SAFETY ENGINEERING OF ANTHROPOGENIC OBJECTS

THE QUANTIFICATION METHOD OF FUNCTIONAL AVAILABILITY MILITARY VEHICLES

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Abstract

The availability is one of the most important feature of a technical object which shapes its operational quality. The paper undertakes the issue related to the quantification of functional availability of vehicles, with reference to reliability aspects. The conducted exploitation research paved the way for elaborating methods of determining functional availability for vehicles, in particular focusing on reliability. The essential research was conducted using the developed mathematical model based on the probabilistic, stochastic Markov process, which allowed modelling the process of changes in the exploitation states of vehicles.
In view of the above, it is essential to develop methods that enable the most accurate estimation of the functional availability of vehicles. Knowing the values of functional availability, we can estimate the probability of performing assigned tasks and precisely control the vehicle exploitation system, especially in terms of availability, which is important for vehicles.
This work presents a method for determining the functional availability of vehicles, taking into account the limitations introduced by the vehicle exploitation system. Particular attention was paid to the possibility of using probabilistic, stochastic semiMarkov models, whose rules and method of creation were generally described, with regard to the ability to acquire operational data in the vehicle exploitation system.

Key words: Functional availability, safety, reliability, semiMarkov models.

INTRODUCTION

Readiness is one of the most important features shaping the exploitation quality of an object, in particular the readiness of technical facilities operated in the military.

This paper refers to the functional readiness of military vehicles, which shapes the combat ability of every military unit. It is vital to become aware that the essential platforms of the Land Force units (determining the implementation of the main task by a military unit) are either wheeled or tracked. In addition, in all military units the system of logistical support relies on military vehicles. Taking into consideration the above, it should be noted that the readiness of military vehicles affects all aspects affecting execution of tasks by a military unit. The readiness of military vehicles becomes even more relevant during the execution of combat missions, immediately translating into the effectiveness of the achievement of planned objectives, and most importantly soldiers' safety, which depends on the reliability and ability to conduct tasks by means of military equipment.

The relevance of military vehicles, mentioned before, explains the development of the research and development carried out on modern military vehicles, designed to both shape an appropriate level of readiness of vehicles in the Armed Forces of the Republic of Poland and the enhancement of the methodology of its quantification. The need to shape the readiness of technical military objects has been stressed in work, (Kowalski, and Młyńczak, 2009), where it was found that "During armed conflicts (war), objects should be able to reach high readiness and low fallibility. It is important for new objects to achieve low fallibility, in the shortest possible time from their entry into service, which requires an elimination of errors and of damage already in the production phase. The knowledge of the so-called errors and production faults is thus a key issue for the readiness of weapons systems in a possible armed conflict".

1. The method for the determination of functional availability of military vehicles

The functional availability of a vehicle is a vehicle condition, which is in a state of serviceability. Moreover, it is equipped and able to begin the execution of a task at any random time t, however, without estimating the sufficiency of resources and preserving the serviceability until a task completion. The functional availability can be expressed by a probability measure in accordance with the formula described in paper, (Migawa, 2013).

$$K_g(t) = P(X(t) = 1) \# (1)$$

where X(t) denotes the stochastic process with a set of states S = {0.1}, with 1 referring to the state of functional suitability of a technical object, and 0 denoting the opposite state.

In case the time of operation of a technical object equals t→∞, the value of function K_g (t) may move to the boundary value called the stationary value, as (Girtler, and Ślęzak, 2008):

$$K_g = lim[t \to \infty] K_g(t) = \frac{ET}{ET + EU} \# (2)$$

where:

ET - expected value of a random variable of time of technical object functional suitability,

EU - expected value of a random variable of time of technical object functional unsuitability.

