

SLIDING MODE SPEED CONTROL OF AN INDUCTION MOTOR DRIVE USING TIME-VARYING SWITCHING LINE*

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Abstract: An equivalent sliding mode control method of an induction motor speed has been discussed. The control signal consists of two parts: a continuous and discontinuous one. Very good dynamical response of the drive system has been obtained, which however changes influenced by external and parametric disturbances occurring under various operation conditions. In order to ensure identical dynamic performance of the speed transients and system robustness during the switching line reaching phase, regardless of external and parametric disturbances, the time-varying switching line is proposed. The simulation results have been validated by experimental tests of the induction motor drive system.

Keywords: *sliding mode control, equivalent control, induction motor, speed controller, time-varying switching line*

1. INTRODUCTION

Sliding mode control (SMC) has been known for several decades and is still a topic of great interest as well as in control theory in various applications [1]. The SMC uses a high frequency switching control law when states cross a sliding manifold to drive the system states from any initial values to the user specified surface. Once the system reaches the sliding manifold, the equivalent control keeps the system always pointing to the surface. The SMC is one of the control systems with a simple structure and the sliding motion is guaranteed to be insensitive to bounded parametric and functional uncertainties [1]. However, its main drawback – the chattering, i.e., induced high frequency oscillations visible in controlled variables due to high frequency switching control law may be reduced by replacement of the sign function by its nonlinear continuous approximations [2, 3] or selecting a modified control method [4].

The sliding mode control has been successfully used to control various industrial plants, among others, converter fed induction motor (IM) drives [5]. This type of control can directly define transistors on/off signals in a way natural for all commonly used

*Manuscript received: May 6, 2017; accepted: July 5, 2017.

voltage converters. However, in the case of speed or position control, this direct type of control does not ensure controlling internal state variables of the induction motor [6]. Therefore, it is necessary to use a cascade connection of external and internal regulators of suitable state variables.

The sliding mode control ensures robustness to disturbances when the describing point reaches the intersection of switching hyperplanes in the phase space. For simplification reasons, the term reaching the switching line in the phase plane was used in the paper. At the moment of changing a reference value, a change in the position of the above mentioned line takes place, and the system becomes sensitive to any disturbances until the line is reached again. For this reason, time-dependent change of the switching line position has been proposed for a drive system analyzed in this paper.

In the basic solution proposed in [7], this change was supposed to have stepwise character in short time intervals. Due to the fact that between subsequent changes the control system is not robust, a time-continuous switching line was proposed in [8]. Initial research conducted for second order systems was transferred to the third order systems, with simultaneous consideration for limitations of control signals and ensuring optimum system operation with reference to selected quality indexes (ITAE, IAE) [9].

A time-varying switching line was introduced in various electrical drive applications. DC motor position control is shown in [10] and [11]. Switching surface adapting with speed control error for a brushless DC motor is presented in [12]. Shaft position control for a PMSM drive is proposed in [13].

Time-varying switching line can be also successfully applied in the induction motor control. A sliding mode linear feedback control with switched gains was used in connection with indirect field oriented control (IFOC) in [14] and [15]. A switching line changing exponentially in time was used in [16]. Controlling the position of a generalized electric motor (with ideal control applied in a torque loop) with simultaneous use of fuzzy logic and time-varying switching line was presented in [17]. The time-varying switching line was also used in the discrete time sliding mode control for induction motor [18], however only the simulation tests are shown. The time-varying switching line was also used in fault tolerant drive systems to ensure identical dynamics of a six phase induction machine during healthy and faulty (lack of one or more stator phases) operations [19].

All of the above mentioned papers presented the use of the time-varying switching line in the control of the position of the motor shaft (except for the BLDC motor velocity control in [12]). This work proposes the use of the described technique in the control of induction motor speed.

The paper has been organized in seven sections. After an introduction, in the second section, the design process of the sliding mode control systems and its successive stages are described shortly. Next, in Section 3, a mathematical model of an IM is presented as well as direct and cascade speed control concepts with the application of sliding modes. Then the equivalent sliding mode speed control with the time-varying switching

line is proposed and discussed. In the last part of the paper, selected results of simulation and experimental research for the control system of the IM drive with a stationary and a time-varying switching lines are demonstrated. At the end of this article a short summary of the obtained results is presented. Parameters of the tested induction motor drive are given in the Appendix.

