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The analysis of working liquid flow in a hydrostatic line with the use of frequency characteristics

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Abstract. One of the little described problems in hydrostatic drives is the fast changing runs in the hydraulic line of this drive affecting the nature of the formation and intensity of pressure pulsation and flow rate occurring in the drive. Pressure pulsation and flow rate are the cause of unstable operation of servos, delays in the control system and other harmful phenomena. The article presents a flow model in a hydrostatic drive line based on fluid continuity equations (mass conservation), maintaining the amount of Navier-Stokes motion in the direction of flow (x axis), energy conservation (liquid state). The movement of liquids in a hydrostatic line is described by partial differential equations of the hyperbolic type, so modeling takes into account the wave phenomena occurring in the line. The hydrostatic line was treated as a cross with two inputs and two outputs, characterized by a specific transmittance matrix. The product approximation was used to solve the wave equations. An example of the use of general equations is presented for the analysis of a miniaturized hydrostatic drive line fed from a constant pressure source and terminated by a servo mechanism.

Key words: aviation, unmanned aerial vehicle, hydrostatic drive, servomechanism, hydrostatic line.

1. Introduction

A very rapid development of unmanned aerial system can be observed over the recent years. Control system assemblies are a factor restricting their development, especially in the case of unmanned aerial vehicles (UAV) weighing above 150 kg and travelling at a speed of flight of above 100 m/s. These assemblies should generate significant forces and, at the same time, ensure the movement smoothness of control elements. In addition, a drive of UAV control elements should be characterized by high operating speed, high accuracy and optimum energy utilization, with its simultaneous miniaturization.

The drive assembly in UAV control elements is a servomechanism. The most convenient types of drives used for powering servomechanisms of UAV control systems and their controlling are electrical or hydrostatic drives. An important advantage of a hydrostatic drive is, i.a., the possibility to achieve high flux density of the power transmitted within the propulsion system, i.e., small mass per unit of generated or transmitted power [1–4]. The operating nature of a servomechanism in a hydrostatic drive largely depends on the properties of a working liquid flow in a hydraulic line connecting the supply source (hydraulic pomp) with the servomechanism. Due to the small dimensions of the hydraulic pump and servomechanism used in UAV, their connections and hydraulic lines connecting them have internal diameters less than 6 mmm [3, 5, 6]. To ensure the required speed of movement of the UAV actuator, the working fluid flow rate in the pipe must be 25–30 dm^3/min. Due to the required flow rate through the hydraulic lines, its optimal diameter is 5 mm.

With the rapidly changing working liquid pressure and flow rate courses in a hydrostatic drive line, pressure and flow rate pulsation of constant or transient character may appear. Pressure and flow rate pulsation is the cause of unstable servomechanism operation, delays in the control system and other adverse phenomena [7–14]. The nature of these processes depends on certain working liquid properties, i.a., viscosity, density and compressibility, line material compressibility and its geometric dimensions. These properties create unitary line parameters, i.e., resistance – which takes into account the influence of working liquid viscosity, inertance – which takes into account the impact of working liquid inertia, and capacitance – which characterizes the impact of liquid compressibility. In the event of a hydrostatic servomechanism located at a certain distance from the supply assembly, a line shall be treated as a line, in which the effect of inertia of a liquid flowing in the conduit, the compressibility effect of the liquid and the viscous friction effect have a significant influence on the operating stability of the servomechanism. The properties of a hydrostatic drive line are associated with, i.a., pressure or flow rate signal increment speed on the line input or output. The maximum increment speed of these signals corresponds to the maximum frequency of pressure and flow rate waves present in the drive. Knowledge of a hydrostatic drive line properties is essential due to the possibility of an analytical assessment of pressure pulsation intensity present in the drive, hence, the possibility of such a drive design, so that no adverse pulsations are present. Among the

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computation methods applied for solving wave equations of a hydrostatic drive line (conduit), three basic ones may be distinguished: characteristics, variable separation and operator calculus [15–21]. Operational calculus is mostly used for the calculations. Hyperbolic functions present in wave equations are the cause of computational difficulties, hence, in order to facilitate these calculations, approximation expressions are used. Classic approximation involves replacing hyperbolic functions with their expansions in series, while only the first terms of an expansion are taken into account. However, this method is unsatisfactory for two reasons, i.e., approximation function roots do not overlap with the hyperbolic function roots, and the determination of flow stability in a hydraulic line based on such approximation is uncertain, since in many cases, stable models are obtained in the place of non-stable and vice-versa [11, 14, 21]. These disadvantages are not part of product approximation [22].

This paper attempts to describe the flow properties in a miniaturized line of a hydrostatic drive, in response to rapid servomechanism distortion. Fluid movement is described by partial differential equations of the hyperbolic type; therefore, the modelling takes wave phenomena undergoing in a hydrostatic line into account.

2. Flow model in a hydrostatic drive line

In order to derive general equations of a hydrostatic line drive in response to rapid distortion of a servomechanism, the following assumptions were adopted:

- mass forces acting on the working fluid in a hydrostatic line are negligible,
- flow in a hydrostatic drive line is laminar and axially-symmetric,
- working fluid is a Newtonian fluid,
- working fluid temperature and pressure distribution within the line section are constant over the entire length of a hydrostatic drive line,
- line walls are elastic and the line internal radius is constant.

