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Dispersion Analysis of Rounds Fired from a Glauberyt Machine Pistol

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Abstract. Within an enclosed shooting range of the EMJOT company, the process of firing one hundred single bullets from a Glauberyt machine pistol was recorded. The empirical test used 9x19 mm FMJ Luger (Parabellum) ammunition manufactured in the Czech Republic in 2017. As the weapon is dedicated to special forces, the shots were fired by an anti-terrorist operative, at a target located 25 m away. In order to determine bullet dispersion, the results of the experiment were subjected to statistical processing. Mean displacement and mean square displacement relative to the mean hit point, histograms, normal distribution, as well as statistical tests and hypotheses were used for estimation. The shots were recorded with a high speed digital camera Phantom v 9.1. The videos recorded were used to determine the initial kinematic parameters of the bullet trajectory.

Keywords: flight mechanics, machine pistol, empirical study, dispersion, theoretical analysis

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1. INTRODUCTION

The purpose of this paper is to analyse the dispersion of single bullets fired from a Glauberyt machine pistol at a target placed 25 m away. The analysis covers experimental tests performed within an enclosed shooting range and statistical processing of the results. An additional purpose of the analysis is to determine guidelines intended to provide a beneficial modification of the dynamic properties of the Glauberyt machine pistol. This means introducing modifications to the design, physical parameters and principle of operation of the weapon to reduce the recoil of bullets fired in single fire. Estimation of bullet hole dispersion on the target is one of the stages of the task.

2. EXPERIMENTAL TESTS

The experimental tests were conducted within an enclosed, certified shooting range belonging to the EMJOT company in Chorzów (Poland). Consequently, it was assumed that weather and ballistic conditions are constant and do not introduce random interference. One hundred single shots were fired from a Glauberyt machine pistol. The shots were fired at a target 25 m away by an anti-terrorist operative. Fig. 1 shows the shooting stance taken by the operative during the experiment.

Fig.1. Shooting stance taken by the anti-terrorist operative

Sellier & Bellot ammunition with the specification of 9x19 mm FMJ Luger (Parabellum), manufactured in the Czech Republic in 2017, was used for the experiment, as shown in Fig. 2.

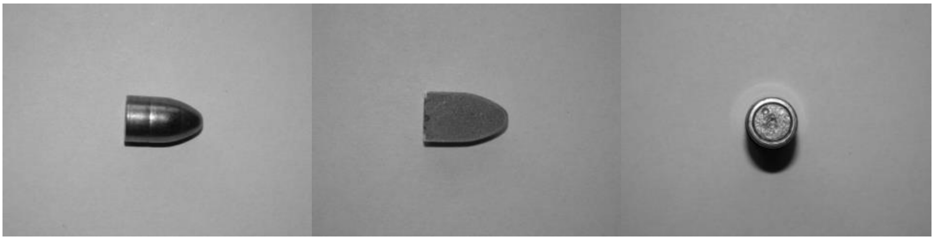

Fig. 2. Bullet after separation, bullet longitudinal cross-section, and bullet bottom

For the purposes of the theoretical analysis, the shots fired were recorded. To this end, a Phantom v.9.1 high-speed digital camera with the necessary accessories and professional lighting was used. The videos recorded were analysed using specialist software. After averaging the results obtained from multiple firing sessions, initial kinematic parameters of bullet flight, among others, were obtained [1]. Fig. 3 shows the moment of firing a round.

Fig. 3. The moment being fired of the bullet

A human being is a complex biological system, whose performance depends on its current psychophysical condition, experience acquired, predispositions, and knowledge. Consequently, its impact on the process of firing shots from a machine pistol is significant. The Glauberyt machine pistol is a weapon dedicated to special forces and, for this reason, an anti-terrorist operative participated in the tests. The analysis is aimed at determining the properties of the man-weapon system [2], not only the weapon itself. Consequently, it was elected not to perform the tests with the weapon mounted in a jig.

Fig. 4 shows a bullet in flight, immediately after leaving the barrel of Glauberyt. Using the specialist TEMA software, designed for analysing recorded video images, it is possible to determine the kinematic parameters characterising bullet behaviour during motion in a gravity field and the Earth’s atmosphere [3].