2. SLIDING MODE CONTROL. THEORETICAL BASIS

The design of the sliding mode control algorithm for any dynamical system can be divided into the following stages:

- defining a mathematical model of the object,
- identification of mathematical model parameters,
- defining a control signal vector and a switching function vector,
- defining the sliding mode control law appropriately for the adopted control signals and switching functions,
 - selection of design parameters of a regulator,
 - stability analysis of the designed system.

Sliding mode control, as an algorithmic control method, requires detailed knowledge of a mathematical model of the control object. Additionally, as it will be shown, the method uses directly the parameters of the mathematical model, therefore they should be properly identified.

The control signal vector, in its general form, can be defined as follows:

$$\mathbf{k} = [k_1 \quad k_2 \quad \dots \quad k_n]^T \quad (1)$$

where n is the number of available control signals.

The next step in the design of sliding mode control algorithms is the choice of the switching function vector:

$$\mathbf{s} = [s_1 \quad s_2 \quad \dots \quad s_n]^T \quad (2)$$

The purpose of the sliding mode system will be reducing all components of the above vector (2) to zero. Next, with so adopted control signal vector (1) and the switching function vector (2), it is necessary to define the sliding mode control law. Three different solutions, which will be considered in the following part of the paper, are demonstrated in Fig. 1.

The solution which is most frequently used in numerous applications is the relay control, for which the control law takes the following form:

$$\mathbf{k} = -\Gamma \operatorname{sign}(\mathbf{s}^*), \quad \mathbf{s}^* = \mathbf{s}^T \mathbf{D} \quad (3)$$

where the gain matrix is

$$\mathbf{\Gamma} = \begin{bmatrix} \gamma_1 & 0 & \dots & 0 \\ 0 & \gamma_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \gamma_n \end{bmatrix} \quad (4)$$

with positive parameters: $\gamma_1, \gamma_2, \dots, \gamma_n > 0$.

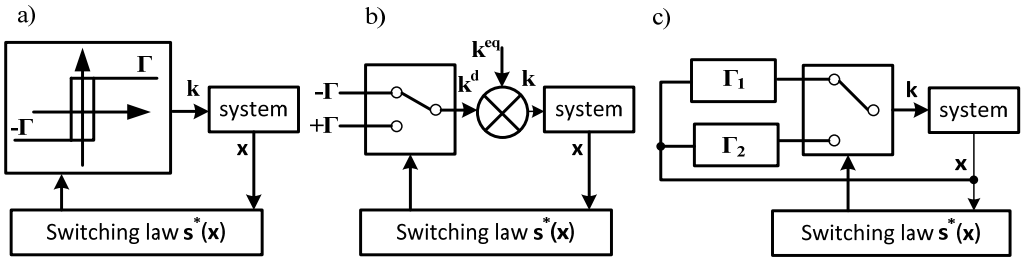


Fig. 1. Three chosen sliding mode control topologies:
a) relay control, b) equivalent control, c) linear feedback control with switched gains

Matrix \mathbf{D} appears during the division of the derivative of a switching function vector (2) into

$$\dot{\mathbf{s}} = \mathbf{f}_1 + \mathbf{f}_2 + \mathbf{D}\mathbf{k} \quad (5)$$

where \mathbf{f}_1 can be determined using available variables, \mathbf{f}_2 is an unknown part, depending on disturbances (in the case of the drive system it is mainly the load torque and, among others, parameter uncertainties).

The information of the value of the vector \mathbf{f}_1 can be used to significantly reduce oscillation in reference signals, thanks to the use of equivalent control method (Fig. 1b). In this method, the control signal (1) consists of two components

$$\mathbf{k} = \mathbf{k}^{eq} + \mathbf{k}^d \quad (6)$$

Continuous component \mathbf{k}^{eq} is calculated according to the following equation

$$\dot{\mathbf{s}} = \mathbf{f} + \mathbf{D}\mathbf{k}^{eq} = \mathbf{0} \quad (7)$$

and discontinuous component \mathbf{k}^d , defined like in case of the relay control, as

$$\mathbf{k}^d = -\mathbf{\Gamma}^d \text{sign}(\mathbf{s}^*)^T \quad (8)$$

In the case of the third control method, the linear feedback control with switched gains (Fig. 1c), the control law is defined as follows:

$$\mathbf{k} = \Gamma(\mathbf{x})\mathbf{x} \quad (9)$$

The gain matrix depends on the switching function vector

$$\Gamma(\mathbf{x}) = \begin{cases} \Gamma_1 & \text{when } \mathbf{s}(\mathbf{x})\mathbf{x} \geq 0 \\ \Gamma_2 & \text{when } \mathbf{s}(\mathbf{x})\mathbf{x} < 0 \end{cases} \quad (10)$$

where Γ_1, Γ_2 have similar forms to (4).