According to the adopted assumptions, dynamic properties of a hydrostatic drive line are fully described by the following system of partial differential equations in cylindrical coordinates [4, 6, 17–19]:

- fluid continuity equation (mass conservation)

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\rho r v_r + \frac{\partial}{\partial x}\rho v_x = 0, \tag{1}$$

where:

ρ – working liquid density,

r – hydrostatic drive line (conduit) current radius,

x – axial coordinate of a hydrostatic drive line,

v_r, v_x – speed components in the radial and axial directions;

- Navier-Stokes motion amount preservation equation in the flow direction (axis x)

$$\begin{aligned}\rho\left(\frac{\partial v_x}{\partial t} + V_r\frac{\partial v_x}{\partial r} + V_x\frac{\partial v_x}{\partial x}\right) &= \\ = -\frac{\partial p}{\partial x} + \mu\frac{\partial}{\partial x}\left[-\frac{2}{3r}\frac{\partial}{\partial r}(rv_r) + \frac{4}{3}\frac{\partial v_x}{\partial x}\right] &+ \\ + \frac{\mu}{r}\frac{\partial}{\partial r}r\left(\frac{\partial v_x}{\partial r} + \frac{\partial v_r}{\partial x}\right)&,\end{aligned} \tag{2}$$

where: μ – dynamic viscosity coefficient, p – working fluid pressure;

- energy conservation (liquid state) equation [4]

$$\frac{\partial \rho}{\rho} = \frac{\partial p}{B}, \tag{3}$$

where: B – fluid compressibility modulus.

The aforementioned equations may be linearized. Equation (1) after linearization for the condition $\frac{2\pi|a|}{\omega} \gg r_w$ and with the provision that $\bar{v}_x \ll |c|$ [4, 6] assumes the form

$$\frac{\partial \rho'}{\partial t} + \bar{\rho}\frac{\partial v_x'}{\partial x} = 0, \tag{4}$$

where:

$\bar{\rho}$, ρ' – density of fluid, its determined value and deviation for determined value,

$\bar{v}_x$, v_x' – axial direction speed components, its determined value and deviation from the determined value,

r_w – hydrostatic drive line (conduit) internal radius,

a – speed of wave propagation in the line,

ω – pressure or flow rate wave frequency in the line.

The hydraulic pulse transmission speed a in a line impacts the operational delay of a servomechanism, which should not exceed 0.05 s. Speed c of transmitting a hydraulic pulse in a line filled with working liquid may be calculated from the Żukowski equation [1]

$$a = \frac{1}{\sqrt{\frac{\rho}{K} + \frac{\rho k}{En}}}, \tag{5}$$

where:

K – working liquid bulk modulus [MPa],

E – elasticity modulus of a hydrostatic drive line material [Pa],

k – hydrostatic drive line diameter [m],

n – hydrostatic drive line wall diameter [m].

For an ideally rigid hydrostatic drive $(K = \infty)$, the $a = \sqrt{K/\rho}$ may be assumed, which means it equals the sound propagation speed within a given working liquid medium. The experiments show that for a line with an internal diameter of 5 mm, made of 20XA steel (a material normally used for hydrostatic drives), transmitting a pulse at a distance of 10 m through oil with a viscosity of $\mu = 0.012$ Pa·s, working liquid density

$\rho = 850$ kg/m^3, takes place regardless of the pressure, within 0.018–0.02 s, which corresponds to 800–970 m/s. For oil viscosity $\mu = 0.561$ $Pa{\cdot}s$, the impulse transmission speed is reduced to 560–680 m/s.

By substituting the value of density differential to equation (4), from the equation of state (3), we get

$$\frac{\overline{\rho}}{B}\frac{\partial p'}{\partial t} + \overline{\rho}\frac{\partial v'_x}{\partial x} = 0. \tag{6}$$

With maintained condition $\dfrac{2\pi|a|}{\omega} \gg r_w$ the equation (2) is simplified to

$$\overline{\rho}\frac{\partial v'_x}{\partial t} = -\frac{\partial p'}{\partial x} + \frac{4}{3}\mu\frac{\partial^2 v'_x}{\partial x^2} + \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v'_x}{\partial r}\right). \tag{7}$$

Assuming that a sinusoid wave of pressure propagates in a conduit, the instantaneous pressure value at point x is expressed with a relationship $p' = p_0 \sin\omega\left(t - \dfrac{x}{a}\right)$.

