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# Computer-aided prediction of the renewal and availability characteristics of complex technical systems

**Keywords**

reliability, large system, availability, renewal , prediction

**Abstract**

The procedures of defining the parameters of the availability of the multistate systems in variable operation conditions are shown. Furthermore, the renewal and availability characteristics of the complex technical system are defined. The computer program for prediction of these characteristics capabilities with description are presented. As the application of this computer program, the prediction the renewal ad availability characteristics of the exemplary complex technical system is shown.

## 1. Introduction

Analysis of availability and renewal processes of technical systems often relates to the complex systems. Operation processes of these systems are complicated and have a significant impact of their availability and renewal processes. The solution of this problem is a combination of semi-Markov [2], [5] operation process model with multistate reliability and availability model of systems [3], [4]. This way of solution to the problem leads to construct a general model of total system reliability and availability of complex systems [4], taking into account the operation process of these systems [6].

## 2. Availability of multistate systems in variable operations conditions

To estimate the renewal and availability of the complex technical systems it should be fixed following reliability characteristics of these systems [1], [4]:

i) in the both cases of the system renovation:
- the number of the system reliability states,
- the system critical reliability state *r*,
- the mean value $\mu(r)$ of the unconditional lifetime of the system in the reliability states subset not worse than the system critical reliability state *r*,
- the standard deviation $\sigma(r)$ of the unconditional lifetime of the system in the reliability states subset not worse than the system critical reliability state *r*;

ii) in the case when the system renovation time is non-ignored
- the $r$-th coordinate $\boldsymbol{R}(t,r)$, $t \geq 0$ of the system unconditional reliability function $\boldsymbol{R}(t,\cdot)$.

In addition, the following input renewals parameters of these systems are needed [1], [4]:

i) in the case when the system renovation time is ignored
- the number *N* of the system exceeding the critical reliability state *r* and renovations,
- the system renewal process duration time *t*;

ii) in the case, when the system renovation time is non-ignored
- the mean value $\mu_0(r)$ of the system renovation time,
- the standard deviation $\sigma_0(r)$ of the system renovation time,

- the number $N$ of the system exceeding the critical reliability state $r$ and renovations,
- the system renewal process duration time $t$;
- the length of the system availability interval $\tau$.

## 3. Prediction of the renewal and availability characteristics of complex technical systems

When the data described in section 2 is given and according to the theorems from [1], [4] the following characteristics of the renovation and availability of the complex technical are determine:

i) in the case when the system renovation time is ignored

- the time $S_N(r)$ until the *Nth* exceeding by the system critical reliability state *r*, for sufficiently large *N*, has approximately normal distribution $N(N\mu(r), \sqrt{N}\sigma(r))$, i.e.

$$F^{(N)}(t,r) = P(S_N < t) \cong F_{N(0,1)}(\frac{t - N\mu(r)}{\sqrt{N}\sigma(r)}), \quad (1)$$

$t \in (-\infty, \infty),\ r \in \{1,2,...,z\};$

- the expected value and the variance of the time $S_N(r)$ until the *Nth* exceeding by the system the reliability critical state *r*, are respectively given by

$$E[S_N(r)] \cong N\mu(r), \quad (2)$$

$$D[S_N(r)] \cong N\sigma^2(r),\ r \in \{1,2,...,z\}; \quad (3)$$

- the number $N(t,r)$ of exceeding by the system the reliability critical state *r* up to the moment $t, t \geq 0$, for sufficiently large *t*, approximately has the distribution of the form

$$P(N(t,r) = N) \cong F_{N(0,1)}(\frac{(N+1)\mu(r) - t}{\sigma(r)\sqrt{\frac{t}{\mu(r)}}})$$

$$- F_{N(0,1)}(\frac{N\mu(r) - t}{\sigma(r)\sqrt{\frac{t}{\mu(r)}}}), \quad (4)$$

$N = 0,1,...,\ r \in \{1,2,...,z\};$

- the expected value and the variance of the number $N(t,r)$ of exceeding by the system the reliability critical state *r* up to the moment $t, t \geq 0$, for sufficiently large *t*, are respectively given by

$$H(t,r) \cong \frac{t}{\mu(r)}, \quad (5)$$

$$D(t,r) \cong \frac{t}{\mu^3(r)}\sigma^2(r),\ r \in \{1,2,...,z\}; \quad (6)$$

ii) in the case when the system renovation time is non-ignored

- the time $\bar{S}_N(r)$ until the *Nth* exceeding by the system the reliability critical state *r*, for sufficiently large *N*, has approximately normal distribution $N(N\mu(r) + (N-1)\mu_o(r)), \sqrt{N\sigma^2(r) + (N-1)\sigma_o^2(r)})$, i.e.,

$$\bar{F}^{(N)}(t,r) = P(\bar{S}_N(r) < t)$$

$$\cong F_{N(0,1)}(\frac{t - N(\mu(r) + \mu_o(r)) + \mu_o(r)}{\sqrt{N(\sigma^2(r) + \sigma_o^2(r)) - \sigma_o^2(r)}}), (7)$$