Due to a large number of the parameters in (10), this control method and its application to the IM drive control will not be considered in the following part of the paper. However, this control method has been applied with the time-varying switching line in the position control of the induction motor drive [15].

Smaller number of parameters which have to be selected in the regulator design process (a significant difference in comparison with the linear feedback control with switched gains) and the lowest oscillation level (especially in relation to relay control) has decided, that the equivalent control method has been used in further research.

The last design stage of the sliding mode regulators should be the stability analysis of the designed system. The standard positive-defined Lyapunov function can be used, which takes the following form

$$L = \frac{1}{2} \mathbf{s}^T \mathbf{s} = \frac{1}{2} (s_1^2 + s_2^2 + \dots + s_n^2) > 0 \quad (11)$$

while its derivative is

$$\dot{L} = \mathbf{s}^T \dot{\mathbf{s}} = s_1 \dot{s}_1 + s_2 \dot{s}_2 + \dots + s_n \dot{s}_n \quad (12)$$

A negative value of the Lyapunov function derivative (12) ensures asymptotic convergence of the vector of switching functions (2) which, as it has already been mentioned, is the purpose of the considered control system.

3. DIRECT AND CASCADE CONTROL STRUCTURE OF THE INDUCTION MOTOR SPEED

3.1. MATHEMATICAL MODE OF INDUCTION MOTOR

The mathematical model of the IM can be derived using commonly known simplifications, in stationary α - β frame and per unit system (see the Appendix):

- voltage equations

$$\mathbf{u}_s = r_s \mathbf{i}_s + T_N \frac{d}{dt} \boldsymbol{\Psi}_s \quad (13)$$

$$\mathbf{0} = r_r \mathbf{i}_r + T_N \frac{d}{dt} \boldsymbol{\Psi}_r - j \omega_m \boldsymbol{\Psi}_r \quad (14)$$

- current-flux equations

$$\boldsymbol{\Psi}_s = l_s \mathbf{i}_s + l_m \mathbf{i}_r \quad (15)$$

$$\boldsymbol{\Psi}_r = l_r \mathbf{i}_r + l_m \mathbf{i}_s \quad (16)$$

- equation of motion and electromagnetic torque:

$$\frac{d\omega_m}{dt} = \frac{1}{T_M} (m_e - m_o) \quad (17)$$

$$m_e = \text{Im}(\boldsymbol{\Psi}_s^* \mathbf{i}_s) = \Psi_{s\alpha} i_{s\beta} - \Psi_{s\beta} i_{s\alpha} \quad (18)$$

where: $\mathbf{u}_s = u_{s\alpha} + j u_{s\beta}$ – stator voltage, $\mathbf{i}_s = i_{s\alpha} + j i_{s\beta}$ – stator current, $\mathbf{i}_r = i_{r\alpha} + j i_{r\beta}$ – rotor current, $\boldsymbol{\Psi}_s = \psi_{s\alpha} + j \psi_{s\beta}$ – stator flux, $\boldsymbol{\Psi}_r = \psi_{r\alpha} + j \psi_{r\beta}$ – rotor flux, r_s – stator resistance, $l_s = l_m + l_{s\sigma}$ – stator inductance, r_r – rotor resistance, $l_r = l_m + l_{r\sigma}$ – rotor inductance, $l_{s\sigma}$ – stator leakage inductance, $l_{r\sigma}$ – rotor leakage inductance, l_m – magnetizing inductance, T_M – mechanical time constant of the drive, ω_m – motor speed, m_e – electromagnetic torque, m_o – load torque, $T_N = 1/(2\pi f_{sN})$ – nominal time constant (which results from the per unit system usage), f_{sN} – nominal frequency, $\sigma = 1 - l_m^2/(l_s l_r)$ – motor leakage factor.

