FEM mesh consisting of 6000 regular finite elements (HX24L) was created, with 7986 nodes and 23958 degrees of freedom.

3. Results

3.1. Static analysis

3.1.1. Autodesk Robot 2D model

After static analysis in Autodesk Robot, extreme value parameters of the calculations were summarized in Table 2. Figure 2 shows the principal stress maps.

Table 2. Summary of extreme values of analysed calculation parameters – Autodesk Robot (*own study*)

Calculated value*	Horizontal direction X		Vertical direction Z	
Displacement δ [mm]	–0.9	1.0	–1.0	1.0
Normal stresses $\sigma_{x,,y}$ [MPa]	–1.042	0.664	–1.489	0.640
Principal stresses $\sigma_{1,2}$ [MPa]	**1**		**2**	
	–0.514	0.668	–1.725	0.082

* A minus sign at stress values indicates that they are compressive. A minus sign at displacement values indicates that they are opposite to the assumed coordinate axes.

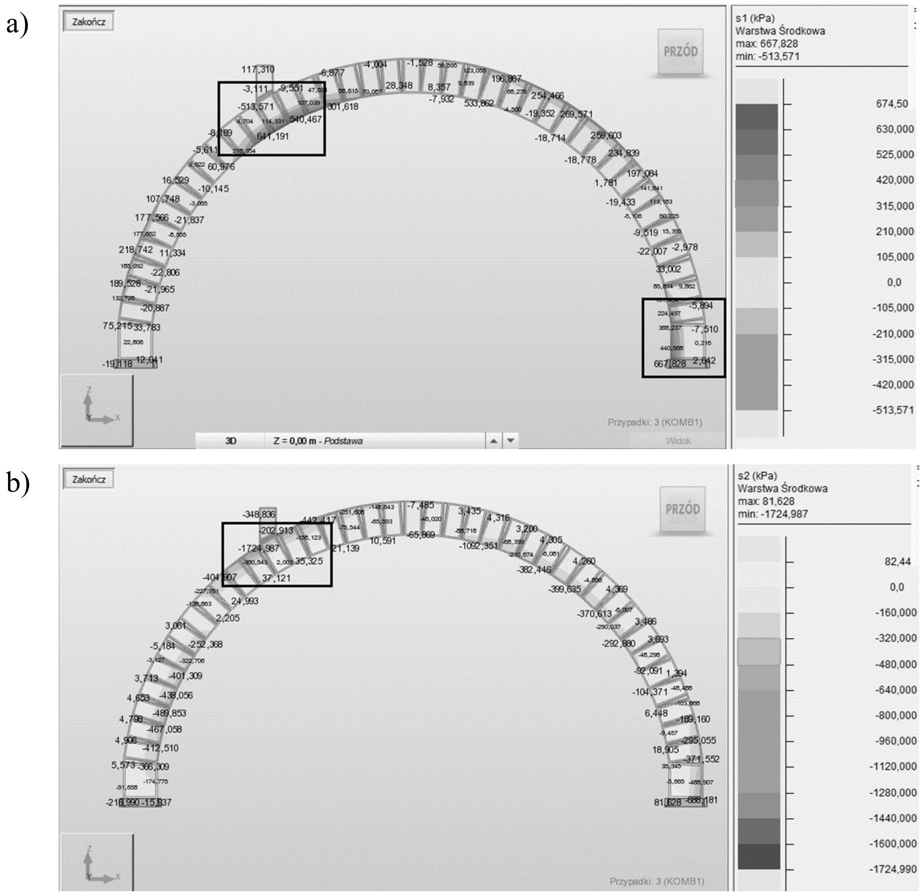


Fig. 2. Principal stress plot: a) σ_1, b) σ_2 (*own study*)

3.1.2. DIANA 2D model

Table 3 shows the extreme values of the parameters analysed, and the main stress maps are shown in Figure 3.