If we assume the condition $\dfrac{4}{3}\mu\dfrac{\omega}{a^2\bar{\rho}} \ll 1$ [4], then in the equation (7) the term $\dfrac{4}{3}\mu\dfrac{\partial^2 V'_x}{\partial x^2} \ll \dfrac{\partial p'}{\partial x}$ and as a result, the equation (7) takes the form

$$\bar{\rho}\frac{\partial v'_x}{\partial t} = -\frac{\partial p'}{\partial x} + \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v'_x}{\partial r}\right). \tag{8}$$

Using a Laplace transformation relative to time, to the equation (8), with zero initial conditions $p(x,0) = 0$ and $v_x(x,r,0) = 0$, we get

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{s}{\nu}\left(v + \frac{1}{\bar{\rho}s}\frac{\partial P}{\partial x}\right) = 0. \tag{9}$$

By introducing to the equation (9) a new variable

$$U = v + \frac{1}{\bar{\rho}s}\frac{\partial P}{\partial x}, \tag{10}$$

we get an equation with a form

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} - \frac{s}{\nu}U = 0. \tag{11}$$

Equation (11) is a modified Bessel equation of order zero. Its solution is an order zero Bessel function, while we only consider a first kind function, which is finite for $r = 0$

$$U = F(x,s)\left[J_0\left(j\cdot r\cdot\sqrt{\frac{s}{\nu}}\right)\right]. \tag{12}$$

Substituting U from the equation (12) to the equation (10), we get

$$V = F(x,s)\left[J_0\left(j\cdot r\cdot\sqrt{\frac{s}{\nu}}\right)\right] - \frac{1}{\bar{\rho}s}\frac{\partial P}{\partial x}. \tag{13}$$

The equation (13) should satisfy a boundary condition $V = 0$ for $r = r_w$. It happens if

$$\frac{\partial P}{\partial x} = \bar{\rho}\cdot s\cdot F(x,s)\left[J_0\left(j\cdot r_w\cdot\sqrt{\frac{s}{\nu}}\right)\right]. \tag{14}$$

Hence

$$V = F(x,s)\left[J_0\left(j\cdot r\cdot\sqrt{\frac{s}{\nu}}\right) - J_0\left(j\cdot r_w\cdot\sqrt{\frac{s}{\nu}}\right)\right]. \tag{15}$$

Multiplying both sides of the equation (15) by $2\pi r$ and integrating relative to r in the boundaries from 0 to r_w, we obtain a relationship for the flow rate in the line

$$Q = 2\pi\int_0^{r_w} V\cdot r\cdot \mathrm{d}r = \pi\cdot r_w^2\cdot F(x,s)\cdot \left[\frac{2}{j\cdot\sqrt{s/\omega_0}}J_1\left(j\sqrt{\frac{s}{\omega_0}}\right) - J_0\left(j\sqrt{\frac{s}{\omega_0}}\right)\right], \tag{16}$$

where: $\omega_0 = \dfrac{\nu}{r_w^2}$ is a characteristic frequency of pressure or flow rate wave in the line.

By determining $F(x,s)$ from the equation (14) and substituting to the equation (16), we get

$$L(s)Q(x,s) = -\frac{\partial P(x,s)}{\partial x}, \tag{17}$$

where:

- $P(x,s)$ – Laplace transform of a hydraulic line current pressure,
- $Q(x,s)$ – Laplace transform of a hydraulic line current flow rate,
- $L(s)$ – serial impedance per line length unit,

and

$$L(s) = \frac{L_0 s}{1 - \dfrac{2}{j\cdot\sqrt{s/\omega_0}}\cdot\dfrac{J_1\left(j\cdot\sqrt{s/\omega_0}\right)}{J_0\left(j\cdot\sqrt{s/\omega_0}\right)}}, \tag{18}$$

whereas $L_0 = \dfrac{\bar{\rho}}{\pi r_w^2}$ is the inertance (fluid inertia) per line length unit.

By performing a Laplace transformation relative to time for zero initial conditions $p(x,0) = 0$ and $V_x(x,r,0) = 0$, and then multiplying by $2\pi r$ and integrating them relative to r within the boundaries from 0 to r_w, the following equation is obtained

$$Y(s)P(x,s) = -\frac{\partial Q(x,s)}{\partial x}, \tag{19}$$

where: $Y(s) = C_0 s$ is the by-pass admittance per line length unit, whereas $C_0 = \dfrac{\pi r_w^2}{a^2\bar{\rho}} = \dfrac{\pi r_w^2}{B}$ is the capacitance (liquid compressibility) per line length unit.

In order to utilize the theory of automatic adjustment for analysis and synthesis, based on relationships (17) and (19), a hydrostatic drive line may be described by an equation in matrix shape with a bi-parameter (quadripole) form

$$\frac{\partial}{\partial x}\begin{bmatrix} P(x,s) \\ Q(x,s) \end{bmatrix} = -\begin{bmatrix} 0 & L(s) \\ Y(s) & 0 \end{bmatrix}\begin{bmatrix} P(x,s) \\ Q(x,s) \end{bmatrix}, \tag{20}$$

One signal from each pair of the quadripole signals is a flow parameter (similarly to flow rate), and the second one is a pressure parameter (generalized pressure).