$t \in (-\infty, \infty),\ r \in \{1,2,...,z\};$

- the expected value and the variance of the time $\bar{S}_N(r)$ until the *Nth* exceeding by the system the reliability critical state *r*, for sufficiently large *N*, are respectively given by

$$E[\bar{S}_N(r)] \cong N\mu(r) + (N-1)\mu_o(r), \quad (8)$$

$$D[\bar{S}_N(r)] \cong N\sigma^2(r) + (N-1)\sigma_o^2(r), \quad (9)$$

$r \in \{1,2,...,z\};$

- the number $\bar{N}(t,r)$ of exceeding by the system, the reliability critical state *r* up to the moment $t, t \geq 0$, for sufficiently large *t*, has approximately distribution of the form

$$P(\bar{N}(t,r) = N)$$

$$\cong F_{N(0,1)}(\frac{(N+1)(\mu(r) + \mu_o(r)) - t - \mu_0(r)}{\sqrt{\frac{t + \mu_0(r)}{\mu(r) + \mu_0(r)}(\sigma^2(r) + \sigma_o^2(r))}})$$

$$- F_{N(0,1)}(\frac{N(\mu(r) + \mu_o(r)) - t - \mu_0(r)}{\sqrt{\frac{t + \mu_0(r)}{\mu(r) + \mu_0(r)}(\sigma^2(r) + \sigma_o^2(r))}}), \quad (10)$$

$N = 0,1,...,\ r \in \{1,2,...,z\};$

- the expected value and the variance of the number $\overline{N}(t,r)$ of exceeding by the system the reliability critical state $r$ up to the moment $t, t \geq 0,$ for sufficiently large $t$, are respectively given by

$$\overline{H}(t,r) \cong \frac{t+\mu_0(r)}{\mu(r)+\mu_o(r)}, \quad (11)$$

$$\overline{D}(t,r) \cong \frac{t+\mu_0(r)}{(\mu(r)+\mu_o(r))^3}(\sigma^2(r)+\sigma_o{}^2(r)), \quad (12)$$

$r \in \{1,2,...,z\};$

- the time $\overline{\overline{S}}_N(r)$ until the *Nth* system's renovation, for sufficiently large $N$, has approximately normal distribution

$N(N(\mu(r)+\mu_o(r)), \sqrt{N(\sigma^2(r)+\sigma_o^2(r))})$, i.e.,

$$\overline{\overline{F}}^{(N)}(t,r) = P(\overline{\overline{S}}_N(r) < t)$$

$$\cong F_{N(0,1)}\left(\frac{t-N(\mu(r)+\mu_o(r))}{\sqrt{N(\sigma^2(r)+\sigma_o^2(r))}}\right), \quad (13)$$

$t \in (-\infty,\infty), \ r \in \{1,2,...,z\};$

- the expected value and the variance of the time $\overline{\overline{S}}_N(r)$ until the *Nth* system's renovation, for sufficiently large $N$, are respectively given by

$$E[\overline{\overline{S}}_N(r)] \cong N(\mu(r)+\mu_o(r)), \quad (14)$$

$$D[\overline{\overline{S}}_N(r)] \cong N(\sigma^2(r)+\sigma_o^2(r)), \quad (15)$$

$r \in \{1,2,...,z\};$

- the number $\overline{\overline{N}}(t,r)$ of system's renovations up to the moment $t, t \geq 0,$ for sufficiently large $t$, has approximately distribution of the form

$$P(\overline{\overline{N}}(t,r) = N) \cong$$

$$F_{N(0,1)}\left(\frac{(N+1)(\mu(r)+\mu_o(r))-t}{\sqrt{\frac{t}{\mu(r)+\mu_0(r)}(\sigma^2(r)+\sigma_o^2(r))}}\right)$$

$$-F_{N(0,1)}\left(\frac{N(\mu(r)+\mu_o(r))-t}{\sqrt{\frac{t}{\mu(r)+\mu_0(r)}(\sigma^2(r)+\sigma_o^2(r))}}\right), \quad (16)$$