Fig. 4. Bullet in flight

The obtained experimental results enabled travel speed to be determined. For the purpose of the tracking process performed by the software, a characteristic point located on the bullet was chosen. The correlation feature was used, which – by matching the shades of pixels in the fragment of an image, enabled a chosen point to be tracked. Additionally, image filters were used to increase contrast. The flight recordings of the rounds fired enabled determining the actual linear velocity and angular velocity in the round's tilt immediately after leaving the barrel. The determined linear velocity is 368 m/s, and angular velocity is 8979 rad/s, which are consistent with the ammunition used for the experiment.

3. BASIC ESTIMATORS

To determine the weapon's dispersion, the target shooting results were processed statistically. Initially, basic estimators, namely mean displacement and mean square displacement relative to mean hit point, and target hit histograms, were applied [4]. Calculations were performed using Scilab software [5]. The results are shown in Fig. 5 and Fig. 6.

One hundred single rounds were fired at a target placed 25 m away. Bullet hole parameters, defined in a Cartesian coordinate system *0xy*, are shown as two vectors used in the simulation program:

x=[-1.1, -1.6, 0.3, 0.5, 2.3, 1.5, 1.7, 4.2, 4.8, 3.7,
6.2, 5.9, 7.8, 6.1, 8.0, 7.6, 11.6, 7.2, 7.6, 5.3,
0.5, 1.3, 1.5, 2.5, 2.3, 2.4, 2.3, 3.6, 4.0, 3.8,
4.3, 3.3, 5.2, 6.5, 7.4, 9.1, 9.1, 10.5, 11.5, 2.1,
-2.5, 1.0, 1.7, 3.5, 2.2, 3.0, 4.0, 2.0, 1.4, 3.2,
5.7, 6.6, 6.2, 7.1, 6.5, 6.7, 9.3, 11.6, 9.8, 14.3,
2.1, 2.9, 2.0, 6.3, 5.6 ,6.2, 5.4, 7.9, 10.2, 9.5,
8.5, 9.8, 6.3, 8.5, 7.3, 6.9, 11.8, 13.6, 5.7, 6.2,
-1.0, 2.7, 5.2, 6.7, 8.3, 10.0, 7.9, 5.3, 5.6, 4.8,
7.9, 8.0, 4.3, 5.7, 3.2, 6.8, 2.0, 6.2, 7.9, 2.9]

y=[-12.2, -12.5, -8.9, -15.6, -8.4, -6.4, -1.4, 3.4, -3.8, -10.4,
-10.7, -9.1, -11.9, -6.0, -6.3, -2.6, -5.5, -6.2, -6.2, -3.8,
-6.8, 1.9, -3.5, -3.5, -4.9, -8.9, -9.9, -7.7, -10.6, -4.8,
-4.0, -1.4, 5.4, -8.3, -7.9, -9.0, -10.3, -3.7, -10.9, -17.1,
-13.2, -7.0, -6.2, -5.1, -8.6, -8.5, -8.2, 1.8, -13.6, -17.4,
-12.2, -8.8, -8.1, -7.6, -2.5, -1.7, -3.9, -1.2, -11.3, -13.5,
0.3,- 5.7, -7.6, -3.3, -5.5, -6.2, -6.6, -4.2, -2.0, -7.1,
-8.2, -8.3, -12.6, -13.6, -15.9, -17.6, -12.5, -13.2, -6.2, -6.6,
-9.6, 0.3, -3.9, -1.2, -0.2, 4.4, -6.0, -7.9, -8.8, -9.7,
-8.6, -10.2, -11.6, -11.4, -12.7, -12.5, -15.1, -14.1, -14.3, -19.9]

Vector elements are stated in the unit of measure – centimetres. The error of their location is of the order of tenths of a centimetre.