Integrating the equation (20) relative to x within the boundaries from 0 to l, we obtain a solution to the hydrostatic drive line equation in the fixed variable plane

$$\begin{bmatrix} P(0,s) \\ Q(0,s) \end{bmatrix} = \begin{bmatrix} \cosh T(s) & L_c(s)\sinh T(s) \\ \frac{1}{L_c(s)}\sinh T(s) & \cosh T(s) \end{bmatrix} \cdot \begin{bmatrix} P(l,s) \\ Q(l,s) \end{bmatrix}, \tag{21}$$

where l is hydrostatic drive line length, and

$$T(s) = l\sqrt{L(s)\cdot Y(s)} \tag{22}$$

is a propagation operator, and

$$L_c = \sqrt{\frac{L(s)}{Y(s)}} \tag{23}$$

is the wave impedance.

Substituting to relationships (22) and (23) relationships (18) and (20), we obtain a propagation operator and wave impedance in the form

$$T(s) = \frac{T_0 s}{\sqrt{1 - \frac{2}{j\cdot\sqrt{s/\omega_0}}\cdot\frac{J_1\left(j\cdot\sqrt{s/\omega_0}\right)}{J_0\left(j\cdot\sqrt{s/\omega_0}\right)}}}, \tag{24}$$

$$L_c(s) = \frac{L_{c0} s}{\sqrt{1 - \frac{2}{j\cdot\sqrt{s/\omega_0}}\cdot\frac{J_1\left(j\cdot\sqrt{s/\omega_0}\right)}{J_0\left(j\cdot\sqrt{s/\omega_0}\right)}}}, \tag{25}$$

where:

$$T_0 = l\sqrt{L_0 C_0} = \frac{l}{a},$$

$$L_{c0} = \sqrt{\frac{L_0}{C_0}} = \frac{a\bar{\rho}}{\pi r_w^2}.$$

Equation (21) may be also presented in the form of quadripole transmittance matrix, in the form:

– admittance

$$\begin{bmatrix} Q(0,s) \\ Q(l,s) \end{bmatrix} = \frac{1}{L_c(s)\sinh T(s)}\begin{bmatrix} \cosh T(s) & -1 \\ 1 & -\cosh T(s) \end{bmatrix} \cdot \begin{bmatrix} P(0,s) \\ P(l,s) \end{bmatrix}, \tag{26}$$

– impedance

$$\begin{bmatrix} P(0,s) \\ P(l,s) \end{bmatrix} = \frac{L_c(s)}{\sinh T(s)}\begin{bmatrix} \cosh T(s) & -1 \\ 1 & -\cosh T(s) \end{bmatrix} \cdot \begin{bmatrix} Q(0,s) \\ Q(l,s) \end{bmatrix}, \tag{27}$$

3. Frequency characteristics of a hydrostatic drive line

The main emphasis when discussing the model was put on determining frequency responses of the systems. Frequency characteristics are obtained by substituting to operator transmittances $s = j\omega$, therefore, by replacing a Laplace transformation by a Fourier transformation. Spectral forms of the propagation operator and wave impedance for a model with variable resistance are as follows

$$T(j\omega) = \frac{jT_0\omega}{\sqrt{1 - \frac{2}{j^{3/2}\sqrt{\omega/\omega_0}}\cdot\frac{J_1\left(j^{3/2}\sqrt{\omega/\omega_0}\right)}{J_0\left(j^{3/2}\sqrt{\omega/\omega_0}\right)}}} = \alpha + j\beta, \tag{28}$$

$$L_c(j\omega) = \frac{L_{c0}}{\sqrt{1 - \frac{2}{j^{3/2}\sqrt{\omega/\omega_0}}\cdot\frac{J_1\left(j^{3/2}\sqrt{\omega/\omega_0}\right)}{J_0\left(j^{3/2}\sqrt{\omega/\omega_0}\right)}}} = \gamma + j\delta, \tag{29}$$

where:

$$\alpha = T_0\omega\frac{\sin\left(\frac{1}{2}\arctan\frac{c}{b}\right)}{\sqrt[4]{b^2+c^2}},$$

$$\beta = T_0\omega\frac{\cos\left(\frac{1}{2}\arctan\frac{c}{b}\right)}{\sqrt[4]{b^2+c^2}},$$

$$\gamma = L_{c0}\frac{\cos\left(\arctan\frac{c}{b}\right)}{\sqrt[4]{b^2+c^2}},$$

$$\delta = -L_{c0}\frac{\sin\left(\arctan\frac{c}{b}\right)}{\sqrt[4]{b^2+c^2}},$$

and

$$b = 1 + \frac{2(df - eg)}{\sqrt{\frac{\omega}{\omega_0}}(d^2 + e^2)},$$

$$c = 1 + \frac{2(de - ef)}{\sqrt{\frac{\omega}{\omega_0}}(d^2 + e^2)},$$

$$d = ber\sqrt{\omega/\omega_0} + bei\sqrt{\omega/\omega_0},$$

$$e = ber\sqrt{\omega/\omega_0} - bei\sqrt{\omega/\omega_0},$$

$$f = ber'\sqrt{\omega/\omega_0} - bei'\sqrt{\omega/\omega_0},$$

$$g = ber'\sqrt{\omega/\omega_0} + bei'\sqrt{\omega/\omega_0}.$$

Product approximation shall be applied in further calculations [22]. Using product approximation, we obtain a spectral form of a propagation operator and wave impedance

$$T(j\omega) \approx jT_0\omega\left[1 + \left(\frac{\omega_0}{j\omega}\right)^{1/2} + \frac{\omega_0}{j\omega} + \frac{7}{8}\left(\frac{\omega_0}{j\omega}\right)^{3/2}\right], \quad (30)$$

$$L_c(j\omega) \approx \frac{L_{c0}}{1 - \left(\frac{\omega_0}{j\omega}\right)^{1/2} + \frac{1}{8}\left(\frac{\omega_0}{j\omega}\right)^{3/2}}, \quad (31)$$