3.2. SPEED AND TORQUE CONTROL STRUCTURES

Generally, the IM speed can be controlled directly or in a cascade structure (Fig. 2). In the case of the former solution, the control algorithm defines directly, on the basis of speed control error, the control signals (on/off) of the transistors of a voltage inverter supplying IM. This type of control can also be designed using sliding modes [6].

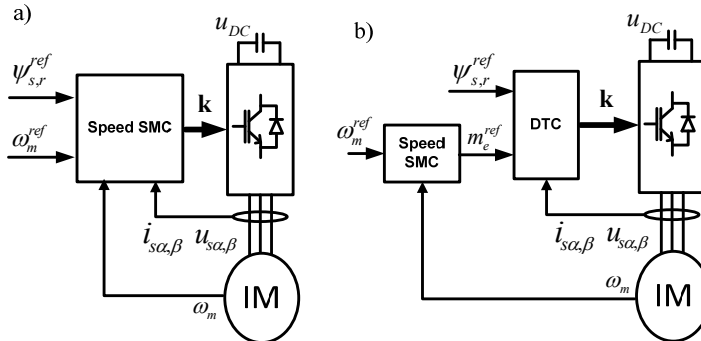


Fig. 2. Sliding mode concepts of the IM speed regulation: a) direct method, b) cascade control

In the latter approach, with a cascade (serial) connection of regulators, the superior speed regulator defines a reference value of the electromagnetic torque which in turn is an input signal of the internal torque regulator. Its task is ensuring an appropriate value of the electromagnetic torque of a motor.

The main difference between direct sliding mode control and cascade structure control is lack of motor torque supervision in the first structure [6]. Additionally, the direct control system leads to significant steady state errors in speed regulation [20]. For this reason, in further research, the system with a serial connection between speed and torque regulators is used. It is assumed that the torque regulator ensures that a torque control loop dynamics can be analyzed as a first order inertia unit, with the following transfer function:

$$\frac{m_e(p)}{m_e^{\text{ref}}(p)} = \frac{1}{T_{me}p + 1} \quad (19)$$

where p – Laplace transfer function variable, and T_{me} – replacement time constant of the torque regulation. Superscript ref indicates a reference value.

The above task can be done, among others, using a sliding mode regulator of the motor torque [21] or one of the regulators with a voltage modulator [22]. The former solution is used in simulation and experimental research presented in this article.

4. SLIDING MODE APPLICATION IN INDUCTION MOTOR DRIVE. EQUIVALENT CONTROL IN CASCADE STRUCTURE

To design the sliding mode regulator of the IM speed, it is necessary to go through the stages specified in the preceding part. The mathematical model of the motor is presented in the third section of this work, and parameters of the motor tested in simulation and experiments can be found in the Appendix. In the cascade control structure, with the superior speed controller (sliding mode type), its output signal vector is reduced to a scalar form, namely to the reference value of the internal motor electromagnetic torque control loop

$$\mathbf{k} = \left[m_e^{\text{ref}} \right] \quad (20)$$

In the described case, the switching function vector becomes also a scalar and it can be proposed in the following way

$$\mathbf{s} = \left[s_\omega \right] = \omega_m^{\text{ref}} - \omega_m - T_c \dot{\omega}_m \quad (21)$$

where T_c is a time constant which can be used to define required speed dynamics.

When the above switching function (21) reaches the zero value, the system behaves analogically to an ideal first order inertia unit with a time constant equal T_c . Two parts of the control signal, the continuous and the discontinuous ones, can be determined according to the method given in the previous part, as:

Block diagram of the control system is presented in Fig. 3. The cascade structure ensures constrain of the reference torque at the value m_e^{\max} . It is assumed that the electromagnetic torque of the motor m_e (Eq. (22)) is determined by an appropriate estimator. This estimator provides also necessary signals for the internal torque control loop – sliding mode direct torque control (SM-DTC) structure [21].

5. TIME-VARYING SWITCHING LINE APPLIED TO THE INDUCTION MOTOR DRIVE SPEED CONTROL

5.1. GENERAL REMARKS

Usually, the switching function in the sliding mode control is constant, independent of time. Its position changes in a step mode, according to the step changes of the reference value of a controlled variable (e.g., speed reference value for a drive system). When the reference dynamics of a controlled variable is changing, the slope of the switching line changes as well. The idea of the step change of the switching line position is demonstrated in Fig. 4: a parallel step change in Fig. 4a, and a slope step change in Fig. 4b, respectively. Only two changes of the switching line have been illustrated in this figure – in times t_1 and t_2 . Certainly, in a real application there are much more of such steps, and the time period between these changes $\Delta t = t_{i+1} - t_i$ should be as small as possible.

