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Development of the IANS Chain Using a Satellite System

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ABSTRACT: This paper presents ideas for creating and developing Interactive Navigational Structures (*IANS*) by using satellite systems. The paper presents the possibility of adapting the contemporary methods of robust estimation that are used in geodesy for the purpose of performing selected marine navigation tasks. *IANS* utilise modern *M*-estimation methods. Interactive structures can assist human beings in carrying out special tasks at sea, for example, in identifying objects at sea and on land without the need to approach them. Moreover, these structures can also be used in the process of navigation carried out by underwater vehicles or unmanned watercraft. This paper presents the mathematical methods that are essential for creating *IANS.* The theoretical assumptions are illustrated with an example of how *IANS* can be used while performing a typical navigation task. The paper closes with information about research studies which deal with the practical aspects of using this set of mathematical methods.

1 INTRODUCTION

Satellite positioning systems are indispensable in carrying out basic marine navigation tasks. Navigation is understood as a process of conducting a vehicle from one point to another in a safe and effective way in the appropriate physical and geographical environment (Kopacz, Urbański, 1998; Czaplewski, Morgaś, Kopacz, 2010; Czaplewski, 2014). However, these systems are not enough to solve the problem of accessibility to reliable and highly accurate information about an object's position, in particular in relation to navigation marks that are located on land and at sea. This is particularly important when performing many navigation tasks as part of special works as well as submarine navigation tasks. Furthermore, it is not always possible to precisely determine the position of objects other than vessels when carrying out hydrographic works by using satellite technology. For example, it is very difficult to precisely position navigation marks at sea. It is also not easy to locate the position of navigational hazards, such as wrecks, underwater and above-water rocks as well as other natural and artificial obstructions to navigation. This is because it is impossible for measurement teams or hydrographic survey vessels to directly approach the objects whose positions are being determined. Additionally, it is incredibly important for the proper performance of tasks at sea that the relative positions of vessels be precisely maintained. This is particularly important for:

- hydrographic survey vessels during bathymetric measurements and hydrographic sweep surveys;
- research vessels, for example, during seabed exploration;
- warships during the search for submarines, minesweeping, or the performance of common tasks involving the use of artillery and rockets.

The above-mentioned issues and other similar problems are of vital significance for maritime economy and national defence. In order to solve these problems one must not only use satellite navigation systems but also terrestrial navigation (geo-navigation).

If geo-navigation methods are to be used today, they must be developed in terms of techniques and technology as well as the methods of analysing measurement results. Currently, classical geo-navigation uses procedures that do not meet accuracy standards. Geo-navigation should refer to the well-established methods of modern geodesy that are still being developed in order to be able to meet these standards. Additionally, it is necessary to automate calculations because this will allow one to implement them in integrated navigation systems. Such implementation should make it possible to simultaneously use different navigation systems (so-called combined systems) while employing new positioning methods. As for data analysis, one should use robust *M*-estimation methods which are new but well-established theoretically and which are currently being developed. This approach to the issue of determining the positions of a moving object at sea and a stationary object that is being observed makes it possible to create dynamic, interactive navigational structures, which are dealt with in this paper. The term 'Interactive Navigational Structures' (*IANS*) (Czaplewski, 2004) should be understood as the existing systems of navigation marks, the other stationary objects that are observed and craft carrying out navigation tasks (also submarine navigation), which allow one to establish the coordinates of the remaining elements of the structure due to the (geometrical and physical) interdependencies between them. The fact that the structure is interactive means that it can be constantly altered by the navigator (observer).

2 BASIC ASSUMPTIONS

From a geometric perspective, the Interactive Navigational Structure is made up of a set of points $\mathfrak{Z} = \left\{ Z_j : j = 1,\ldots,n_z \right\}$ with known coordinates (e.g. (X,Y)) in a known coordinate system as well as subsets of points that are being determined, i.e. $\mathfrak{P} = \left\{ P_i : i = 1,\ldots,n_p \right\}$ and $\mathfrak{R} = \left\{ R_l : l = 1,\ldots,n_R \right\}$.