The coefficient of stationary availability, presented above, can be computed in different ways depending on the operational data. It may be calculated analytically using the following dependence (3) given in work, (Simiński, 2013), provided the following data are known:

- S_u - average mileage in-between damage,
- T_o - average duration of vehicle corrective service/ repair,
- Q - the intensity of vehicle operation;

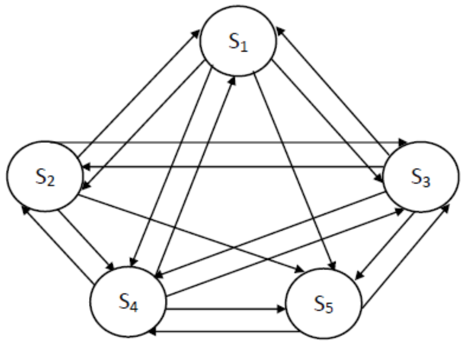
$$K_g = \frac{S_u}{S_u + T_0 q} \#(3)$$

Examining the functional availability of military vehicles is usually conducted on the basis of operational data recorded in the exploitation documents, kept in a military unit. Unfortunately, the guidelines for conducting the documentation do not take into account the need to register data that fully allow the estimation of functional availability in accordance with dependence (3), e.g. it is very difficult to determine the average time of corrective maintenance of a vehicle/vehicle repair. Taking into account the above, the probability semi-Markov stochastic models appear to be very useful in this case.

The semi-Markov model of changes of vehicle exploitation may be presented taking into account a different number of operational states, depending on the needs, as presented in work, (Simiński, Kończak, and Przybysz 2018).

In the first model (Fig. 1.), the author observed the extended times of vehicle stoppage during its upgrading or repair. In practice, this means vehicle inoperability and exclusion from planned tasks due to low efficiency of the subsystem logistics - supplying in technical material resources (including the extended acquisition time, maintaining an inappropriate assortment of spare parts), (Simiński, Kończak, and Przybysz 2018).

In the second model (Fig. 1), the author found it necessary to monitor the condition of vehicles constituting an equipment group to be stored. These vehicles are already adequately prepared for long-term storage, i.e. they do not constitute war stock. It is essential that after the operations associated with the re-activation (extended scope of technical maintenance), the vehicles will become operational in accordance with their design intent in times of a crisis or a war threat, (Simiński, Kończak, and Przybysz 2018).



Model 1

Exploitation states:

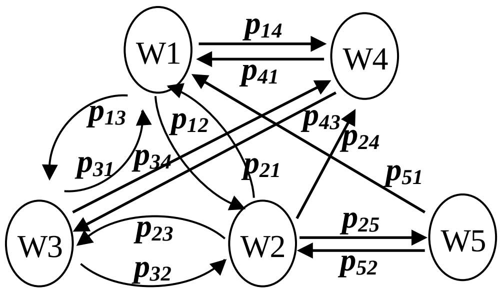
S1 - operation

S2 – stoppage

S3 – maintenance

S4 – repair

S5 - stoppage due to repair



Model 2

Exploitation states:

W1 - operation

W2 - conservation

W3 - stoppage

W4 - repair

W5 - long-term storage

Figure 1. *Grafs of use military vehicle*

The various vehicle states have been created with regard to the suitability of a military vehicle to carry out tasks, i.e. in accordance with the criterion of task readiness.

The distinguished operational phases form a collection of S phases = {s1, s2, s3, s4, s5}, which are both a collection of values of the stochastic process $\{W(t): t \in T\}$ concerning the implementations which are fixed and right-hand intervals. Finding the probabilities of the semi-Markov process in previously specified states narrows down to designating the limiting distributions.

The transitions between the states, presented in Fig. 1, constitute a reductible, homogenous and ergodic Markov chain, which occurs with determined probability p_{ij} and determined intensity of transitions λ_{ij}. Moreover, the conditional probabilities of transition in n-fold step from state "i" to state "j" do not depend on the number of steps n. The parameters of the transition characteristics form the stochastic matrix of transition. For model no 1, depicted in Fig. 1, the matrix of transitions can be presented as follows:

$$M_p = [p_{ij}]_{5x5} \begin{bmatrix} 0 & p_{12} & p_{13} & p_{14} & p_{15} \\ p_{21} & 0 & p_{23} & p_{24} & p_{25} \\ p_{31} & p_{32} & 0 & p_{34} & p_{35} \\ p_{41} & p_{42} & p_{43} & 0 & p_{45} \\ 0 & 0 & p_{53} & p_{54} & 0 \end{bmatrix} \#(4)$$

The nonzero elements of the matrix (4) shown above is the conditional probability of transition p_{ij} from the phase "i" at time "0" to state “j” at time "t" defined as below (5):

$$P_{ij}(t) = P\{X(t) = j \mid X(0) = i\}, i, j \in S \#(5)$$

The probabilities of exploitation state changes may be determined by taking into account the fact that each graph arc in Fig. 1, combining two process states, corresponds to the probability value p_{ij} of the process state change X(t) from state $i \in S$ to state $j \in S$ during the determined observation time. The above probability can be estimated by the frequency of exploitation changes in state transitions for a vehicle due to the following relationship:

$$\omega_{ij} = \frac{n_{ij}}{n_i}, : i, j = 1, 2, 3, i \neq j \#(6)$$

where:

$n_i = \sum_j n_{ij}$ – total number of transitions from state s_i;

n_{ij} – number of transitions from state „i” to state „j” in an examined period of time.