4. An example of the application of general equations for the analysis of a hydrostatic drive miniaturized line supplied from a constant pressure source and terminated with a servomechanism

The most typical hydrostatic drive in aviation control systems, including in unmanned aerial vehicles, is a drive containing a servomechanism powered from a pressure source (positive displacement pump). A major problem when designing such a drive is ensuring the constancy of servomechanism input pressure. Pressure pulsation can be mitigated by adding a hydraulic accumulator into the line. A simplified mathematical model may be presented as a line supplied from a constant pressure source, terminated with a valve with a variable choking surface. The valve simulates a servomechanism.

The dynamics of a line supplied from a constant pressure source, terminated with a valve with a variable choking surface was described using an equation of a hydrostatic drive line with resistance depending on the frequency in the form

$$\begin{bmatrix} P(l,s) \\ Q(l,s) \end{bmatrix} = \begin{bmatrix} \cosh T(s) & -L_c(s)\sinh T(s) \\ -\frac{1}{L_c(s)}\sinh T(s) & \cosh T(s) \end{bmatrix} \cdot \begin{bmatrix} P(0,s) \\ Q(0,s) \end{bmatrix}, \quad (32)$$

For a hydrostatic drive line with resistance depending on the frequency, the boundary conditions have the following form [18, 19]:

– for $x = 0$ $p(0,t) = \text{const}$,

– for $x = l$ $q(l,t) = \left[1 + \frac{f'(t)}{\bar{f}(t)}\right] G\sqrt{p(l,t)} + \frac{V_a(t)}{p(l,t)}\frac{dp(l,t)}{dt}$,

where:

G – the valve (servomechanism) turbulent conductivity,

$\bar{f}$, f' – the determined value and deviation from the determined value of the valve (servomechanism) choking gap cross-section surface,

V_a – the volume of the gaseous space of a hydraulic accumulator.

For the condition $x = l$, the above equations may be written in the operator form

$$Q(l,s) = K_w F(s) + K_z[1 + \zeta_a s]P(l,s),$$

where:

$K_w = \frac{G}{\bar{f}(t)}\sqrt{\bar{p}(l,t)}$ is a valve reinforcement coefficient (servomechanism),

$K_z = \frac{G}{2 \cdot \sqrt{\bar{p}(l,t)}}$ is the valve (servomechanism) substitute conductivity,

$\zeta_a = \frac{\bar{V}_a(t)}{K_z \bar{p}(l,t)}$ is the hydraulic accumulator time constant.

Substituting the above boundary conditions to the equation (32), we obtain hydrostatic drive line transmittance with a resistance depending on the frequency

$$\frac{Q(0,s)}{F(s)} = \frac{K_w}{\cosh T(s) + K_z[1 + \zeta_a s]L_c(s)\sinh T(s)}, \quad (33)$$

$$-\frac{P(l,s)}{F(s)} = \frac{K_w L_c(s)\sinh T(s)}{\cosh T(s) + K_z[1 + \zeta_a s]L_c(s)\sinh T(s)}, \quad (34)$$

where $T(s)$ and $L_c(s)$ describe the relationships (24) and (25).

A minus sign before transmittance (34) means that the valve (servomechanism) choking surface is inversely proportional to pressure (pressure decreases, when the surface increases).

The transform (34), after the application of product approximation has the form

$$-\frac{P(l,s)}{F(s)} = \left\{\frac{K_w L_{c0}}{T_0 s}T^2(s)\prod_{n=1}^{\infty}\left[1 + \frac{T^2(s)}{(n\pi)^2}\right]\right\} \Big/ \Big/ \left\{\prod_{n=0}^{\infty}\left[1 + \frac{T^2(s)}{((2n+1)/2)^2\pi^2}\right] + + \frac{K_z L_{c0}}{T_0 s}(1 + \zeta_a s)T^2(s)\prod_{n=1}^{\infty}\left[1 + \frac{T^2(s)}{(n\pi)^2}\right]\right\}. \quad (35)$$