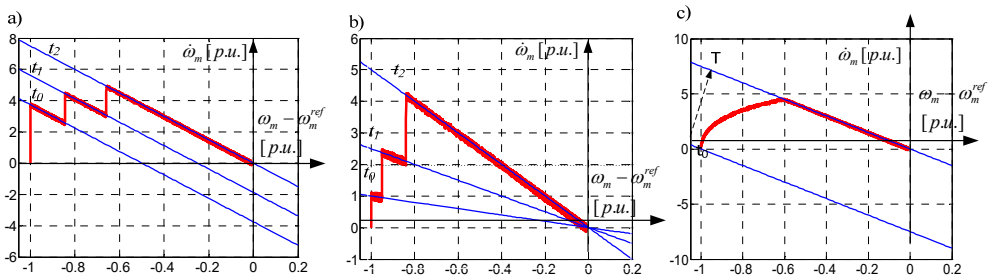


Fig. 4. Phase trajectory of the equivalent sliding mode control of IM speed in the case of step change of a switching line: a) parallel change of the switching line position, b) slope change of the switching line ($t_0 < t_1 < t_2$), c) continuous change of the switching line

However, the sliding control system becomes robust to external and parametric disturbances, when its phase trajectory reaches the switching line. Thus, the system is not resistant to disturbances during the reaching phase of the mentioned line, which takes place during each change of the reference value.

To minimize the disturbance influence during the reaching phase, the continuous change of the switching line has been proposed, in exponential way [23] or in a step

way [7]. Anyway, due to a step type operation after the reaching phase, the system is not fully robust to different disturbances; it is still sensitive between consecutive changes of the switching line.

The generalization of the method enabling the robustness of the sliding mode control in the whole movement range has been the application of the continuous time-varying switching line [8]. The idea of such control is presented in Fig. 4c. The switching line movement begins at t_0 , in the position containing the initial point of the system phase trajectory. Next, the switching line goes in a continuous way to its final position, which is reached after T period. The final position of the switching line corresponds to the given reference value of the controlled state variable (speed in the considered case). The switching line movement can be realized in various ways: with a constant velocity, uniformly accelerated or changing exponentially. In this article, the first approach is taken into account.

5.2. APPLICATION OF THE TIME-VARYING SWITCHING LINE TO SPEED CONTROL OF THE IM DRIVE

As it was stated above, the sliding control drive system becomes robust to external and parametric disturbances, when the describing point on the phase plane lies on the switching line. This is equivalent to reaching the zero value by the switching function (21). Thus, the system is not resistant to disturbances during the reaching phase of the mentioned line, which takes place during each change of a reference value. To make the stage of reaching the switching line independent on disturbances, one may propose its change in time. A parallel change (in relation to the final position) of this line with constant speed requires a modification of the switching function (21):

$$s_\omega = \omega_m^{\text{ref}} - \omega_m - T_c \dot{\omega}_m + \begin{cases} At + B & \text{when } t \leq T \\ 0 & \text{when } t > T \end{cases} \quad (27)$$

where: T – switching line movement time, A , B – parameters, that need to be selected with regard to the continuation of motion:

$$A = -\frac{B}{T}, \quad B = -\omega_m^{\text{ref}}(0) + \omega_m(0) + T_c \dot{\omega}_m(0) \quad (28)$$

The discontinuous part of the equivalent control remains unchanged (23), while, in accordance with the method given in the previous section, the continuous part is modified:

$$m_e^{\text{ref}, eq} = \frac{T_M T_{me}}{T_c} \left(\dot{\omega}_m^{\text{ref}} + \frac{T_c - T_{me}}{T_M T_{me}} m_e + \begin{cases} A & \text{when } t \leq T \\ 0 & \text{when } t > T \end{cases} \right) \quad (29)$$

6. SIMULATION AND EXPERIMENTAL RESULTS

Extensive research was conducted to examine the robustness of the tested system to external (variable loading torque) and parametric disturbances (change of mechanical time constant T_M , which is equivalent to change of moment of inertia J).