So far, it has been proved that the ergodic probability can be calculated from the boundary of the transition matrix in n steps $M_n = M_p^n$, by solving a set of linear equations or an equivalent matrix equation, passing from the continuous time t to discrete time n, which is the number of the next vehicle observation experience at time Δt, as (Żurek, Mitkow, and Ziółkowski, 2006) and (Żurek, Ziółkowski, and Borucka 2017):

$$\bigwedge_j p_j \lim_{n\to\infty} p_{ij}(n) = \sum_i p_i p_{ij} \Leftrightarrow$$
$$M_p^t [p_j] = [p_j], przy \sum_j \mathrm{p_j} = 1 \#(7)$$

where:

$\mathrm{M_p^t}$ – transposed matrix of transition $\mathrm{M_p}$,

$[\mathrm{p_j}]$ – vector of limiting probabilities,

$[\mathrm{p_{ij}}]$ – probability of transition from state „i" to state „j";

the transpose of the transition matrix (4) will look as follows:

$$M_p^t = \begin{bmatrix} 0 & p_{21} & p_{31} & p_{41} & 0 \\ p_{12} & 0 & p_{32} & p_{42} & 0 \\ p_{13} & p_{23} & 0 & p_{43} & p_{53} \\ p_{14} & p_{24} & p_{34} & 0 & p_{54} \\ p_{15} & p_{25} & p_{35} & p_{45} & 0 \end{bmatrix} \#(8)$$

Using the above-mentioned relationship (7), the ergodic probability will be computed from the following set of equations:

$$\begin{cases} \begin{bmatrix} 0 & p_{21} & p_{31} & p_{41} & 0 \\ p_{12} & 0 & p_{32} & p_{42} & 0 \\ p_{13} & p_{23} & 0 & p_{43} & p_{53} \\ p_{14} & p_{24} & p_{34} & 0 & p_{54} \\ p_{15} & p_{25} & p_{35} & p_{45} & 0 \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{bmatrix} \\ \sum_{j=1}^{5} p_j = 1 \end{cases} \#(9)$$

After multiplying the matrix of the above state of equations (9) and taking into account the normalization condition, it is possible to obtain a system of linear equations:

$$\begin{cases} p_{21} \cdot p_2 + p_{31} \cdot p_3 + p_{41} \cdot p_4 = p_1 \\ p_{12} \cdot p_1 + p_{32} \cdot p_3 + p_{42} \cdot p_4 = p_2 \\ p_{13} \cdot p_1 + p_{23} \cdot p_2 + p_{43} \cdot p_4 + p_{53} \cdot p_5 = p_3 \\ p_{14} \cdot p_1 + p_{24} \cdot p_2 + p_{34} \cdot p_3 + p_{54} \cdot p_5 = p_4 \\ p_{15} \cdot p_1 + p_{25} \cdot p_2 + p_{35} \cdot p_3 + p_{45} \cdot p_4 = p_5 \\ p_1 + p_2 + p_3 + p_4 + p_5 = 1 \end{cases} \#(10)$$

By solving the above set of equations (10), it is possible to obtain formulas for limiting probabilities. Having used the conditional probabilities of transition $\mathrm{p_{ij}}$, calculated on the basis of the exploitation data by means of dependence (6), one may calculate the values of probabilities of a vehicle remaining in particular operating states for the author's Markov chain.

The index of stationary availability, expressed by momentary probability of an event, means that a vehicle at any time t is ready to undertake work, being equivalent to the fact that at any time t it will be in a state of functional suitability, as expressed by the following formula:

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$$K_G = \frac{p_1 + p_2}{\sum_{j=1}^{5} p_j} \#(11)$$

where:

p_1 – probability of a vehicle in operation;

p_2 – probability of a vehicle at a rest (ready to start an operation).