Sets $\mathfrak{Z}$ can be made up of visual navigation marks, elements of radar navigation systems, radio-navigation stations and DGPS (Differential Global Positioning System) reference stations. Set $\mathfrak{P}$ consists of a vessel's positions P_i at sea that are being determined or the positions of groups of vessels that carry out a particular task together (e.g. a hydrographic sweep survey). Subset $\mathfrak{R}$ is made up of points whose coordinates are determined based on the positions that belong to set $\mathfrak{P}$, which may complement set $\mathfrak{Z}$ under certain conditions.

Let us assume that, at stage (k), there is a given set $\mathfrak{Z}^{(k)}$ which is insufficient to determine a vessel's position at stage $(k+1)$. Based on a set of points $\mathfrak{Z}^{(k)}$, a vessel's positions $\mathfrak{P}^{(k)}$ are determined, and the coordinates of points $\mathfrak{R}^{(k)}$ are determined based on these positions. Then, a set of points $\mathfrak{Z}^{(k+1)} = \left\{ \mathfrak{Z}^{(k)}, \mathfrak{R}^{(k)} \right\}$ is available at stage $(k+1)$; this set can be supplemented with other elements of navigation systems. As for the adopted accuracy criteria, the *IANS* chains can take different forms. General relations within *IANS* are shown in Figure 1.

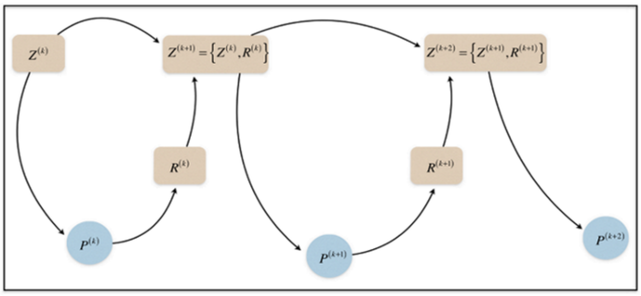

Figure 1. Concept of the Interactive Navigational Structure's chain (own work).

The sets presented above have been connected to form one observational arrangement, within which geometric quantities are measured.

Let us assume that a vessel's position P_i was determined by using the bearings and distances from point P_i to the elements of set $\mathfrak{Z}$; at the same time its position was established by using the DGPS. Moreover, the bearing and distance between a vessel's position and point *R* belonging to set $\mathfrak{R}$ were measured. For such a measurement arrangement, we can create *IANS*'s basic element (Fig. 2).

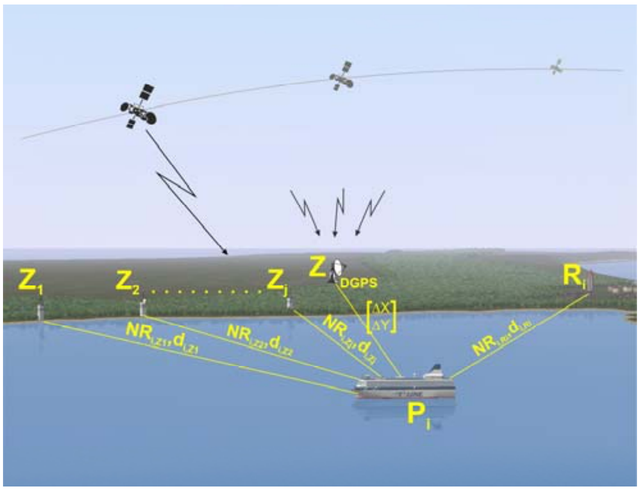

Figure 2. *IANS*'s basic element (own work).