Computational formulas for frequency characteristics as per transmittance (34), i.e. hydrostatic drive line supplies from a constant pressure source terminated with a servomechanism and a hydraulic accumulator, are as follows

$$M\left[-\frac{1}{K_w}\frac{P(l,j\omega)}{F(j\omega)}\right]=20\log\sqrt{\xi^2+\chi^2}+$$
$$-20\log\sqrt{(\varepsilon+K_z\xi-K_z\zeta_a\omega\chi)^2+(\eta+K_z\chi+K_z\zeta_a\omega\xi)^2},$$
$$arg\left[-\frac{1}{K_w}\frac{P(l,j\omega)}{F(j\omega)}\right]=\arctan\frac{\chi}{\xi}-\arctan\frac{\eta+K_z\chi+K_z\zeta_a\omega\xi}{\varepsilon+K_z\xi-K_z\zeta_a\omega\chi},$$

where

$$\varepsilon=\cosh\alpha\cdot\cos\beta,$$
$$\eta=\sinh\alpha\cdot\sin\beta,$$
$$\xi=\gamma\cdot\sinh\alpha\cdot\cos\beta-\delta\cdot\cosh\alpha\cdot\sin\beta,$$
$$\chi=\delta\cdot\sinh\alpha\cdot\cos\beta-\gamma\cdot\cosh\alpha\cdot\sin\beta.$$

Using product approximation, the α, β, γ, δ coefficients are calculated from the below formulas

$$\alpha=T_0\cdot\omega\left[\frac{1}{\sqrt{2}}\left(\frac{\omega_0}{\omega}\right)^{1/2}+\frac{\omega_0}{\omega}+\frac{7\sqrt{2}}{8}\left(\frac{\omega_0}{\omega}\right)^{3/2}\right],$$
$$\beta=T_0\cdot\omega\left[1+\frac{1}{\sqrt{2}}\left(\frac{\omega_0}{\omega}\right)^{1/2}+\frac{\omega_0}{\omega}-\frac{7\sqrt{2}}{8}\left(\frac{\omega_0}{\omega}\right)^{3/2}\right],$$
$$\gamma=\left\{L_{c0}\left[1-\frac{1}{\sqrt{2}}\left(\frac{\omega_0}{\omega}\right)^{1/2}-\frac{\sqrt{2}}{8}\left(\frac{\omega_0}{\omega}\right)^{3/2}\right]\right\}\Big/$$
$$\Big/\left\{\left[1-\frac{1}{\sqrt{2}}\left(\frac{\omega_0}{\omega}\right)^{1/2}-\frac{\sqrt{2}}{8}\left(\frac{\omega_0}{\omega}\right)^{3/2}\right]^2+\right.$$
$$\left.+\left[\frac{1}{\sqrt{2}}\left(\frac{\omega_0}{\omega}\right)^{1/2}-\frac{\sqrt{2}}{8}\left(\frac{\omega_0}{\omega}\right)^{3/2}\right]^2\right\},$$
$$\delta=\left\{L_{c0}\left[-\frac{1}{\sqrt{2}}\left(\frac{\omega_0}{\omega}\right)^{1/2}+\frac{\sqrt{2}}{8}\left(\frac{\omega_0}{\omega}\right)^{3/2}\right]\right\}\Big/$$
$$\Big/\left\{\left[1-\frac{1}{\sqrt{2}}\left(\frac{\omega_0}{\omega}\right)^{1/2}-\frac{\sqrt{2}}{8}\left(\frac{\omega_0}{\omega}\right)^{3/2}\right]^2+\right.$$
$$\left.+\left[\frac{1}{\sqrt{2}}\left(\frac{\omega_0}{\omega}\right)^{1/2}-\frac{\sqrt{2}}{8}\left(\frac{\omega_0}{\omega}\right)^{3/2}\right]^2\right\}.$$

For the purpose of detailed calculations, it was assumed that the line is a steel conduit with an internal diameter of 5 mm and a length of 2 m, filled with working liquid, which in a temperature of 25°C exhibits the following parameters: dynamic viscosity $\mu=0.012$ kg/m·s, working liquid density $\rho=850$ kg/m^3, elasticity modulus 1400 MPa. Hydraulic line inlet pressure is 10 MPa. The following coefficient values correspond to the so adopted line parameters: $T_o=1.18\cdot10^{-2}$ s, $\omega=1.45$ s^{-1}, $K_w=9.670$ cm/s, $K_z=0.692$ cm^5/N·s, $L_{c0}=0219$ N·s/cm^5. In the case of the hydrostatic drive in question, in order to determine a response to control signals (with a narrow frequency band), it is enough to apply product approximation of the degree n = 0, whereas to determine a response to pump-generated flow rate pulsation, an approximation of the degree n = 2 should be adopted. Fig. 1 shows the frequency characteristics corresponding to transmittance (34) and product approximation frequency characteristics corresponding to transmittance (35) degree n = 2 for a case without a hydraulic accumulator ($\varsigma_a=0$).