$N = 0,1,..., \ r \in \{1,2,...,z\};$

- the expected value and the variance of the number $\overline{\overline{N}}(t,r)$ of system's renovations up to the moment $t, t \geq 0,$ for sufficiently large $t$, are respectively given by

$$\overline{\overline{H}}(t,r) \cong \frac{t}{\mu(r)+\mu_o(r)}, \quad (17)$$

$$\overline{\overline{D}}(t,r) \cong \frac{t}{(\mu(r)+\mu_o(r))^3}(\sigma^2(r)+\sigma_o{}^2(r)), \quad (18)$$

$r \in \{1,2,...,z\};$

- the steady availability coefficient of the system at the moment $t, t \geq 0,$ for sufficiently large $t$, is given in form

$$A(t,r) \cong \frac{\mu(r)}{\mu(r)+\mu_o(r)}, \quad t \geq 0, \ r \in \{1,2,...,z\}; \quad (19)$$

- the steady availability coefficient of the system in the time interval $< t, t+\tau), \tau > 0,$ for sufficiently large $t$, is given by

$$A(t,\tau,r) \cong \frac{1}{\mu(r)+\mu_o(r)}\int_\tau^\infty \boldsymbol{R}(t,r)dt, \quad t \geq 0, \quad (20)$$

$\tau > 0, \ r \in \{1,2,...,z\}.$

## 4. Description of the computer program for prediction of complex technical systems renewal and availability

The presented computer program is based on methods of identification the complex technical system availability and renewal processes presented in Section 2 and 3, given in [1], [4]. The computer program is written in Java language with using SSJ V2.1.3 library. The SSJ library is a Java library, developed in the Department d'Informatique et de Recherche Operationelle (DIRO) at the Universite de Montreal, gives the support of stochastic simulations. The on-line documentation of SSJ can be found at the website http://www.iro.umontreal.ca/~simardr/ssj/indexe.html.

The computer program is composed of five panels. The first panel "Choosing the renovation type" is used for choosing the renovation type and for reading reliability data for systems renewal and availability, i.e. the reading data is:

– the number, $z + 1$, of the system reliability states,
– the system critical reliability state $r$,
– the mean value of the unconditional lifetime of the system in the reliability states subset not worse than the system critical reliability state $r$,
– the standard deviation of the unconditional lifetime of the system in the reliability states subset not worse than the system critical reliability state $r$.

Next, according to the type of renovation, the computer program go to either panel „Time ignored" or panel „Time non-ignored".

When the renovation with ignored time is chosen, there it should be give the basic input parameters as follows:

– the number $N$ of the system exceeding the critical reliability state ,
– the system renewal process duration time.

In the case when the renovation time is non-ignored, the following parameters are additionally needed:

– unconditional reliability function,
– the mean value of the system renovation time,
– the standard deviation of the system renovation time,
– the number $N$ of the system exceeding the critical reliability state $r$ and renovations,
– the system renewal process duration time,
– the length of the system availability interval.

When the Reading data is finished, the computer program automatically goes to next panel. According to the type of renovation, it can be panel „Results – ignored time" or "Results – non-ignored time".

The computer program is predicting the following characteristics of the complex technical system renewal and availability:

i) in the case when the system renovation time is ignored

– the distribution of the time $S_N(r)$ until the *Nth* exceeding of reliability critical state $r$ of this system, $r \in \{1,2,...,z\}$,
– the expected value and the variance of the time $S_N(r)$, $r \in \{1,2,...,z\}$, until the *Nth* exceeding the reliability critical state $r$ of this system,
– the distribution of the number $N(t,r)$ of exceeding the reliability critical state $r$ of this system up to the moment $t, t \geq 0$, $r \in \{1,2,...,z\}$,
– the expected value and the variance of the number $N(t,r)$ of exceeding the reliability critical state $r$ of this system at the moment $t, t \geq 0$, $r \in \{1,2,...,z\}$;