It is assumed that the interactive character of navigational structures makes it possible to transfer the elements of set $\mathfrak{R}$ to set $\mathfrak{Z}$, which requires creating a spatial arrangement that would allow one to control the determinations. The element of *IANS* that is presented in Figure 2 does not meet these requirements. The basic element of the Interactive Navigational Structure does not ensure reliable assessment of the accuracy of the position of points whose coordinates are being determined, and such a structure is not very robust to gross errors. In order to solve these problems, the elements of *IANS* can be connected to form *IANS* chains. Figure 3 presents a fragment of such a chain.

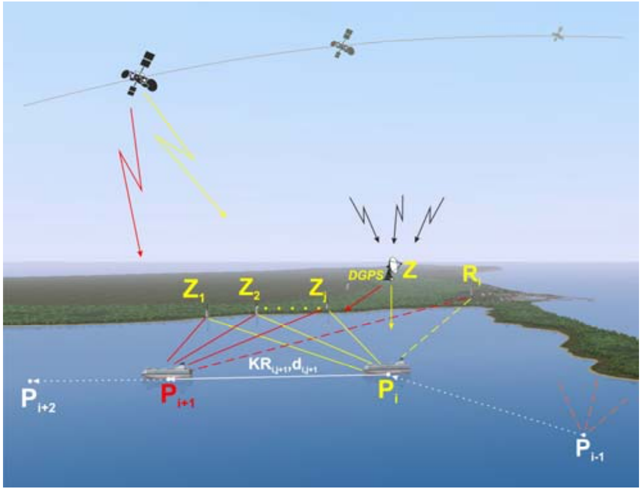

Figure 3. Fragment of an *IANS* chain (own work).

The navigational structure that is presented here is not only interactive, but also integrative in character. This is because it combines different kinds of observations from various navigation systems. It should be assumed that any kinds of connections between the elements of sets $\mathfrak{Z}, \mathfrak{P}, \mathfrak{R}$ will be used when putting *IANS* into practice. It is also possible that observations will be fraught with such levels of error that they will have to be rejected or they will influence the final determinations to a lesser extent. The methodology for establishing an adjustment problem for each of the stages of developing *IANS* as well as for using attenuation to reduce the influence of gross errors on the final determination is presented later in the paper.

3 ADJUSTMENT PROBLEM FOR THE BASIC ELEMENT OF IANS AND THE SOLUTION

By using functional models which are typical of geodesy (e.g. Wiśniewski, 2009; Baran, 1999) and which have been adopted for the purpose of creating Interactive Navigational Structures, one can present a robust and decision adjustment problem (1) for the basic element of *IANS*:

$$\left.\begin{array}{l}
\left.\begin{array}{l}
\mathbf{V}_{\boldsymbol{x}_i} = \mathbf{A}_{P_i}\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}} + \mathbf{A}^{(i)}_{R}\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}} + \mathbf{L}_{\boldsymbol{x}_i} \\
\mathbf{V}_{\mathbf{X}_i^{DGPS}} = \hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}} + \mathbf{X}^0_{P_i} - \mathbf{X}_{P_i}^{DGPS}
\end{array}\right\}\text{functional models} \\
\left.\left.\begin{array}{l}
\mathbf{C}_{\boldsymbol{x}_i} = \sigma_0^2\mathbf{Q}_{\boldsymbol{x}_i} = \sigma_0^2\mathbf{P}^{-1}_{\boldsymbol{x}_i} \\
\mathbf{C}_{\mathbf{X}_{P_i}^{DGPS}} = \sigma_0^2\mathbf{Q}_{\mathbf{X}_{P_i}^{DGPS}} = \sigma_0^2\mathbf{P}^{-1}_{\mathbf{X}_{P_i}^{DGPS}}
\end{array}\right\} \rightarrow
\begin{array}{l}
\tilde{\tilde{\mathbf{C}}}_{\boldsymbol{x}_i} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}^{-1}_{\boldsymbol{x}_i} \\
\tilde{\tilde{\mathbf{C}}}_{\mathbf{X}_{P_i}^{DGPS}} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}^{-1}_{\mathbf{X}_{P_i}^{DGPS}}
\end{array}\right\}\text{statistic models} \\
\min_{\Omega}\Phi^{D-R}\left(\mathbf{d}^{(i)}_{\mathbf{X}_{P_i}}, \mathbf{d}^{(i)}_{\mathbf{X}_{R_i}}\right) = \Phi^{D-R}\left(\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}}, \hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}}\right) = \mathbf{V}^T_{\boldsymbol{x}_i}\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_i}\mathbf{V}_{\boldsymbol{x}_i} + \mathbf{V}^T_{\mathbf{X}_{P_i}^{DGPS}}\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_{P_i}^{DGPS}}\mathbf{V}_{\mathbf{X}_{P_i}^{DGPS}} \\
\Omega = \left(\mathbf{d}^{(i)}_{\mathbf{X}_{P_i}}, \mathbf{d}^{(i)}_{\mathbf{X}_{R_i}}\right)
\end{array}\right\} \quad (1)$$