Using the same process model of a vehicle operation visualized in Fig. 1, it is also possible to create a system of differential equations, the so-called Kolmogorov equations, as (Bobrowski, 1985) and (Grabski, 2002), using the following mnemonic rule: "the probability derivative of the process remaining in the highlighted state $P_i'(t)$ for state "i" is equal to an algebraic sum of terms created by a product of probability of this state, from which the branch (arc) departs and the intensity of the transition corresponding to a given branch. The number of the sum terms is equal to the number of directed branches, connecting state "i" with other graph branches. If a branch (arc) is directed towards state "i", the term has a plus sign. In a reverse case, it has a minus sign". Taking into account the above, the system of equations for the transition graph under consideration will look as follows, according to dependence (12):

$$\frac{dP_1}{dt} = -\lambda_{12}P_2\,(t) - \lambda_{13}P_3\,(t) - \lambda_{14}P_4\,(t)$$
$$-\lambda_{15}P_5\,(t) + \lambda_{21}P_2\,(t) + \lambda_{31}P_3\,(t) + \lambda_{41}P_4\,(t)$$
$$\frac{dP_2}{dt} = -\lambda_{21}P_1\,(t) - \lambda_{23}P_3\,(t) - \lambda_{24}P_4\,(t)$$
$$-\lambda_{25}P_5\,(t) + \lambda_{12}P_1\,(t) + \lambda_{32}P_3\,(t) + \lambda_{42}P_4\,(t)$$
$$\frac{dP_3}{dt} = -\lambda_{31}P_1\,(t) - \lambda_{32}P_2\,(t) - \lambda_{34}P_4\,(t)$$
$$-\lambda_{35}P_5\,(t) + \lambda_{13}P_1\,(t) + \lambda_{23}P_2\,(t) \qquad (12)$$
$$+\lambda_{43}P_4\,(t) + \lambda_{53}P_5\,(t)$$
$$\frac{dP_4}{dt} = -\lambda_{41}P_1\,(t) - \lambda_{42}P_2\,(t) - \lambda_{43}P_3\,(t)$$
$$-\lambda_{45}P_5\,(t) + \lambda_{14}P_1\,(t) + \lambda_{24}P_2\,(t)$$
$$+\lambda_{34}P_3\,(t) + \lambda_{54}P_5\,(t)$$
$$\frac{dP_5}{dt} = -\lambda_{53}P_3\,(t) - \lambda_{54}P_4\,(t) + \lambda_{15}P_1\,(t)$$
$$+\lambda_{15}P_1\,(t) + \lambda_{25}P_2\,(t) + \lambda_{35}P_3\,(t) + \lambda_{45}P_4\,(t)$$

The above set of equations (12) can be expressed in a matrix form:

$$P'(t) = \Lambda P(t)\#(13)$$

Where:

$$\Lambda = \begin{bmatrix} -\lambda_{11} & \lambda_{12} & \lambda_{13} & \lambda_{14} & \lambda_{15} \\ \lambda_{21} & -\lambda_{22} & \lambda_{23} & \lambda_{24} & \lambda_{25} \\ \lambda_{31} & \lambda_{32} & -\lambda_{33} & \lambda_{34} & \lambda_{35} \\ \lambda_{41} & \lambda_{42} & \lambda_{43} & -\lambda_{44} & \lambda_{45} \\ 0 & 0 & \lambda_{53} & \lambda_{54} & -\lambda_{55} \end{bmatrix} \qquad (14)$$

The matrix Λ - drawn up on the basis of the discussed model in Fig. 1 is called the intensity matrix of the process transitions.

For the ergodic stochastic process of a finite number of states, the differential equations are converted to algebraic equations and can be calculated by means of the following dependence, as (Grabski, 2002):

$$\Lambda^T \cdot [p_j] = 0\#(15)$$

where:

$[p_j]$ – vector of limiting probabilities.