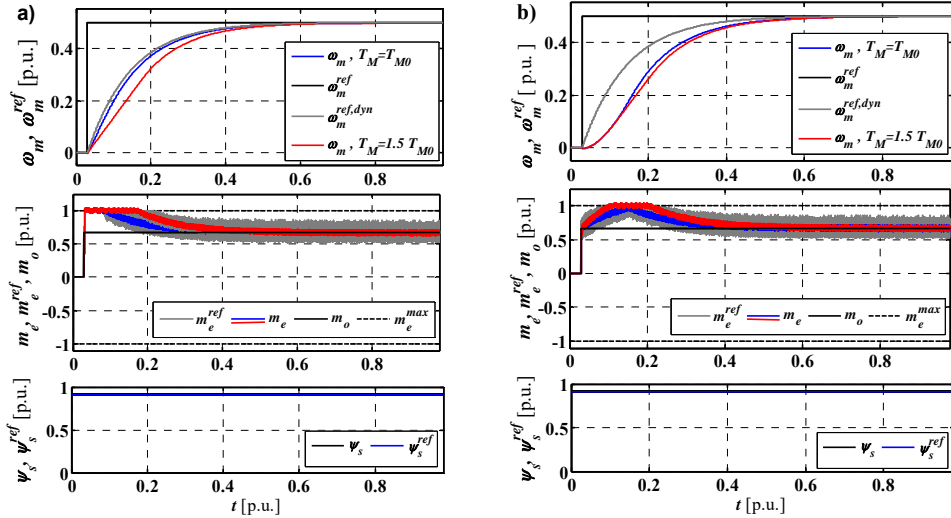


Fig. 5. Equivalent sliding mode control of induction motor angular speed: a) without a time-varying switching line, b) with a time-varying switching line, in the case of operation with rated mechanical time constant (blue) and mechanical constant increased to 150% (red); given and real speed (first row), given and real torque (second row); given and real stator flux amplitude, $m_o = m_N$; simulation research

Using the simulation model developed in the Matlab Simulink environment, tests were conducted (Fig. 5) to show the control process of induction motor angular speed with a variable mechanical time constant of the drive (change up to 150% of the nominal value T_{M0}). In a system with a stationary switching line (Fig. 5a, the first row) speed transients differ from one another and the dynamics of the motor speed for the increased value of a mechanical constant deviates from the reference (grey). The electromagnetic torque in both cases is limited on the maximum value (Fig. 5a, the second row), and the amplitude of rotor flux is maintained at a given, nominal value (Fig. 5a, the third row). By using the time-varying switching line (Fig. 5b, the first row), however, nearly identical speed transients can be achieved, and in this sense, the system can be considered robust. In consequence, the system dynamics significantly deviated from the required one. Nevertheless, one may assume that settling times are approximately equal to the given one.

For the purpose of verification of the presented control algorithms, and the results of simulation tests, experimental tests were conducted. Tests were conducted for 3 kW induc-

tion motor (for its parameters, cf. Appendix), in a laboratory system with digital signal processor dSpace DS1103. Sampling time was 100 μ s (10 kHz). During the tests (Figs. 6, 7) the system robustness to load torque changes was examined.

Figure 6 shows the start-up of the IM without load torque (blue line) and with nominal torque (red line). Similarly to simulation results, system operation was tested with stationary (Fig. 6a) and time-varying switching line (Fig. 6b). The obtained speed transients are almost identical to the ones determined in the simulation study (Fig. 5). Unlike for the stationary line, in the case of the moving switching line, the motor speed dynamics is nearly the same for load and no load operations. It can be seen also on the phase plane (Fig. 6, the second row). The lines are identical only in the case of the moving switching line (Fig. 6b). Robustness of the system moving line is also shown in Fig. 6, the third row – zero value of the switching function indicates that the dynamics of the system is controlled and preserved during the whole start-up process. In both cases (Fig. 6, fourth row), the electromagnetic torque is controlled perfectly.