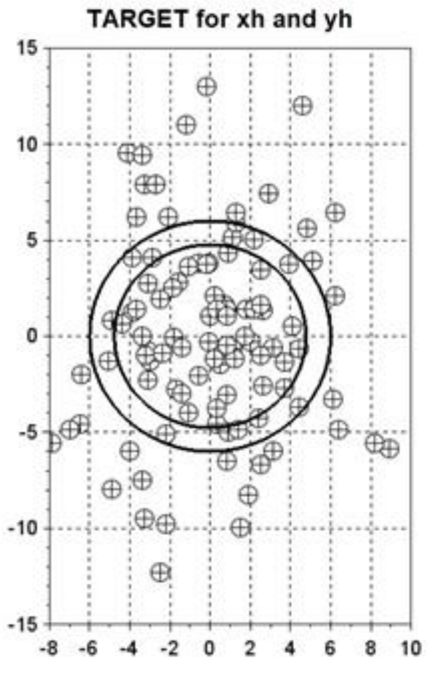

Fig. 5. Bullet hole distribution in the target

Fig. 5 shows the bullet hole distribution in the target. The point with coordinates [*xh*=0 cm, *yh*=0 cm] is the centre of the Cartesian coordinate system *0 xh yh*, which is displaced relative to the aiming point [6] by coordinates defined by the mean displacement vector $\overrightarrow{Us}[xs, ys]$. This vector determines the position of the mean hit point.

Having vectors *x* and *y*, *xs* and *ys* were calculated using function:

xs = mean(*x*)

ys = mean(*y*)

Relations defining the mean displacement vector $\overrightarrow{Us}[xs, ys]$:
Vector module

$$Us = \sqrt{xs^2 + ys^2}$$

Vector angle

$$\sin(\varphi s) = \frac{ys}{Us}$$

The calculated mean hit point is:

$$\overrightarrow{Us} = [0.0539, -0.0763]$$
$$Us = 0.0934\ m$$
$$\varphi s = -54.74\ deg$$

Having vectors *x* and *y* and the mean displacement vector $\overrightarrow{Us}[xs, ys]$, coordinates of the mean displacement vector relative to the mean hit point *xss* and *yss* were calculated using function:

xss = mean(abs(*x*-*xs*))
yss = mean(abs(*y*-*ys*))

Relations defining the mean displacement vector relative to the mean hit point $\overrightarrow{Uss}[xss, yss]$:
Vector module

$$Uss = \sqrt{xss^2 + yss^2}$$

Vector angle

$$\sin(\varphi ss) = \frac{yss}{Uss}$$

The basic estimate is:

$$\overrightarrow{Uss} = [0.0274, 0.0391]$$
$$Uss = 0.0477\ m$$
$$\varphi ss = 54.99\ deg$$

The result is shown in Fig. 4 as the smaller diameter circle.

Having vectors *x* and *y* and the mean displacement vector $\overrightarrow{Us}[xs, ys]$, coordinates of the mean square displacement vector relative to the mean hit point *xsk* and *ysk* were calculated using function:

xsk = mean((x- xs)^2)
ysk = mean((y- ys)^2)

Relations defining the mean square displacement vector relative to mean hit point $\overrightarrow{Usk}[xsk, ysk]$:
Vector module

$$Usk = \sqrt{xsk^2 + ysk^2}$$

Vector angle

$$\sin(\varphi sk) = \frac{ysk}{Usk}$$

The basic estimate is:

$$\overrightarrow{Usk} = [0.0337, 0.0496]$$

$$Usk = 0.0600\, m$$
$$\varphi sk = 55.84\, deg$$

The result is shown in Fig. 4 as the larger diameter circle.

If symbol R_{50} means the radius of the circle containing half of all the bullet holes, then the following equation is obtained for the tests performed:

$$Uss < R_{50} < Usk$$

where: $R_{50} = 0.054$ m

Fig. 6 shows target hits as two histograms, where hit point coordinates [*xh*,*yh*] are random variables. The x-axis represents the *xh* [cm] or *yh* [cm] coordinate. The *y*-axis represents the value of probability for the given range. Ten ranges for the number of one hundred measurements for the *xh* or *yh* random variable.