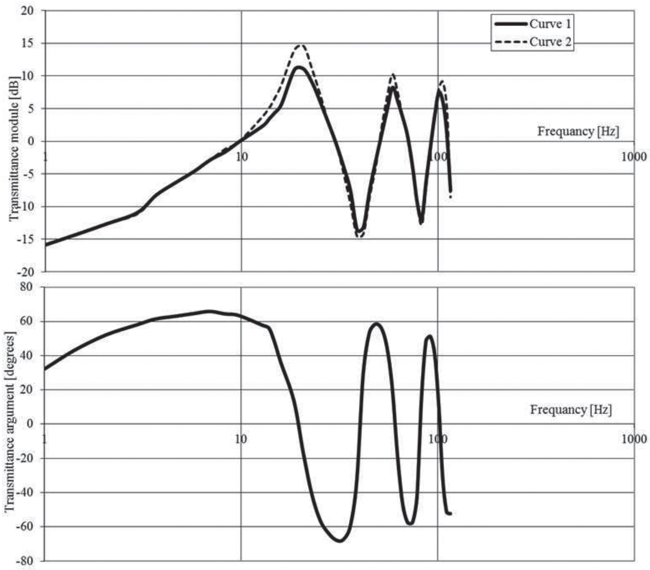

Fig. 1. Frequency characteristics corresponding to transmittance (34) and (35) for a case without a hydraulic accumulator ($\varsigma_a=0$). Curve 1: approximation degree n = ∞; curve 2: approximation degree n = 2

Fig. 2 shows the frequency characteristics corresponding to transmittance (34) and product approximation frequency characteristics corresponding to transmittance (35) degree n = 0 for a case of a line by-passed with a hydraulic accumulator.

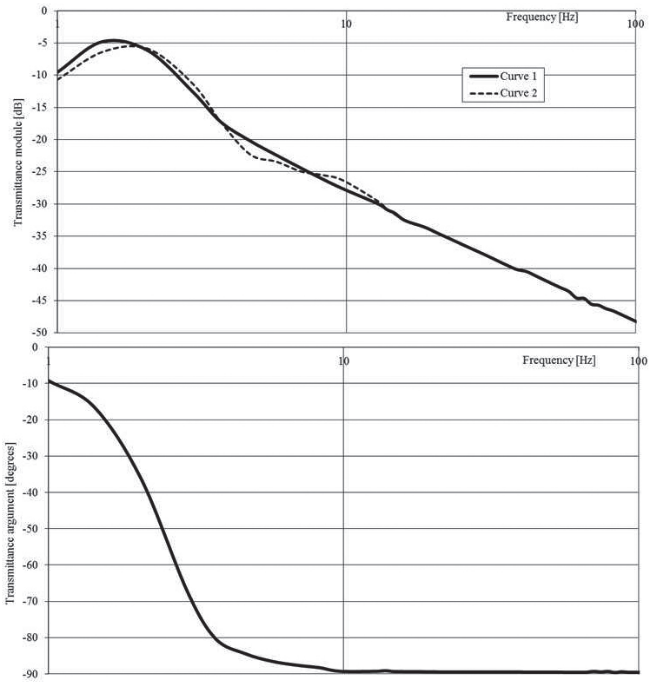

Fig. 2. Frequency characteristics corresponding to transmittance (34) and (35) for a line by-passed with a hydraulic accumulator. Curve 1: approximation degree n = ∞; curve 2: approximation degree n = 0

A comparison of the characteristics from Fig. 1 and Fig. 2 indicates that a hydraulic accumulator significantly damps pressure pulsation in a drive, acting as a low-pass filter. Fig. 1 and Fig. 2 also indicate that in a case, where hyperbolic functions are present in the numerator and the denominator, a lower degree of product approximation is enough for obtaining approximation accuracy similar to the case of transmittance, with hyperbolic functions only in the denominator.

5. Conclusions

The article presents the description and analysis of the phenomena occurring during the flow of the working liquid in the hydraulic line supplying the hydraulic servomechanisms, at their rapid overloading, i.e. standard operation of the hydraulic servomechanisms on UAV. Fluid movement in a hydrostatic line is described by partial differential equations of the hyperbolic type, therefore, the modelling takes into account wave phenomena undergoing in the line. The basic parameters characterizing a hydrostatic line are the propagation operation and characteristic impedance. The first of these parameters describes the delay time in transmitting signals along the line and damping and dispersion of pressure waves and flow rate, whereas the second one is the internal impedance of the line. Product approximation was applied in order to solve wave equations.

A hydrostatic line was characterized by hydraulic impedance components (fixed resistance): serial impedance (consisting of inertance and resistance per length unit, which takes into account the effect of inertia and viscous friction of the fluid), and by-pass admittance per length unit (characterized by capacitance), which takes into account the fluid compressibility effect. Main emphasis was on determining frequency responses. It results from the following:

- calculating frequency characteristics in the case of transmittance, with present hyperbolic functions, causes less problems than calculating transient responses;
- numerous input signals (interference), acting in hydrostatic drives, are of the sinusoid character (in a general case, a sum of trigonometric functions). This group includes, i.a., flow rate pulsations generated by hydraulic pumps. In these cases, frequency characteristics provide direct information on the dynamic properties of a hydrostatic drive.