Table 3. Summary of extreme values of analysed calculation parameters – DIANA (*own study*)

Calculated value[*]	Horizontal direction X		Vertical direction Z	
Displacement δ [mm]	–0.004	0.672	–0.573	0.278
Normal stresses $\sigma_{x.,y}$ [MPa]	–0.631	0.381	–0.655	0.439
Principal stresses $\sigma_{1,2}$ [MPa]	**1**		**2**	
	–0.873	0.506	–0.932	0.440

[*] A minus sign at stress values indicates that they are compressive. A minus sign at displacement values indicates that they are opposite to the assumed coordinate axes

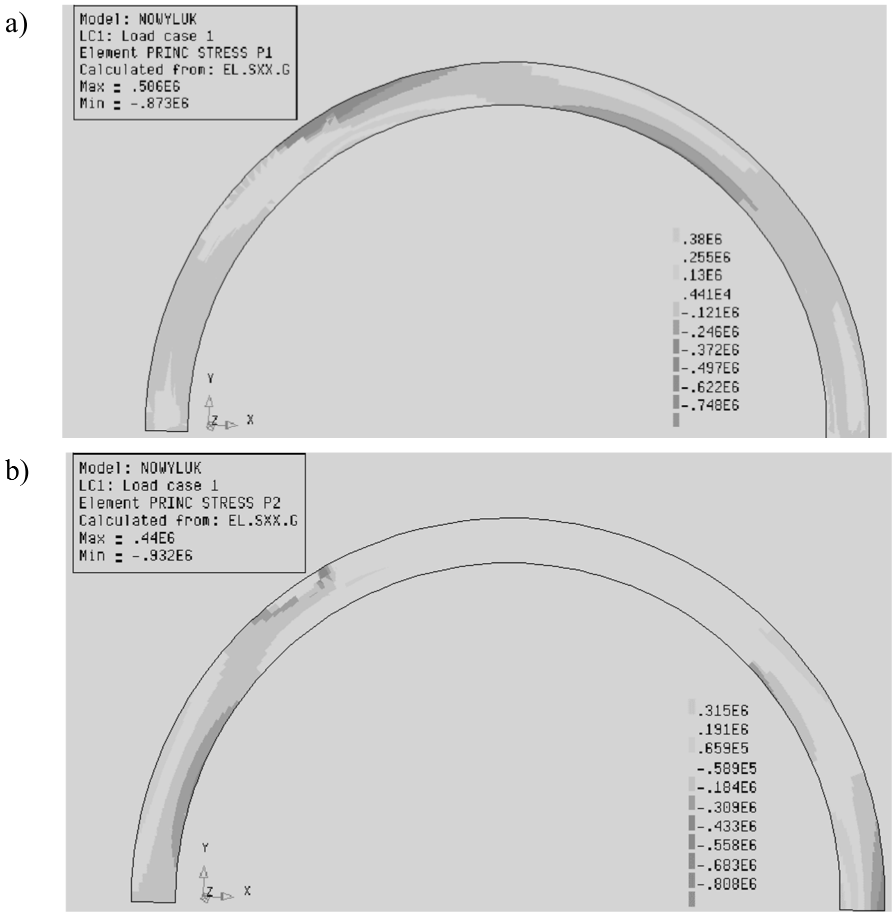


Fig. 3. Principal stress plot: a) $\boldsymbol{\sigma_1}$, b) $\boldsymbol{\sigma_2}$ (*own study*)

3.1.3. DIANA 3D model

For comparison with the results of the 2D analysis, calculations were made for the 3D model in DIANA. Table 4 summarizes extreme values, and Figure 4 shows their numerical distribution.

Table 4. Summary of extreme values of analysed calculation parameters – DIANA (*own study*)

Calculated value[*]	Horizontal direction X		Vertical direction Z	
Displacement δ [mm]	–0.007	0.679	–0.600	0.281
Normal stresses $\sigma_{x,,y}$ [MPa]	–0.760	0.383	–0.014	0.450
Principal stresses $\sigma_{1,2}$ [MPa]	**1**		**2**	
	–0.904	0.507	–0.016	0.498

[*] A minus sign at stress values indicates that they are compressive. A minus sign at displacement values indicates that they are opposite to the assumed coordinate axes

