GEOMETRICAL ASPECTS OF AN INSOLATION OF A MULTI-SIDE ROOF'S SURFACE

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Abstract: This paper aims at formulation of universal method of evaluating the amount of solar energy that is absorbed by building's roof which is necessary to determine the optimum value of solar energy depending on roof's geometry and its location in respect to the four quarters of the globe.
The paper presents an interesting application of Monge's method of appointing shade geometry with a help of AutoCAD software. The concept of direct surface insolation indicator is introduced in order to determine the volume of solar energy.

Keywords: Descriptive Geometry, multi-side roof's geometry, solar energy solar radiation components, geometry of shade and shadows

1 Introduction

This paper is the first stage of the analysis whose aim is to formulate a universal method of evaluation of the amount of solar energy that is absorbed by building's roof which is necessary for optimum evaluation depending on roof's geometry and its location in respect to the four quarters of the globe. The main aim of this paper is formulation of general assumptions and research methods of the influence of a detached house multi-sided roof's shape (geometry) on the amount of solar energy consumption. Due to the fact that this issue is very broad, the first assumptions concern direct radiation. The first main problem is determination of the size of sunlit roof's surface at a given time depending on the season with given precision (day, hour, minute, etc.). A surface for a given roof, as a specified multi-sided surface, may be defined at a given moment with the help of classical descriptive geometry methods, for example Monge's method.

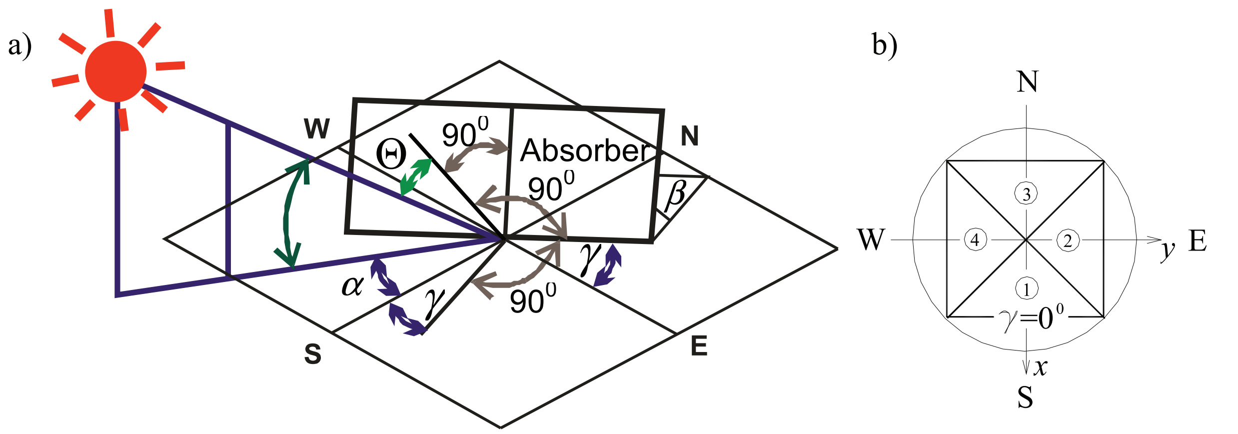

Figure 1

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The following parameters were assumed: a specified shape of building's roof at a given point and changing, at certain time intervals, the Sun's location. For that geometrical situation roof's (building's) own shade was found and the area of a section was calculated. For this purpose the ratio of direct solar radiation was used.

2 Formulation of the problem

As a starting point for discussion we take an empirical formula of solar radiation components: direct ray I_b, diffused ray I_d and reflected ray I_h:

$$I = I_b \cos\theta + \frac{I_d(1+\cos\beta)}{2} + \frac{I_h(1+\cos\beta)}{2} \quad [W/m^2] \tag{1}$$

where: θ stands for an angle enclosed between two lines: one that is determined by the direction of direct solar radiation and the other normal to the absorber's surface (see fig.1), β is the angle of collector's surface inclination in relation to ground level. In formula (1) component of direct radiation depends on the solar rays' angle of incidence $-\theta$. We may formulate *wnb* index which describes the value of insolation:

$$wnb = \Sigma\, \boldsymbol{P}_i \cdot \cos\theta_i\ [m^2], \tag{2}$$

where $\boldsymbol{P}_i$ stands for sunlit surface area $\mathbf{P}_i$, and θ_i is the solar rays' angle of incidence on the surface $\mathbf{P}_i$, (i=1,2,3,4).