As part of the above adjustment problem, these equivalent covariance matrices were presented in the statistical model: $\tilde{\tilde{\mathbf{C}}}_{\mathbf{x}_i} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}^{-1}_{\mathbf{x}_i}$ and $\tilde{\tilde{\mathbf{C}}}_{\mathbf{X}_{P_i}^{DGPS}} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}^{-1}_{\mathbf{X}_{P_i}^{DGPS}}$, where:

$$\begin{array}{l}
\tilde{\tilde{\mathbf{P}}}_{\mathbf{x}_i} = \tilde{\mathbf{T}}(\mathbf{x}_i, \mathbf{V}_{\mathbf{x}_i})\mathbf{P}_{\mathbf{x}_i} \\
\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}} = \tilde{\mathbf{T}}(\mathbf{X}_{P_i}^{DGPS}, \mathbf{V}_{\mathbf{X}_i^{DGPS}})\mathbf{P}_{\mathbf{X}_i^{DGPS}}
\end{array} \quad (2)$$

are equivalent weight matrices that have been determined based on decision and attenuation matrices:

$$\begin{array}{l}
\tilde{\mathbf{T}}(\mathbf{x}_i, \mathbf{V}_{\mathbf{x}_i}) = \mathfrak{T}(\mathbf{x}_i)\mathbf{T}(\mathbf{V}_{\mathbf{x}_i}) \\
\tilde{\mathbf{T}}(\mathbf{X}_{P_i}^{DGPS}, \mathbf{V}_{\mathbf{X}_i^{DGPS}}) = \mathfrak{T}(\mathbf{X}_{P_i}^{DGPS})\mathbf{T}(\mathbf{V}_{\mathbf{X}_i^{DGPS}})
\end{array} \quad (3)$$

where:

$\mathbf{T}(\mathbf{V}_{\mathbf{x}_i}) = Diag\{t(\upsilon_1), t(\upsilon_2), \ldots, t(\upsilon_n)\}$ – attenuation function

$\mathfrak{T}(\mathbf{x}_i) = Diag\left(\mathfrak{T}(\mathbf{x}_{Z_i}), \mathfrak{T}(\mathbf{x}^{(i)}_{R_i})\right)$ – decision function

and

$$\mathfrak{T}(\mathbf{x}_{Z_i}) = Diag\left(\mathfrak{t}(N_{i,1}), \ldots, \mathfrak{t}(N_{i,n_{Z_i}}), \mathfrak{t}(d_{i,1}), \ldots, \mathfrak{t}(d_{i,n_{Z_i}})\right)$$

$$\mathfrak{T}(\mathbf{x}^{(i)}_{R_i}) = Diag\left(\mathfrak{t}(NR^{(i)}_{i,1}), \ldots, \mathfrak{t}(NR^{(i)}_{i,n_R}), \mathfrak{t}(dR^{(i)}_{i,1}), \ldots, \mathfrak{t}(dR^{(i)}_{i,n_R})\right)$$