In the light of dependence (15) mentioned above and the normalization condition, it is possible to calculate ergodic probabilities of the considered Markov process:

$$\begin{cases} \begin{bmatrix} -\lambda_{11} & \lambda_{21} & \lambda_{31} & \lambda_{41} & 0 \\ \lambda_{12} & -\lambda_{22} & \lambda_{32} & \lambda_{42} & 0 \\ \lambda_{13} & \lambda_{23} & -\lambda_{33} & \lambda_{43} & \lambda_{53} \\ \lambda_{14} & \lambda_{24} & \lambda_{34} & -\lambda_{44} & \lambda_{54} \\ \lambda_{15} & \lambda_{25} & \lambda_{35} & \lambda_{45} & -\lambda_{55} \end{bmatrix} . \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ \sum_{j=1}^{5} p_j = 1 \end{cases} \#(16)$$

Having multiplied the matrix of the above-mentioned set of equations (16) and taking into account the normalization condition, the following set of linear equations is obtained:

$$\begin{cases} -\lambda_{11} \cdot p_1 + \lambda_{21} \cdot p_2 + \lambda_{31} \cdot p_3 + \lambda_{41} \cdot p_4 = 0 \\ \lambda_{12} \cdot p_1 - \lambda_{22} \cdot p_2 + \lambda_{32} \cdot p_3 + \lambda_{42} \cdot p_4 = 0 \\ \lambda_{13} \cdot p_1 + \lambda_{23} \cdot p_2 - \lambda_{33} \cdot p_3 + \lambda_{43} \cdot p_4 + \lambda_{53} \cdot p_5 = 0 \\ \lambda_{14} \cdot p_1 + \lambda_{24} \cdot p_2 + \lambda_{34} \cdot p_3 - \lambda_{44} \cdot p_4 + \lambda_{55} \cdot p_5 = 0 \\ \lambda_{15} \cdot p_1 + \lambda_{25} \cdot p_2 + \lambda_{35} \cdot p_3 + \lambda_{45} \cdot p_4 - \lambda_{55} \cdot p_5 = 0 \\ p_1 + p_2 + p_3 + p_4 + p_5 = 1 \end{cases} \#(17)$$

By solving the above set of equations (17), similar to the previously presented set of equations (10), it is possible to obtain formulas for the limiting probabilities of the Markov process, this time however expressed by the transition intensity.

The intensity of transitions λ_{ij} and diagonal intensities λ_{ii} can be determined on the basis of operating data, by designating the average times of remaining in state "i" before moving to state "j" - $\bar{t}_{ij}$ and the average duration times of state s_j, prior to all states s_j of a vehicle in a determined observation time, according to the following dependence:

$$\lambda_{ij} = \frac{1}{\bar{t}_{ij}} \#(18)$$

$$\lambda_{ij} = \frac{1}{\bar{t}_{ii}} \#(19)$$

previously presented set of equations (10), it is possible to obtain formulas for the limiting probabilities of the Markov process, this time however expressed by the transition intensity.

CONCLUSIONS

The fundamental purpose of the exploitation system of military vehicles is primarily to maintain a high level of functional availability, which determines the possibility of undertaking assigned tasks, both in stationary and field conditions, during the execution of combat tasks and in warfare.

In view of the above, it is justified to develop methods that allow a very accurate estimate of vehicle functional availability. Knowing the values of functional availability, one can estimate the probability of executing assigned tasks as well as precisely controlling the system of vehicle operation, particularly with respect to their readiness, which is of crucial importance in case of military vehicles.

A necessary condition for the optimization process of vehicle operations is to possess a necessary collection of data with regard to the operational events. Quite frequently, the quantity and quality of the information determines the choice of a proper mathematical model in order to estimate the functional readiness.

The specificity of military vehicles operation and the rules of conducting operational records set a collection of operating data that can be used in mathematical models. Limited performance data, mostly regarding operational events, tremendously limit the capabilities of using many mathematical models. It was noted that the models based on semi-Markov processes can be used in the strategies of maintaining military vehicles, despite considerably reduced and registered exploitation data. The use of Markovian processes is also connected with certain limitations. It is not desirable to create operational models, stressing too many exploitation states, despite detailed data to allow a development of exploitation graphs, shown in Fig. 1. The development of the semi-Markov model entails more difficulties in solving a set of equations of limiting distribution, in which a larger number of distinguished states, the functional matrix will also become more complex. Obviously, it is possible to use the common numerical methods, as stated in articles, (Girtler, 2008), (Girtler and Ślęzak, 2012). However, these methods do not make it possible to determine the probability values of the occurrence of particular process states for high values of time (theoretically $t \to \infty$).

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