ii) in the case when the system renovation time is non-ignored

– the distribution function of the time $\overline{S}_N(r)$ until the *Nth* exceeding the reliability critical state $r$ of this system, $r \in \{1,2,...,z\}$,
– the expected value and the variance of the time $\overline{S}_N(r)$ until the *Nth* exceeding the reliability critical state $r$ of this system, $r \in \{1,2,...,z\}$,
– the distribution of the number $\overline{N}(t,r)$ of exceeding the reliability critical state $r$ of this system up to the moment $t, t \geq 0$, $r \in \{1,2,...,z\}$,
– the expected value and the variance of the number $\overline{N}(t,r)$ of exceeding the reliability critical state $r$ of this system up to the moment $t, t \geq 0$, $r \in \{1,2,...,z\}$,
– the distribution function of the time $\overline{\overline{S}}_N(r)$ until the *Nth* system's renovation, $r \in \{1,2,...,z\}$,
– the expected value and the variance of the time $\overline{\overline{S}}_N(r)$ $r \in \{1,2,...,z\}$, until the *Nth* system's renovation,
– the distribution of the number $\overline{\overline{N}}(t,r)$ of system's renovations up to the moment $t, t \geq 0$, $r \in \{1,2,...,z\}$,
– the expected value $\overline{\overline{H}}(t,r)$ and the variance $\overline{\overline{D}}(t,r)$ of the number $\overline{\overline{N}}(t,r)$ of system's renovations up to the moment $t, t \geq 0$, $r \in \{1,2,...,z\}$,
– the steady availability coefficient of the system at the moment $t$,
– the steady availability coefficient of the system in the time interval $< t, t + \tau)$, $\tau > 0$.

## 5. Prediction of exemplary complex technical system renewal and availability

We consider the exemplary system $S$. The system is composed of two parallel subsystems $S_1$, $S_2$ forming a series structures (*Figure 1*).

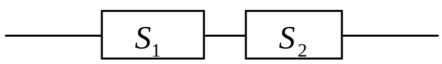


*Figure 1*. General scheme of system S reliability structure

Subsystem $S_1$ is composed of 2 components $E_{ij}^{(1)}$, $i$ = 1,2, $j$ = 1, with the structure given in *Figure 2*.

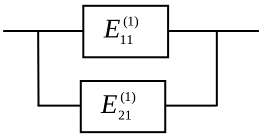


*Figure 2*. General scheme of subsystem $S_1$ reliability structure

Subsystem $S_2$ is composed of 4 components, $E_{ij}^{(2)}$, $i$ = 1,2,3, 4, $j$ = 1, with the reliability structure given in *Figure 3*.

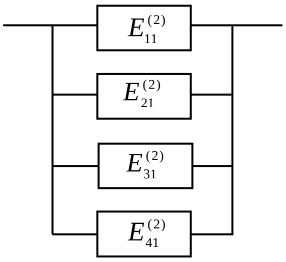


*Figure 3*. General scheme of subsystem $S_2$ reliability structures

We assume that the changes of the operation states of the system operation process have an influence on its reliability structure and its multi-state components reliability as well.
Arbitrary, we distinguish two following operation states:

- operation state $z_1$ – system is composed of parallel subsystem $S_1$ (*Figure 2*),
- operation state $z_2$ – system is composed of parallel subsystem $S_2$ (*Fgure 3*).

Furthermore, we assume following probabilities of operation states $p_1 = 0.25$, $p_2 = 0.75$.

Next, at all operation states $z_b$, $b = 1,2$, we distinguish the following four reliability states of the system and its components:

- reliability state 3 – the system is fully safe,
- reliability state 2 – is less safe because of the system aging,
- reliability state 1 – is less safe because of the system aging and more dangerous,
- reliability state 0 – the system is destroyed.

The system and components critical reliability state is $r$=2 and it is assumed that there are possible the transitions between the components reliability states only from better to worse ones [3], [4].
Thus, the system S has the $r$-th coordinate $\boldsymbol{R}(t,r)$, $t \geq 0$ of the system unconditional reliability function $\boldsymbol{R}(t,\cdot)$ [1], [4] in the form

$$R(t,2) = 0.25 \cdot [R(t,2)]^{(1)} + 0.75 \cdot [R(t,2)]^{(2)}, \qquad (21)$$
$$t \geq 0,$$

where its components have the exponential reliability functions, different in particular operation states $z_b$, $b$ = 1,2 given by

$$[R(t,2)]^{(1)} = 2\exp[-2t] - \exp[-4t], \qquad (22)$$

and

$$[R(t,2)]^{(2)} = 4\exp[-2t] - 6\exp[-4t] + 4\exp[-6t] - \exp[-8t], \qquad (23)$$
$$t \geq 0.$$

After reading all necessary data, and after choosing the renovation with ignored time, the computer program gives the results presented in *Figure 4*.