Detailed rules for creating decision functions and attenuation functions are described, for example, in the papers written by Yang (1994), Yang, Song and Xu (2002), and Wiśniewski (2002), whereas the methodology for creating a decision and attenuation function (3) is described in detail in Czaplewski's paper (2004). In order to simplify expression (1), the following designations can be introduced:

$$\mathbf{V}_i = \begin{bmatrix} \mathbf{V}_{\mathbf{x}_i} \\ \mathbf{V}_{\mathbf{X}_i^{DGPS}} \end{bmatrix}, \quad \mathbf{A}^{(i)}_i = \begin{bmatrix} \mathbf{A}_{P_i} & \mathbf{A}^{(i)}_R \\ \mathbf{I}_{(2)} & \mathbf{0} \end{bmatrix},$$

$$\mathbf{L}_i = \begin{bmatrix} \mathbf{L}_{\mathbf{x}_i} \\ \mathbf{x}^0_{P_i} - \mathbf{X}_{P_i}^{DGPS} \end{bmatrix}, \quad \hat{\mathbf{d}}_{\mathbf{X}_i} = \begin{bmatrix} \hat{\mathbf{d}}_{\mathbf{X}_{P_i}} \\ \hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}} \end{bmatrix},$$

$$\tilde{\tilde{\mathbf{C}}}_i = Diag\left(\tilde{\tilde{\mathbf{C}}}_{\mathbf{x}_i}, \tilde{\tilde{\mathbf{C}}}_{\mathbf{x}_{P_i}^{DGPS}}\right), \quad \tilde{\tilde{\mathbf{P}}}_i = Diag\left(\tilde{\tilde{\mathbf{P}}}_{\mathbf{x}_i}, \tilde{\tilde{\mathbf{P}}}_{\mathbf{x}_{P_i}^{DGPS}}\right)$$

Thus, the final adjustment problem is expressed in the following, classical form:

$$\left.\begin{aligned} &\mathbf{V}_i = \mathbf{A}_i \hat{\mathbf{d}}_{\mathbf{X}_i} + \mathbf{L}_i \\ &\tilde{\tilde{\mathbf{C}}}_i = \sigma_o^2 \tilde{\tilde{\mathbf{P}}}_i^{-1} \\ &\min_{\mathbf{d}_{\mathbf{X}_i}} \Phi^{D-R}\left(\mathbf{d}_{\mathbf{X}_i}\right) = \min_{\mathbf{d}_{\mathbf{X}_i}} \Phi^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}_i}\right) = \mathbf{V}_i^T \tilde{\tilde{\mathbf{P}}}_i \mathbf{V}_i \end{aligned}\right\} \tag{4}$$

and the solution is as follows (Baran, 1999; Wiśniewski, 2009):

$$\hat{\mathbf{d}}_{\mathbf{X}_i} = -\left(\mathbf{A}_i^T \tilde{\tilde{\mathbf{P}}}_i \mathbf{A}_i\right)^{-1} \mathbf{A}_i^T \tilde{\tilde{\mathbf{P}}}_i \mathbf{L}_i \tag{5}$$

Therefore, the matrix of adjusted coordinates takes the following form:

$$\hat{\mathbf{X}}_i = \mathbf{X}_i^o + \hat{\mathbf{d}}_{\mathbf{X}_i} = \begin{bmatrix} \mathbf{X}_{P_i}^o \\ \mathbf{X}_{R_i}^o \end{bmatrix} + \begin{bmatrix} \hat{\mathbf{d}}_{\mathbf{X}_{P_i}} \\ \hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i)} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{X}}_{P_i} \\ \hat{\mathbf{X}}_{R_i}^{(i)} \end{bmatrix} \tag{6}$$

with an estimator of the covariance matrix (Yang, 1997):

$$\hat{\mathbf{C}}_{\hat{\mathbf{X}}_i} = \sigma_o^2 \left(\mathbf{A}_i^T \tilde{\tilde{\mathbf{P}}}_i \mathbf{A}_i\right)^{-1} = \sigma_o^2 \mathbf{Q}_{\hat{\mathbf{X}}_i} = \sigma_o^2 \mathbf{P}_{\hat{\mathbf{X}}_i}^{-1} \tag{7}$$