The problem concerning the description of the amount of solar energy dimension absorbed by a given surface in the course of a certain period of time (*wnbt* index will be formulated with an appropriate integral after taking into consideration the time) is very complex. It requires a formulation of a general algorithm of shade evaluation or to be more precise solar rays' illumination for a specific class of geometrical surfaces. We will restrict our analysis to chosen multi-sided surfaces (roofs) on selected days of the year. The established method remains quite universal and indicates research directions in order to find a solution for a general issue.

In the mathematical description of this phenomenon we will make use of a *spherical system,* whose centre is an observation post, main circle is the horizon and the main direction - geographical direction of South (S). We assume that the Earth at an observation post, is flat, and the horizon seen from that point is motionless and describes a perfect circle. Then, solar coordinates are: *solar azimuth* α defined as the angle between the equator's semi-plane in the southern hemisphere and a semi-plane of a vertical circle crossing the Sun, and *astronomical altitude* ε that is the angle between the horizontal surface and the direction of solar rays' incidence. For their description *declination* δ*,* that is an angle between vector connecting the Earth with the Sun and the equator's surface is needed. That declination can be calculated from Cooper's formula:

$$\delta=23{,}45^{\circ}\sin\left(360^{\circ}\frac{284+n}{365}\right), \tag{3}$$

where n stands for the number of the day during a year.

Subsequently we appoint the so called "hourly Sun's angle" calculated as:

$$\omega = 15^{\circ}\,(12 - t_s), \tag{4}$$

where t_s stands for the time calculated in hours. At 12 p.m. the dimension of an angle ω equals 0°. An hourly angle changes at the speed of 15 degrees per hour.

3 Samples calculation for selected four days in Białystok

Calculations will be conducted for Bialystok at four selected days of the year: 21st March, 22nd June, 23rd September and 22nd December. For Bialystok, whose latitude φ equals to 53^{o}08', we can distinguish the following time intervals of the Sun operating on the horizon: 21.03–(6:28,18:42); 22.06–(3:59, 20:59); 23.09–(6:13,18:25); 22.12–(8:38,16:43).

By assuming a step of 20 minutes we create a sequence of angles from the formula:

$$\varepsilon = \arcsin(\sin\varphi\sin\delta + \cos\varphi\cos\delta\cos\omega) \tag{5}$$

with a step of 5^{o} as a base for a table of elevation angles.

An exemplary shade calculation of hipped roof end for two variants of roof's arrangements is as follows:

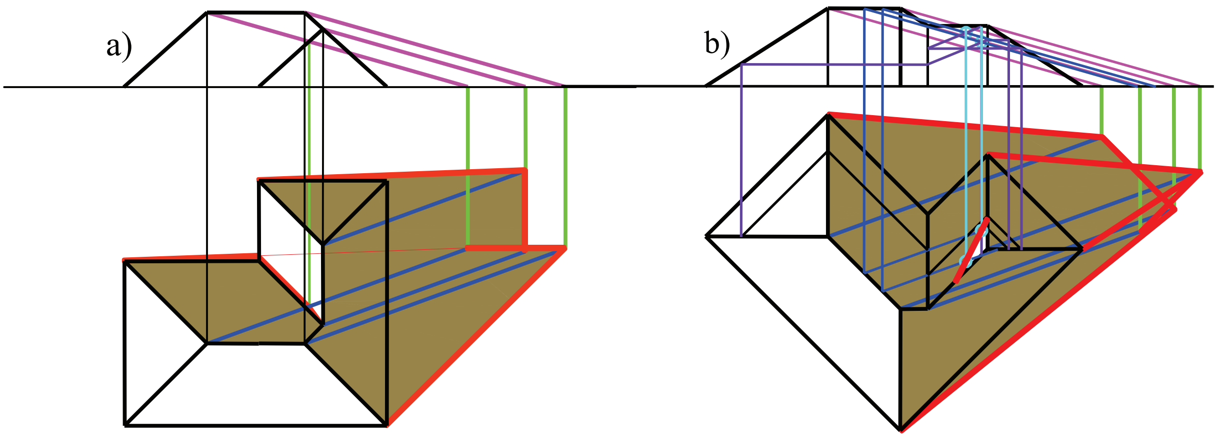

Figure 2

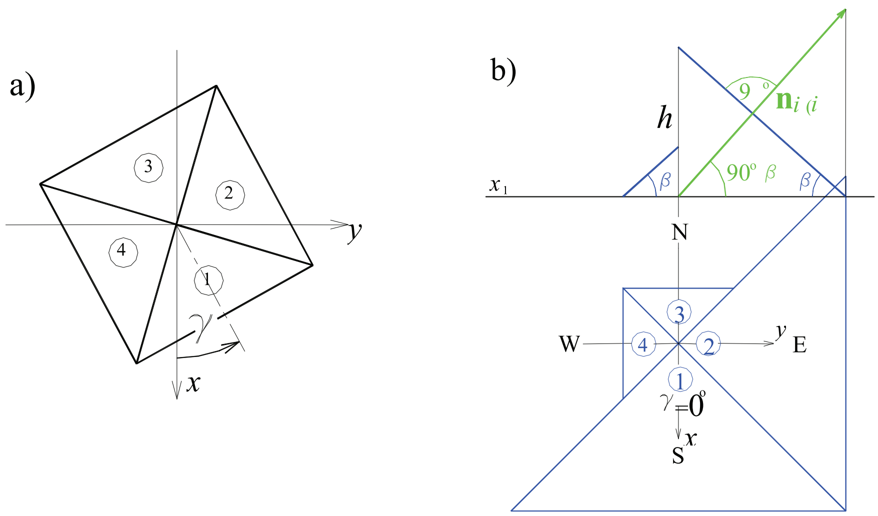

Figure 3

We appoint the azimuth angle α from the formula:

$$\alpha = \arccos\left(\frac{\sin\varepsilon\sin\varphi - \sin\delta}{\cos\varepsilon\cos\varphi}\right). \tag{6}$$

After determination of the shades, reading of an insolated surface takes place (fig. 2a,b). By putting into the table the available data we receive (from astronomical tables): 35 pairs of angles (ε,α) for 21.03, 50 pairs of angles 22.06, 36 for 23.09 and 22 for 22.12. The data as well as the values of angels calculated from formulas (2), (3), (4), (5), (6) values were inserted into the table using software Excel (see table 1). In order to determine the angle of solar rays' incidence on any hipped roof end and over any right-angled polygon we can take a tentorial roof with an angle of inclination β. This way we limit our discussion to the roof

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spread over a rectangular polygon. We determine the position of a building by means of the angle γ related to the selected hipped roof end.

As we can see in figure 3, the selected hipped roof end, which defines the position, is hipped roof end 1. All hipped roof ends have exactly four (i=1,2,3,4) different locations in respect to the four cardinal points. According to those assumptions we may estimate the unitary vector of solar ray $\mathbf{n}(\alpha,\varepsilon)$ as a $\mathbf{n}(\alpha,\varepsilon) = [\cos\alpha \cos\varepsilon, \sin\alpha \cos\varepsilon, \sin\varepsilon]$. The unitary vector $\mathbf{n}_i(\beta,\gamma)$ which is perpendicular to the hipped roof end i, (i=1,2,3,4) has the following coordinates $\mathbf{n}_i(\beta,\gamma) = [\sin\beta \cos(\gamma+(i-1)90^{o}), \sin\beta \sin(\gamma+(i-1)90^{o}), \cos\beta]$. Therefore the angle's cosine equals scalar product $\cos\theta_i = \mathbf{n}(\alpha,\varepsilon)\circ\mathbf{n}_i(\beta,\gamma)$ or

$$\cos\theta_i = \cos\varepsilon \sin\beta \left(\cos\alpha \cos(\gamma+(i-1)90^{o}) + \sin\alpha \sin(\gamma+(i-1)90^{o})\right) + \sin\varepsilon \cos\beta. \quad (7)$$

Table 1

		eps	alpha				
		2	127				
		5	123				
		7	120				
		10	116				
		13	112				
		16	108				
		19	105				
eps	alpha	21	101	eps	alpha		
6	82	24	97	3	86		
9	78	27	93	6	82		
12	74	30	89	9	78		
15	70	33	85	12	74		
17	65	36	81	15	70		
20	61	39	76	17	65		
23	56	42	72	20	61	eps	alpha
25	51	45	67	23	56	2	45
27	46	48	61	25	51	4	41
29	41	50	55	27	46	6	36
31	36	53	49	29	41	8	32
33	30	55	42	31	36	9	28
34	24	57	35	33	30	10	23
35	19	58	27	34	24	12	19
36	12	59	18	35	19	12	14
37	6	60	9	36	12	13	9
37	0	60	0	37	6	13	5
37	6	60	9	37	0	13	0
36	12	59	18	37	6	13	5
35	19	58	27	36	12	13	9
34	24	57	35	35	19	12	14
33	30	55	42	34	24	12	19
31	36	53	49	33	30	10	23
29	41	50	55	31	36	9	28
27	46	48	61	29	41	8	32
25	51	45	67	27	46	6	36
23	56	42	72	25	51	4	41
20	61	39	76	23	56	2	45
17	65	36	81	20	61	0	49
15	70	33	85	17	65	-2	53
12	74	30	89	15	70		
9	78	27	93	12	74		
6	82	24	97	9	78		
3	86	21	101	6	82		
0	90	19	105	3	86		
-3	94	16	108	0	90		
-6	98	13	112	-3	94		
		10	116				
		7	120				
		5	123				
		2	127				
		0	131				
		-2	135				
		-4	139				