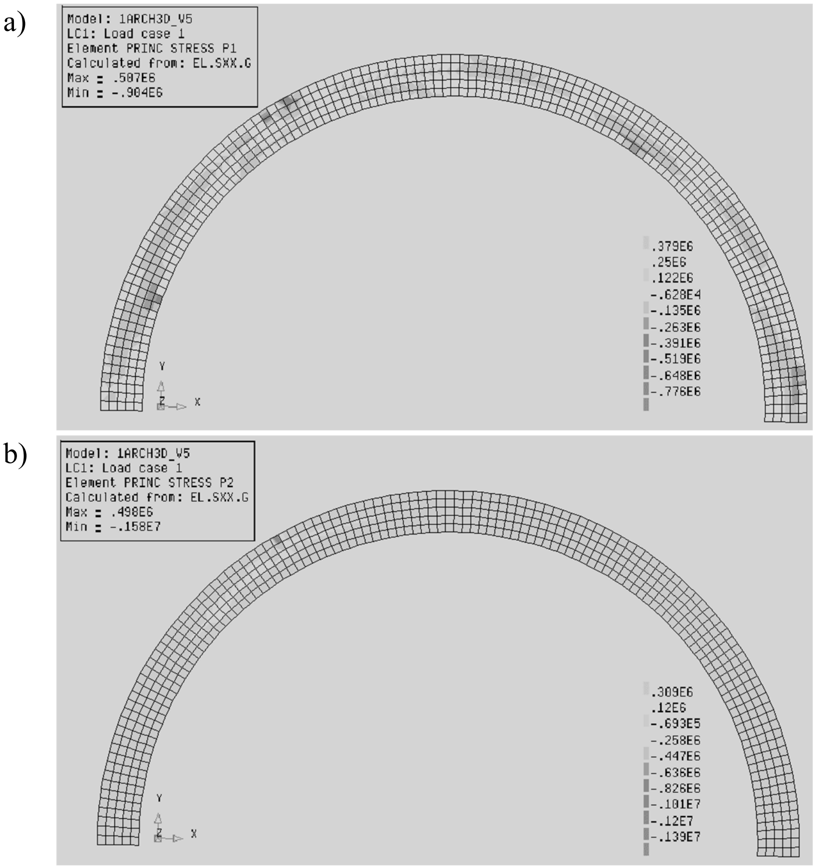


Fig. 4. Principal stress plot: a) σ_1, b) σ_2 (*own study*)

3.2. Dynamic analysis

3.2.1. DIANA 2D and 3D models

Figure 5 shows a comparison of eigenfrequencies and eigenmodes for the 2D and 3D analysis.

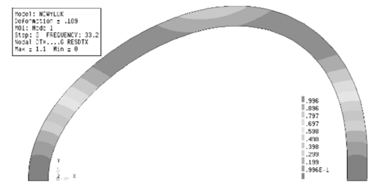
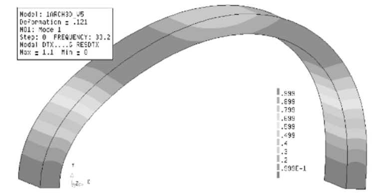
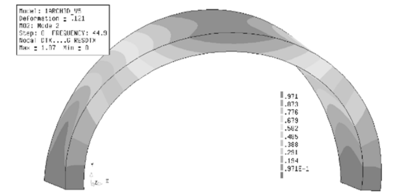
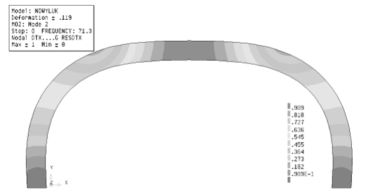
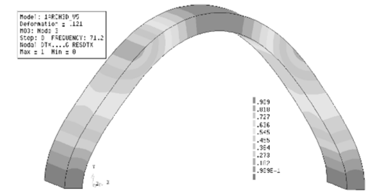
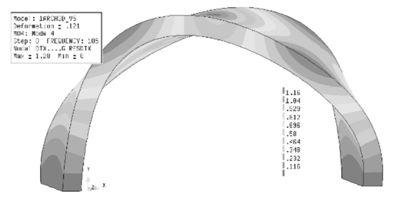
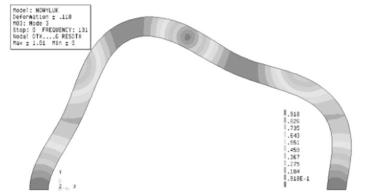
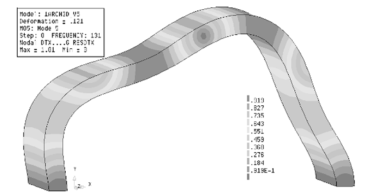
MODEL 2D		MODEL 3D	
Mode 1: 33.2 Hz		Mode 1: 33.2 Hz	
		Mode 2: 44.9 Hz	
Mode 2: 71.3 Hz		Mode 3: 71.2 Hz	
		Mode 4: 105.0 Hz	
Mode 3: 131.0 Hz		Mode 5 : 131.0 Hz	