The process of solving the problem which is expressed as formula (4) has an iterative character. The first (starting) step should be implemented in a classical way, i.e. by using the method of least squares.

4 DEVELOPING THE IANS CHAIN

A single element of *IANS* only allows one to adjust a given vessel's position P_i. Points belonging to $\Re_i$ can only be determined without supernumerary observations at this stage. This is why the *IANS* chain should be developed. For the purpose of presenting the development of the *Interactive Navigational Structure*'s chain, let us assume a navigational situation in which a vessel has moved to position P_{i+1}. The vessel's new position is measured relative to navigation marks that belong to set $\mathcal{Z}_{i+1} \subset \mathcal{Z}$, measurements using the DGPS to set $\mathcal{Z}_{i+1}^{DGPS} \subset \mathcal{Z}$ by using the DGPS, relative to the previously determined points $\Re_i \subset \Re$ and relative to new (for point P_{i+1}) points $\Re_{i+1} \subset \Re$. Let us also assume that the elements of the vessel's path vector (i.e. the course and the distance travelled) are known. The navigation arrangement which is analysed in this paper is presented in the figure below.

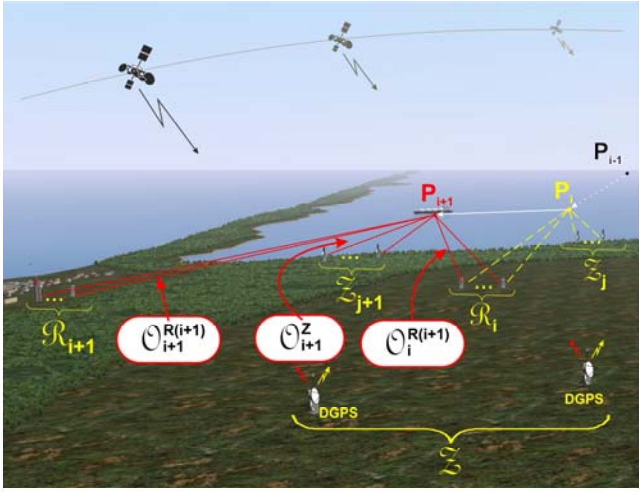

Figure 4. The stage of developing *IANS* at moment $i+1$

Since we assume that also DGPS measurements are made at position P_{i+1} (with the aim of obtaining coordinates $\mathbf{X}_{P_{i+1}}^{DGPS}$ with covariance matrix $\mathbf{C}_{\mathbf{X}_{P_{i+1}}^{DGPS}} = \sigma_o^2 \mathbf{Q}_{\mathbf{X}_{P_{i+1}}^{DGPS}}$), we will write the following:

$$\left.\begin{aligned} &\mathbf{V}_{\mathbf{X}_{i+1}^{DGPS}} = \hat{\mathbf{X}}_{P_{i+1}}^{(i+1)} - \mathbf{X}_{P_{i+1}}^{DGPS} \\ &\hat{\mathbf{X}}_{P_{i+1}}^{(i+1)} = \mathbf{X}_{P_{i+1}}^o + \hat{\mathbf{d}}_{\mathbf{X}_{P_{i+1}}}^{(i+1)} \end{aligned}\right\} \Rightarrow \mathbf{V}_{\mathbf{X}_{i+1}^{DGPS}} = \hat{\mathbf{d}}_{\mathbf{X}_{P_{i+1}}}^{(i+1)} + \mathbf{X}_{P_{i+1}}^o - \mathbf{X}_{P_{i+1}}^{DGPS} \tag{8}$$