Calculations made on the basis of formula (7) in Excel spreadsheet were then tabulated as shown in table 2. The calculations were made for two variants of roof's location. Geometric outcomes of sunlit roof parts were established with the help of AutoCAD software. That made in possible to obtain perfectly determined polygons of sunlit parts as well as precise areas of sunlit roof parts (auto thanks to in AutoCAD software). The obtained values were inserted into the appropriate columns in Excel spreadsheet (see table 2).

Table 2: The determination of direct surface insolation indicator in Excel

The table of calculation indicators of roof's insolation												
area ω_i	student's	June	June	June	June	area ω_1	area ω_2	area ω_3	area ω_4	area ω_5	area ω_6	student's
$\cos\theta$ $\gamma=0^{\circ}$	surname	sin(eps)	eps	cos(alpha)	alpha	$\cos\theta$ $\gamma=0^{\circ}$	$\cos\theta$ $\gamma=0^{\circ}$	$\cos\theta$ $\gamma=0^{\circ}$	$\cos\theta$ $\gamma=0^{\circ}$	$\cos\theta$ $\gamma=0^{\circ}$	$\cos\theta$ $\gamma=0^{\circ}$	surname
$\cos\theta$ $\gamma=45^{\circ}$			degrees		degrees	$\cos\theta$ $\gamma=45$	$\cos\theta$ $\gamma=45$	$\cos\theta$ $\gamma=45$	$\cos\theta$ $\gamma=45$	$\cos\theta$ $\gamma=45$	$\cos\theta$ $\gamma=45^{\circ}$	
	Citko	0,0426	2	-0,6063	127	0,0000	54,1000	18,6700	0,0000	20,3000	0,0000	
		0,7952	0,04	-0,6063	2,22	0,0000	30,4738	8,1588	0,0000	8,8711	0,0000	
						0,0000	54,1000	18,6700	0,0000	46,5300	0,0000	
						0,0000	6,5439	12,9606	0,0000	32,3009	0,0000	
		0,0853	5	-0,551	123	0,0000	54,1000	18,6700	0,0000	20,4100	0,0000	
		0,8345	0,09	-0,551	2,15	0,0000	33,5260	8,0415	0,0000	8,7909	0,0000	
						0,0000	54,1000	18,6700	0,0000	46,1100	0,0000	
						0,0000	10,6571	13,3774	0,0000	33,0386	0,0000	
	Gromada	0,1297	7	-0,4941	120	0,0000	54,6000	18,6700	0,0000	25,8000	0,0000	
		0,8694	0,13	-0,4941	2,09	0,0000	36,7571	7,9192	0,0000	10,9436	0,0000	
						0,0000	54,6000	18,6700	0,0000	47,0000	0,0000	
						0,0000	14,8762	13,7420	0,0000	34,5942	0,0000	
		0,1755	10	-0,4356	116	0,0000	54,6000	18,6700	0,0000	28,2000	0,0000	
		0,9001	0,18	-0,4356	2,02	0,0000	39,4968	7,7930	0,0000	11,7710	0,0000	
						0,0000	54,6000	18,6700	0,0000	47,4000	0,0000	
						0,0000	18,9348	14,0517	0,0000	35,6749	0,0000	
	Kulik	0,2225	13	-0,3758	112	0,0000	54,5600	18,6700	0,0000	30,0700	0,0000	
		0,9267	0,22	-0,3758	1,96	0,0000	42,0033	7,6639	0,0000	12,3435	0,0000	
						0,0000	54,5600	18,6700	0,0000	44,9300	0,0000	
						0,0000	22,8838	14,3041	0,0000	34,4234	0,0000	
		0,2701	16	-0,3147	108	0,0000	54,5600	18,6700	0,0000	33,0000	0,0000	
		0,9492	0,27	-0,3147	1,89	0,0000	44,3173	7,5327	0,0000	13,3144	0,0000	
						0,0000	54,5600	18,6700	0,0000	44,9400	0,0000	
						0,0000	26,7238	14,4973	0,0000	34,8961	0,0000	
	Olechno	0,3181	19	-0,2521	105	0,0000	54,5653	18,6726	0,0000	47,8223	0,0000	
		0,9677	0,32	-0,2521	1,83	0,0000	46,3967	7,4015	0,0000	18,9561	0,0000	
						0,0000	54,5653	18,6726	0,0000	35,0355	0,0000	
						0,0000	30,4145	14,6319	0,0000	27,4540	0,0000	