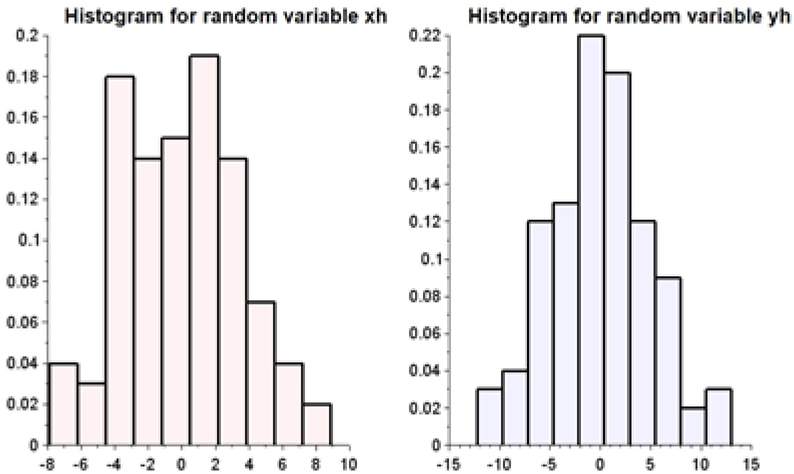

Fig. 6. Histogram for random variables *xh* and *yh*

The target hole distribution histogram enables determining the probability for the calculated ranges and estimating dispersion.

4. CHAUVENET'S CRITERION

To eliminate so-called stray shots, the Chauvenet's criterion was applied [7]. This criterion enables determining whether an observation in a statistical sample enables the result of coarse error occurrence. The principle of this method consists in assessing the probability of achieving a spurious shot result.

Before further statistical analysis is performed, such a result should be discarded. According to the Chauvenet's criterion, the product of the number of spurious results and probability of a result with a value of

$$\left|\bar{x} - x_{pod}\right| > t_s \sigma_x$$

must be lower than 0.5.
Thus, statistics are calculated:

$$t_s = \frac{\left|\bar{x} - x_{pod}\right|}{\sigma_x}$$

where:

- t_s - number of standard deviations, by which a spurious result differs from the mean,
- x_{pod} - value of the spurious result,
- $\bar{x}$ - arithmetic mean,

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

- x_i - measurement value,
- n - number of measurements,
- σ_x - standard deviation,

$$\sigma_x = \frac{1}{n-1}\sum_{i=1}^{n} (x_i - \bar{x})^2$$

For bullet holes defined by coordinate *x*, the following spurious results were assumed, Fig. 7:

$x_{pod} = 8.9$ cm

Calculations resulted in the following:

$t_s = 2.743$

For $t_s = 2.743$ the probability value is $P = 0.991$.
According to the Chauvenet's criterion, the result is not discarded.

For bullet holes defined by coordinate y, the following spurious results were assumed, Fig. 7:

$y_{pod} = -12.3$ cm

Calculations resulted in the following:

$t_s = 2.552$

For $t_s = 2.552$ the probability value is $P = 0.987$.
According to the Chauvenet's criterion, the result is not discarded.

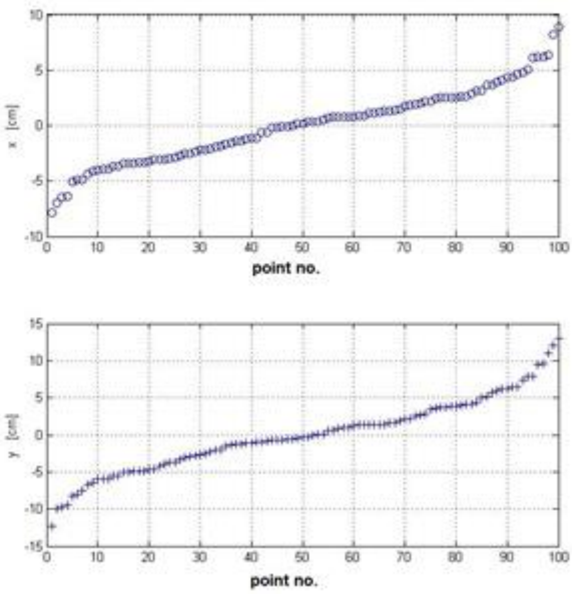

Fig. 7. Bullet hole distribution after sorting for coordinates x and y

5. COMPATIBILITY TEST χ^2 (CHI-SQUARED)

The chi-squared compatibility test χ^2 [8, 9] was used to verify the hypothesis that the bullet holes generated in the target are subject to normal distribution. The statistic was calculated using the following equation:

$$\chi^2 = \sum_{i=1}^{k} \frac{(n_i - nP_i)^2}{nP_i}$$

where:

- χ^2 - test with chi-squared distribution,
- k - number of identified ranges,
- P_i - probability that a bullet hole is within range i,
- n - number of bullet holes,
- n_i - number of bullet holes within range i.

k = 10 ranges were identified, as in Fig. 8. The number of bullet holes in the given range for coordinates x and y are stated in the histograms (frequency charts) shown in Fig. 8.