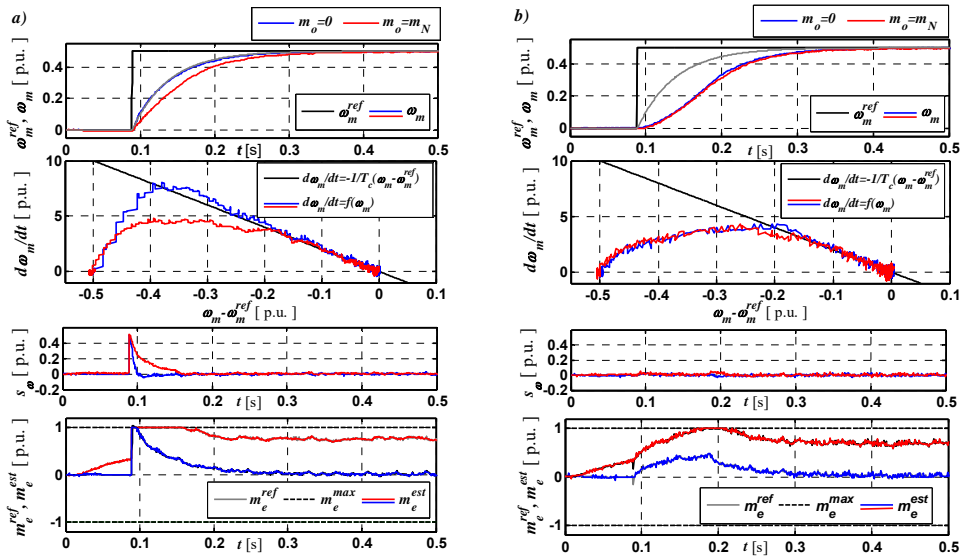


Fig. 6. Equivalent sliding mode control of induction motor angular speed during the start-up: a) with stationary switching line, b) with the time-varying switching line, in the case of operation without (blue) and with (red) rated loading torque; measured and rated speed (first row), speed phase trajectory (second row), switching function (third row), given and real torque (fourth row); experimental research

In Figure 7, speed reversions are applied, and the reference speed is equal to half of the nominal value. First, the drive operates with no load, and then after 5 s the load torque, equal to 0.5, appears. It was supposed that the load torque was passive (its sign was equal to the sign of the speed).

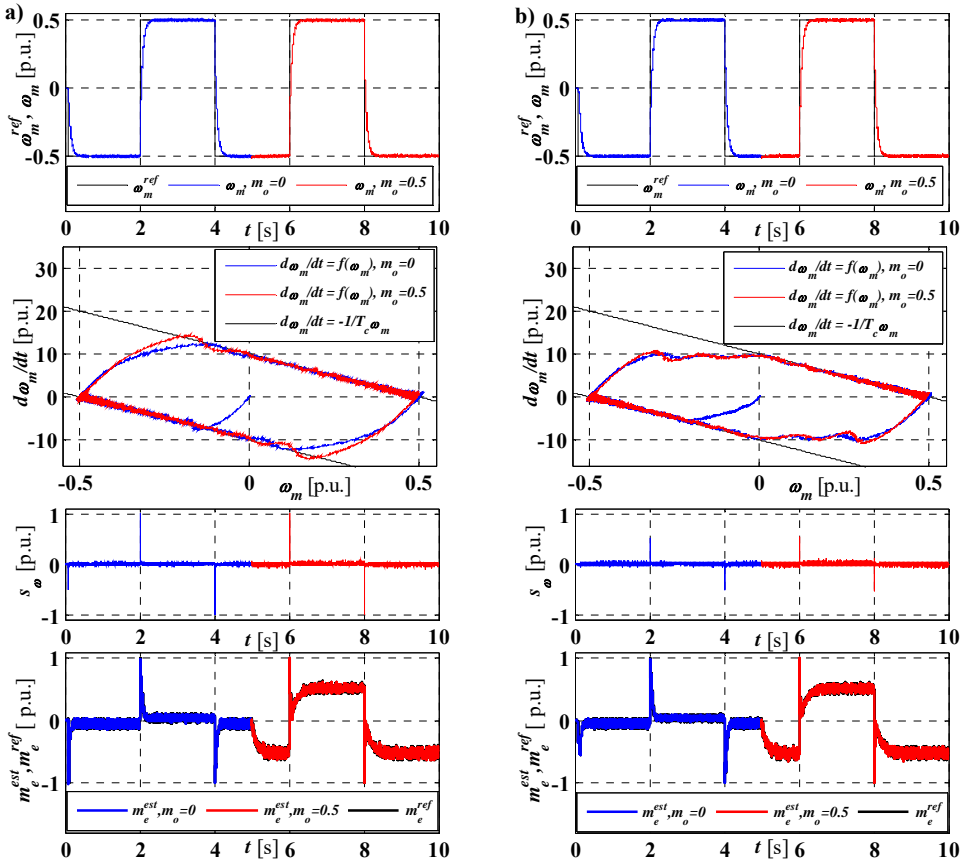


Fig. 7 Equivalent sliding mode control of IM angular speed during speed reversions: a) with stationary switching line, b) with the time-varying switching line, in the case of operation without (blue) and with (red) rated loading torque; measured and rated speed (first row), speed phase trajectory (second row), switching function (third row), given and real torque (fourth row); experimental research