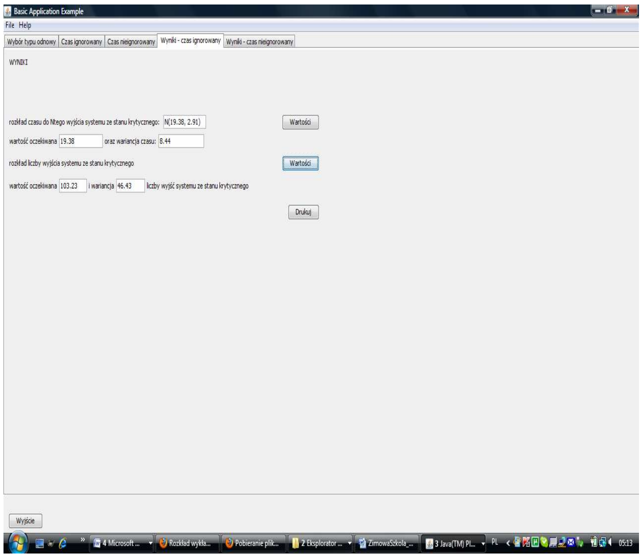


*Figure 4.* The results of the computer program for prediciton of availability and renewal process of complex technical systems in the case of the renovation with ignored time

When the renovation with non-ignored time is chosen, the computer program sets the characteristics show in *Figure 5*.

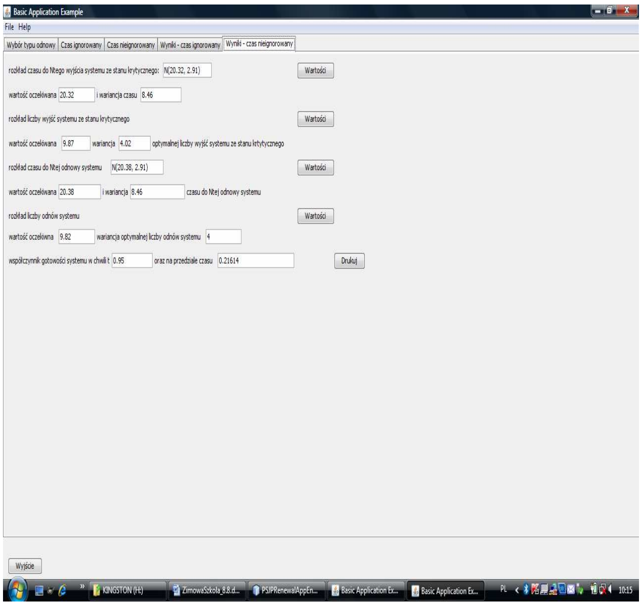


*Figure 5*. The results of predicting the characteristics of the renewal process with non-ignored time

It is possible to generate a table with values of main characteristics of availability and renewal (*Figure 6*).

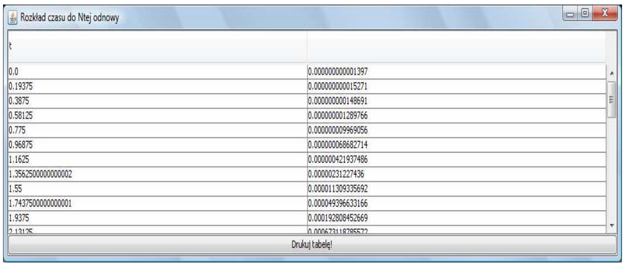


*Figure 6*. The values of the distribution of the time until the *Nth* exceeding of reliability critical state

After receiving the final results and tables, they can be printed and then quit the program or restart.

## 6. Conclusion

The presented computer program for basic characteristics prediction of the renewal process and availability is based on methods and algorithms given in [1]. The computer program determines in case of renovation with ignored time: the distribution, the expected value and the variance of the time until the *Nth* exceeding of reliability critical state of this system, the distribution, the expected value and the variance of the number of exceeding the reliability critical state of this system up to the particular moment. When the non-ignored time of renovation is under consideration, the computer program predicts: the distribution function, the expected value and the variance of the time until the exceeding the reliability critical state of this system, the distribution, the expected value and the variance of the number of exceeding the reliability critical state of this system up to the particular moment, the distribution function, the expected value and the variance of the time until the system's renovation, the distribution, the expected value and the variance of the number of system's renovations up to the particular moment. Additionally, in this type of renovation, the computer program predicts the availability coefficient of the system at the particular moment and in the time interval.

In the article presented program has been used to prognosis unknown characteristics of the exemplary system renewal and availability.

## Acknowledgements

The paper describes the part of the work in the Poland-Singapore Joint Research Project titled "Safety and Reliability of Complex Industrial Systems and Processes" supported by grants from the Poland's Ministry of Science and Higher Education (MSHE grant No. 63/N-Singapore/2007/0) and the Agency for Science, Technology and Research of Singapore (A*STAR SERC grant No. 072 1340050).

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