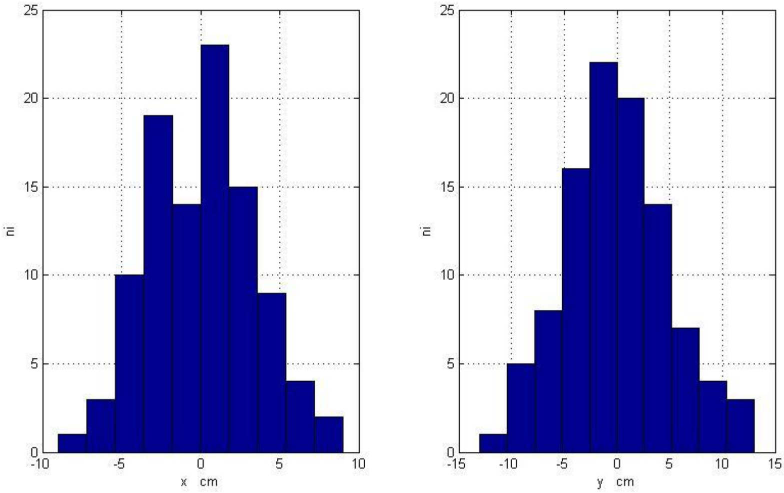

Fig. 8. Histogram prepared for the chi-squared compatibility test

Calculations resulted in the following:
for coordinate x $\chi^2 = 4.640$
for coordinate y $\chi^2 = 3.284$

When verifying statistical hypotheses based on distribution χ^2, the critical area is always located on the right hand side.

Calculate the number of degrees of freedom, which in this case is k-1-2=7. The value of 1 results from the fact that frequency in the last range is defined, because frequencies in the first k -1 ranges are known. The value of 2 results from the fact that normal distribution probability density requires calculating an arithmetic mean and standard deviation. If a relevance level of

$$P(\chi^2 \geq \chi^2_\alpha) = \alpha = 0.05$$

is assumed for the calculated number of degrees of freedom, then the result is $\chi^2_\alpha = 14.067$.

The calculated tests are outside the critical area, so there is no basis for discarding the hypothesis that the bullet holes generated in the target are subject to normal distribution. This confirms the compatibility of the empirical distribution with normal distribution ($\alpha = 0.05$).

6. NORMAL DISTRIBUTION (GAUSSIAN DISTRIBUTION)

After applying the Chauvenet's criterion and chi-squared compatibility test χ^2, we can proceed to determining the normal distribution density function [10, 11], which takes the following form:

$$p(x) = \frac{1}{\sigma_x\sqrt{2\pi}}e^{-\frac{(x-\bar{x})^2}{2\sigma_x^2}}$$

The parameters in this equation have been defined in section 4.

Calculations resulted in the following:

for coordinate x $\bar{x} = 5.392$ cm $\sigma_x = 3.385$ cm

for coordinate y $\bar{y} = -7.628$ cm $\sigma_y = 4.989$ cm

A normal distribution density chart for coordinate *x* (symbol: **o**) and for coordinate *y* (symbol: +) is shown in Fig .9. The random variables are defined in relation to the mean hit point.