The angular speed of the drive is controlled with excellent dynamics, in both cases: with constant (Fig. 7a) and time-varying switching lines (Fig. 7b), for no load and load operation. However, the difference between the drive behavior can be seen in phase portraits (Fig. 7, the second row). In the case of the constant switching line (Fig. 7a), it can be seen that the no load and load runs give various shapes. It is therefore proved that the dynamics of the drive changes with external disturbance. However, this is not true in the case of the time-varying switching line operation. As can be seen in Fig. 7b (the second row), the lines are almost the same, thus the dynamics is independent on the load torque. Changes of the values of switching functions are shown in the third row of Fig. 7. These functions are zero almost all the time, except of sudden changes of reference speed value – this is the purpose of the SMC structure. However, for constant

switching line operation, the overshoots of the switching function are much bigger than for time-varying switching line case. Reference and estimated torques are shown in the fourth row of this figure. Despite some small oscillations, they are almost the same and are limited at the assumed maximum value.

7. CONCLUSIONS

The paper presents the application of a simple and effective sliding mode control method with stationary and time-varying switching lines for IM speed control. Simulations and experimental tests were conducted to verify the proposed theoretical solutions. The equivalent control method, with a relatively small number of design parameters allows to significantly reduce control signal oscillations, and as a result oscillations in state variables. The method ensures perfect motor speed control; however, a drive system is not robust to external and parametric disturbances during the reaching phase of the stationary switching line. Therefore, the use of a time-varying switching line is proposed. The change took place parallel (in a plane), with unvarying motion speed. This slight modification of a switching line equation and the control law allowed obtaining identical speed runs in transients, both in the case of load torque, and mechanical time constant changes. However, the change results in the fact that system dynamics differs from the given one. Because of this, further research is planned with consideration for change of a switching line motion from the parallel motion to a rotary one (variable inclination angle) and the motion speed – from constant to uniformly accelerated one.

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APPENDIX

The parameters of the tested motor are presented in Table 1, its rated data in Table 2, and base parameters, necessary for the process of transformation to relative units in Table 3.

Table 1. Parameters of the tested motor

Parameter	Value	
	[Ω]	[p.u.]
Stator resistance, R_s	7.073	0.071
Rotor resistance, R_r	7.372	0.074
Main reactance, X_m	187.8	1.88
Stator leakage reactance, $X_{s\sigma}$	9.80	0.098

Table 2. Rated data of tested motor

Parameter	Value	
	in physical units	[p.u.]
Power, P_N	3.0 kW	0.625
Torque, M_N	20.46 Nm	0.67
Speed, N_N	1400 rev/min	0.93
Stator voltage, U_{sN}	400 V	0.707
Stator current, I_{sN}	4.0 A	0.707
Frequency, f_{sN}	50 Hz	1
Stator flux, Ψ_{sN}	1.65 Wb	0.9188
Rotor flux, Ψ_{rN}	1.54 Wb	0.8605
Number of pole pairs, p_b	2	2

Table 3. Base parameters

Parameter	Expression	Value
Power	$S_b = 3/2 U_b I_b$	4.8 kW
Torque	$M_b = p_b S_b / \Omega_b$	30.56 Nm
Rotary speed	$N_b = 60 f_{sN} / p_b$	1500 rpm
Stator voltage	$U_{Sb} = (2)^{1/2} U_{sN}$	565.7 V
Stator current	$I_{Sb} = \sqrt{2} I_{sN}$	5.66 A
Frequency	$f_{Sb} = f_{sN}$	50 Hz
Angular velocity	$\Omega_b = 2\pi f_{sN}$	100 π rad/s
Flux	$\Psi_b = U_b / \Omega_b$	1.80 Wb

The inertia moment of the tested motor was equal to $J = 0.0292 \text{ kg}\cdot\text{m}^2$, which corresponds with the equivalent mechanical time constant $T_M = 0.15 \text{ s}$.