The coordinates $\mathbf{X}_{P_i}$ of position P_i have already been adjusted and they are represented by estimator $\hat{\mathbf{X}}_{P_i}^{(i)}$ with covariance matrix $\mathbf{C}_{\hat{\mathbf{X}}_{P_i}^{(i)}} = \sigma_o^2 \mathbf{Q}_{\hat{\mathbf{X}}_{P_i}^{(i)}} = \sigma_o^2 \mathbf{P}_{\hat{\mathbf{X}}_{P_i}^{(i)}}^{-1}$ in the present *IANS* chain. However, positions P_i and P_{i+1} are linked to each other through the elements of a vessel's path vector as well as indirectly, i.e. through points belonging to $\Re_i$ which are common to both elements of *IANS*. Given the above, estimator $\hat{\mathbf{X}}_{P_i}^{(i)}$ is not the final vector of the coordinates of the previous position P_i. Let us assume that vector $\hat{\mathbf{X}}_{P_i}^{(i+1)}$ is the final one. Then, if we treat the previous vector as a pseudo-observation, according to the general principles of sequential adjustment, we can write the following:

$$\left.\begin{aligned} &\hat{\mathbf{X}}_{P_i}^{(i+1)} = \hat{\mathbf{X}}_{P_i}^{(i)} + \mathbf{V}_{\hat{\mathbf{X}}_{P_i}} \\ &\mathbf{X}_{P_i}^o + \hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)} = \mathbf{X}_{P_i}^o + \hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i+1)} + \mathbf{V}_{\hat{\mathbf{X}}_{P_i}} \end{aligned}\right\} \Rightarrow \mathbf{V}_{\hat{\mathbf{X}}_{P_i}} = \hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i+1)} - \hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)} \tag{9}$$

When navigating a single object, coordinates $\hat{\mathbf{X}}_{P_i}^{(i+1)}$ do not have much practical importance for the current position P_{i+1} (although based on the values of the vector of adjustments $\mathbf{V}_{\hat{\mathbf{X}}_{P_i}}$ one can draw additional conclusions about the accuracy of navigation). The adjusted coordinates of the previous position are the result of a joint analysis of all observations that were available for the current situation P_{i+1}. Observations must be analysed jointly so as to obtain the best possible determinations of current coordinates that are represented by $\hat{\mathbf{X}}_{P_i}^{(i+1)}$.

Vector $\hat{\mathbf{X}}_{R_i}^{(i+1)}$ of the coordinates of points $\Re_i$ which was obtained at position P_i will also be treated as a pseudo-observation. If we assume that vector $\hat{\mathbf{X}}_{R_i}^{(i+1)}$ is a joint representation of the coordinates of position P_{i+1}, we can write the following:

$$\left.\begin{aligned}\hat{\mathbf{X}}_{\mathrm{R_i}}^{(\mathrm{i+1})} &= \hat{\mathbf{X}}_{\mathrm{R_i}}^{(\mathrm{i})} + \mathbf{V}_{\hat{\mathbf{X}}_{\mathrm{R_i}}} \\ \mathbf{X}_{\mathrm{R_i}}^{\mathrm{o}} + \hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{R_i}}}^{(\mathrm{i+1})} &= \mathbf{X}_{\mathrm{R_i}}^{\mathrm{o}} + \hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{R_i}}}^{(\mathrm{i})} + \mathbf{V}_{\hat{\mathbf{X}}_{\mathrm{R_i}}}\end{aligned}\right\} \Rightarrow \mathbf{V}_{\hat{\mathbf{X}}_{\mathrm{R_i}}} = \hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{R_i}}}^{(\mathrm{i+1})} - \hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{P_i}}}^{(\mathrm{i})} \qquad (10)$$