General equations describing the flow properties in the hydraulic line were used to analyze the flow in a miniaturized line with a diameter of 5 mm connecting the constant pressure supply source with a hydraulic servomechanism on UAV. The diameter of 5 mm is the minimum diameter ensuring the flow of the working liquid through the conduit with the intensity guaranteeing the required speed of the servomechanism piston rod, i.e. the movement of the UAV actuator. The basic criterion for the optimization of the flow properties through a miniaturized hydrostatic line should be the principle that the load resistance should be equal to the nominal characteristic impedance of the line. However, adopting these principles requires limiting the maximum flow rate of fluid in the line (conduit) and the maximum static line resistance. In the case, in which it is possible to match impedance, pressure pulsation can be mitigated by adding a hydraulic accumulator into the line, while aiming at the lowest possible accumulator capacities. A hydraulic accumulator significantly damps pressure pulsation in a drive, acting as a low-pass filter. However, it should be noted that using pulsation dampers results in decreased operation speed of actuator elements in the system and its decreased rigidity.

References

[1] T.M. Baszta, *Calculations and designs of aircraft air defense devices*, State Scientific and Technical Publishing House, Moskwa, 1961.

[2] D. Lovrec, V. Tic, and T. Tasner, "Dynamic behaviour of different hydraulic drive concepts – comparison and limits", *Int. J. Simul. Model.* 16(3), 448–457 (2017).

[3] W. Kollek *et al.*, *Basics of designing, modelling, operating microhydraulic elements and systems*, Wroclaw University of Technology, Wrocław, 2011.

[4] M. Gullion, *Theory and calculations of hydraulic systems*, Scientific and Technical Publishers, Warszawa, 1966.

[5] Y. Lu, X. Su, and J. Li, "A topology-alterative two-phase flow solver and its validation for a dynamic hydraulic discharge process", *J. Hydraul. Res.* 57(5), 607–622 (2019).

[6] Z. Zarzycki and S. Kudźma, "Computation of transient turbulent flow of liquid in pipe using unsteady friction formula", *Transactions of the Institute of Fluid – Flow Machinery* 116, 27–42 (2005).

[7] L. Li, H. Huang, F. Zhao, M. J. Triebe, and Z. Liu, "Analysis of a novel energy-efficient system with double-actuator for hydraulic press", *Mechatronics* 47, 77–87 (2017).

[8] Z.C. Zhang, H.X. Chen, Z. Ma, J.W. He, H. Liu, and C. Liu, "Research on Improving the Dynamic Performance of Centrifugal Pumps With Twisted Gap Drainage Blades", *J. Fluids Eng.* 141 (9), 3–18 (2019).

[9] D.G. German, J.M. Reese, and Y.L. Zhang, "Vibration of a flexible pipe conveying viscous pulsating fluid flow", *J. Sound Vibr.* 230 (2), 365–387 (2000).

[10] Z. Kudźma, *Damping pressure pulsations and sound in transiet and determined states of hydraulic systems*, Wroclaw University of Technology, Wrocław, 2012.

[11] Z. Wang, Z. Qian, J. Lu and, P. Wu, "Effects of flow rate and rotational speed on pressure fluctuations in a double-suction centrifugal pump", *Energy* 170, 212–227 (2019).

[12] M. Stosiak, *Identification of the impact of vibrations and the method of their mitigation at selected hydraulic valves*, Wroclaw University of Technology, Wrocław, 2015.

[13] L. Grinis, V. Haslavsky, and U. Tzadka, "Self-excited vibration in hydraulic ball check valve", *World Academy of Science Engineering and Technology* 6, 1041–1046 (2012).

[14] A. Kačeniauskas, "Development of efficient interface sharpening procedure for viscous incompressible flows", *Informatica* 19, 487–504 (2008).

[15] P.A. Laski, "Fractional-order feedback control of a pneumatic servo-drive", *Bull. Pol. Ac.: Tech.* 67 (2), 53–59 (2019).

[16] L. Ge, L. Quan, X. Zhang, B. Zhao, and J. Yang, "Efficiency improvement and evaluation of electric hydraulic excavator with speed and displacement variable pump", *Energy Conv. Manag.* 150, 62–71 (2017).

[17] L. Wang, F. Wang, and X. Lei, “Investigation on friction models for simulation of pipeline filling transients”, *J. Hydraul. Res.* 56 (6), 888–895 (2018).

[18] L. Ułanowicz, “Dynamic properties of a hydraulic system as a impedance loaded line”, *Pneumatics Industrial Compressed Air Systems* 4 (73), 30–33 (2009).

[19] S. Błoński, A. Pręgowska, T. Michałek, and J. Szczepański, “The use of Lempel-Ziv complexity to analyze turbulence and flow randomness based on velocity fluctuations”, *Bull. Pol. Ac.: Tech.* 67 (5), 957–962 (2019).

[20] M. Vašina, L. Hružík, and A. Bureček, “Energy and Dynamic Properties of Hydraulic Systems”, *Tehnički Vjesnik* 25 (2), 382–390 (2018).

[21] E. Stupak, R. Kačianauskas, A. Kačeniauskas, V. Starikovičius, A. Maknickas, R. Pacevič, G. Davidavičius, and A. Aidietis, “The geometric model-based patient-specific simulations of turbulent aortic valve floks”, *Arch. Mech.* 69 (4–5), 317–345 (2017).

[22] H. Cormen, E. Leiserson and L. Rivest, *Introduction to algorithms*, Scientific and Technical Publishers, Warszawa, 2000.