Fig. 5. Comparison of eigenfrequencies and eigenmodes for 2D and 3D analysis (*own study*)

4. Discussion

Masonry is a brittle material with almost ten times less tensile strength than compression. Particular attention was paid to the most strenuous areas. From the graphic map, produced from the Autodesk Robot analysis shown in Figure 2a, it can be seen that the maximum tensile stresses, both normal and principal, are formed on the inner side of the arch under the applied load, reaching values of up to 0.664 MPa, as well as at the right support of up to 0.668 MPa. In these places, damage can appear, which shows compliance with the most common mechanisms of arch failure (Hojdys & Janowski, 2011; Hojdys et al., 2014; Masciotta et al., 2016).

The results of the analysis in DIANA gives the most sensitive areas of the arch where the probability of destruction is greatest. Looking at the stress diagrams in Figure 3, it is clear that the most tensile places are located on the inner side of the arch under the applied load, symmetrically to this point but outside and at the right support. Such a scheme reflects classically accepted models of destruction of masonry vaults in the form of a semicircle (Ramos, 2007).

From the 3D static analysis in DIANA, convergent results were obtained to the 2D model, almost identical values of displacements and stresses.

In the dynamic analysis, special attention was paid to the first value of the eigenfrequency, which was decisive in determining the susceptibility of the structure. In the DIANA program, for both the 2D and 3D model, the value of $f_1 = 33.2$ Hz was achieved, which indicates the high stiffness of the arch. In this analysis, there is a big difference between the successive detected eigenfrequencies and eigenmodes. In the 2D model, no intermediate values were obtained, from the 3D analysis. The second and fourth eigenmodes are not visible in the plane, because it is a spatial form. The remaining presented values are almost identical for both models.

Conclusions

Before proceeding with destructive testing, it is very important to numerically determine the places where cracks are predicted. Conducting numerical simulations achieved the basic goal of this work, that is, to show the places of predicted damage to the masonry arch. Thanks to this, it became possible to determine the optimal location of dynamic sensors, giving the possibility of further research on a real arch model. It is worth noting that on each presented stress map the majority stress is compression, which is a typical phenomenon for masonry vaults. The results obtained in the two programs slightly differ, which may be due to the fact that in the first case the model was divided into masonry and mortar elements, and assigned corresponding properties, while in the second program the material was homogenized, giving the whole parameters as for autoclaved aerated concrete bricks. This would explain the smaller calculation values obtained in the DIANA program – lower elastic parameters were specified for masonry elements than for mortar. However, despite these differences, the nature of the damage in both cases is very similar, which can confirm the accuracy of the calculations carried out.

Based on the dynamic analysis, it was found that in this case the 2D model is insufficient. In order to more accurately illustrate the behavior of the arch, a spatial model should be made. It was also possible to observe intermediate figures of a torsional and flexible-torsional nature, occurring in the conditions of actual operation of the structure.

After an in-depth analysis of the static results from the calculations of the 3D model in DIANA, similar results to the 2D analysis were found, while there were differences in the dynamic analysis, which allows us to conclude that in order to more accurately illustrate the behavior of the arch, a spatial model should be made.

The most effective test of the results will be validation by conducting experimental studies on a real model in the laboratory, which may be the next step in conducting a more detailed analysis, and goes beyond the scope of this work.

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