After summing up certain columns from spreadsheet, thus calculation of *wnb* index, we receive

Table 3: The list of components from calculating *wnb*1 and *wnb*2 indicators

		surface w_1	surface w_2	surface w_3	surface w_4	surface w_5	surface w_6	Σ
21 March	cos*q* γ=0°	1516,64	724,34	0,00	293,89	0,00	454,63	
	cos*q* γ=45°	1426,51	1119,08	108,03	35,91	226,03	0,00	
22 June	cos*q* γ=0°	1144,03	1079,80	340,38	510,17	735,33	686,62	
	cos*q* γ=45°	1412,96	1134,20	355,97	388,03	815,51	607,34	
23 September	cos*q* γ=0°	1503,36	750,17	0,00	297,16	0,00	449,54	
	cos*q* γ=45°	1427,30	1133,43	96,72	52,56	196,25	200,74	
22 December	cos*q* γ=0°	957,07	231,59	0	0	0	139,49	
	cos*q* γ=45°	765,10	662,68	0,46	0	0	0,76	
	cos*q* γ=0°	5121,10	2785,91	340,38	1101,22	735,33	1730,29	11814,22
	cos*q* γ=45°	5031,88	4049,40	561,18	476,50	1237,80	808,85	12165,59

Values of *wnb*1= 11814,22 for the first building's location (figure 2a) and *wnb*2= 12165,59 the second building's location (table 3). The conducted calculations demonstrate that the building in figure 2b has higher indicator of roof's insolation. In order to calculate the total solar energy that is absorbed by the roof, we need to calculate the energy taken by different roof surfaces. We calculate them using the integral calculated at time *t*.

4 Proposed partial automation of the calculation

Shade constructions as well as the conducted calculations were obtained with the help of well known and popular software: AutoCAD and Excel. However they were made most of all by means of classic methods. The proposal of partial automation, which will be described in a separate paper, may be shortly described as follow.

The procedure of shade determination on roofs may be partly automated with the use of for example ArchiCAD software. Thanks to its build-in function: "Insolation analysis" geographic location of the analysed roof may be determined as well as the day of analysis may be selected. Then the animation showing shade arrangement from dusk till dawn is obtained. However, in order to retrieve the values of insolated the animation needs to be saved as a movie with a number of frames corresponding to 20 minute step. Next each frame needs to be processed by special software enabling precise determination of the surface and defining the size of insolated surface of each roof slope.

A fully satisfactory solution to the problem of roof insolation at any given time is complete computer automation not only of shade drawing but also determination of insolated roof surfaces for possible general classes of polyhedral surfaces or any surfaces. Such a solution, is possible but only after a thorough research.

5 Summary

The above proposed method of determining roof's insolation implemented in a traditional way and applied to two selected cases illustrates well the presented problem and enables to perform any insolation analysis. It is an interesting example of Descriptive Geometry (also in its traditional form) application for solving technical problems within the construction and environmental engineering range. It indicates the possibility for satisfactory generalizations not only in buildings' shade geometry but also in the use of algorithms.

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GEOMETRYCZNE ASPEKTY NASŁONECZNIENIA POWIERZCHNI DACHU WIELOSPADOWEGO

W pracy podano sposób obliczania wielkości energii słonecznej jaką przyjmuje dach budynku w celu wyznaczenia wartości optymalnej w zależności od kształtu dachu i usytuowania budynku względem stron świata. Do wyznaczania geometrii części oświetlonej dachu (równoważnie - geometrycznego cienia) zastosowano metodę Monge'a z wykorzystaniem programu AutoCAD. Rozważania ograniczono do „pomiaru" promieniowania bezpośredniego i, w celu określenia wielkości energii słonecznej, wprowadzono pojęcie wskaźnika nasłonecznienia bezpośredniego powierzchni.