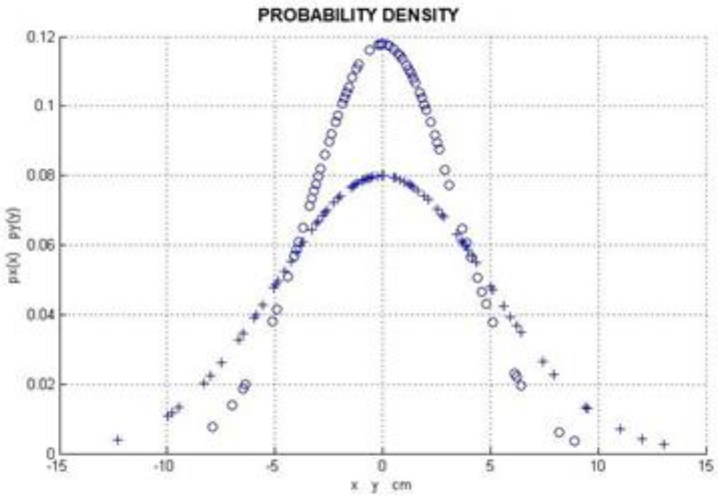

Fig. 9. Normal distribution density for random variables x and y

The probability that a random variable takes a value from the range (x_1, x_2) was calculated from equation:

$$P(x_1 < x < x_2) = \int_{x_1}^{x_2} \frac{1}{\sigma_x\sqrt{2\pi}}e^{-\frac{(x-\bar{x})^2}{2\sigma_x^2}}dx$$

The probabilities for the following ranges take the values of:

for random variable *x*:

$$P(-3.385 < x < 3.385) = 0.6827$$
$$P(-6.770 < x < 6.770) = 0.9545$$
$$P(-10.16 < x < 10.16) = 0.9973$$

for random variable *y*:

$$P(-4.989 < y < 4.989) = 0.6827$$
$$P(-9.978 < y < 9.979) = 0.9545$$
$$P(-14.97 < y < 14.97) = 0.9973$$

7. CONCLUSION

Basic estimators and non-parametric and parametric hypotheses and statistical tests were used to estimate bullet dispersion. The statistical analysis was performed in Scilab and MatLab software [12]. Chauvenet's criterion, χ^2 chi-squared compatibility test and normal distribution density function were used in the analysis.

The mean displacement vector suggests that the Glauberyt machine pistol used for the test requires an adjustment of the aiming system. The weapon shows a tendency to fire slightly to the right and below the aiming point. The analysis performed may serve to properly calibrate the weapon used. Greater shot dispersion is observed on the *y*-axis than along the *x*-axis. Such a result could be expected as it is a consequence of the weapon's recoil during firing. The weapon's horizontal dispersion during firing is lower. The difference of 0.0123 m between the module of mean displacement relative to the mean hit point, and the module of mean square displacement relative to the mean hit point demonstrates the effect of bullet holes located further away from the centre of the Cartesian coordinate system *0 xh yh*.

The determined confidence intervals enable determining the probability of accurately hitting a target located 25 m away

The prepared histograms and normal distributions enable comparing the results obtained for the weapon before and after its dynamical properties are adjusted [13]. They also enable comparing weapons of similar types, e.g. the Glauberyt machine pistol with the Uzi machine pistol. The analysis presented in this paper constitutes a phase of a task intended to enable informed designing of firearms.

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Analiza rozrzutu pocisków wystrzelonych z pistoletu maszynowego Glauberyt

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Streszczenie. Na zamkniętej strzelnicy firmy EMJOT przeprowadzono rejestrację procesu wystrzelenia z pistoletu maszynowego Glauberyt ogniem pojedynczym stu pocisków. Do badań empirycznych użyto amunicję 9x19mm FMJ Luger (Parabellum) produkcji czeskiej z 2017 roku. Ze względu na dedykowanie broni dla oddziałów specjalnych strzały oddane zostały przez antyterrorystę, do tarczy znajdującej się w odległości 25 m. W celu określenia rozrzutu pocisków wyniki eksperymentu poddano obróbce statystycznej. Do estymacji zastosowano uchylenie średnie i uchylenie średnio-kwadratowe względem średniego punktu trafień, histogramy, rozkład normalny, oraz testy i hipotezy statystyczne. Przy wykorzystaniu szybkiej kamery cyfrowej Phantom v.9.1 przeprowadzono rejestrację wykonanych strzelań. Zarejestrowane filmy posłużyły do określenia początkowych kinematycznych parametrów lotu pocisku.

Słowa kluczowe: mechanika lotu, pistolet maszynowy, badania empiryczne, rozrzut, analiza teoretyczna