By using the determinations related to the goal function, which were presented in Czaplewski's paper (2004), as well as the functional and statistical models that were formulated earlier, one can propose the following robust and decision adjustment problem:

$$\left.\begin{aligned}
&\mathbf{V}_{\boldsymbol{x}_{\mathrm{i+1}}} = \mathbf{A}_{\mathrm{P_{i+1}}}\hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{P_{i+1}}}}^{(\mathrm{i+1})} + \mathbf{A}_{\mathrm{R_{i+1}}}\hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{R_{i+1}}}}^{(\mathrm{i+1})} + \mathbf{A}_{\mathrm{R_i}}^{(\mathrm{i+1})}\hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{R_i}}}^{(\mathrm{i+1})} + \mathbf{A}_{\mathrm{P_i}}^{(\mathrm{i+1})}\hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{P_i}}}^{(\mathrm{i+1})} + \mathbf{L}_{\boldsymbol{x}_{\mathrm{i+1}}} \\
&\mathbf{V}_{\mathbf{X}_{\mathrm{i+1}}^{\mathrm{DGPS}}} = \hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{P_{i+1}}}}^{(\mathrm{i+1})} + \mathbf{X}_{\mathrm{P_{i+1}}}^{0} - \mathbf{X}_{\mathrm{P_{i+1}}}^{\mathrm{DGPS}} \\
&\mathbf{V}_{\hat{\mathbf{X}}_{\mathrm{P_i}}} = \hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{P_i}}}^{(\mathrm{i+1})} - \hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{P_i}}}^{(\mathrm{i})} \\
&\mathbf{V}_{\hat{\mathbf{X}}_{\mathrm{R_i}}} = \hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{R_i}}}^{(\mathrm{i+1})} - \hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{R_i}}}^{(\mathrm{i})} \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&\mathbf{C}_{\boldsymbol{x}_{\mathrm{i+1}}} = \sigma_0^2\mathbf{P}_{\boldsymbol{x}_{\mathrm{i+1}}}^{-1} \rightarrow \tilde{\tilde{\mathbf{C}}}_{\boldsymbol{x}_{\mathrm{i+1}}} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{\mathrm{i+1}}}^{-1} \\
&\mathbf{C}_{\mathbf{X}_{\mathrm{i+1}}^{\mathrm{DGPS}}} = \sigma_0^2\mathbf{P}_{\mathbf{X}_{\mathrm{i+1}}^{\mathrm{DGPS}}}^{-1} \rightarrow \tilde{\tilde{\mathbf{C}}}_{\mathbf{X}_{\mathrm{i+1}}^{\mathrm{DGPS}}} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_{\mathrm{i+1}}^{\mathrm{DGPS}}}^{-1} \\
&\mathbf{C}_{\hat{\mathbf{X}}_{\mathrm{P_i}}^{(\mathrm{i})}} = \sigma_0^2\mathbf{P}_{\hat{\mathbf{X}}_{\mathrm{P_i}}^{(\mathrm{i})}}^{-1} \rightarrow \tilde{\tilde{\mathbf{C}}}_{\hat{\mathbf{X}}_{\mathrm{P_i}}^{(\mathrm{i})}} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}_{\hat{\mathbf{X}}_{\mathrm{P_i}}^{(\mathrm{i})}}^{-1} \\
&\mathbf{C}_{\hat{\mathbf{X}}_{\mathrm{R_i}}^{(\mathrm{i})}} = \sigma_0^2\mathbf{P}_{\hat{\mathbf{X}}_{\mathrm{R_i}}^{(\mathrm{i})}}^{-1} \rightarrow \tilde{\tilde{\mathbf{C}}}_{\hat{\mathbf{X}}_{\mathrm{R_i}}^{(\mathrm{i})}} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}_{\hat{\mathbf{X}}_{\mathrm{R_i}}^{(\mathrm{i})}}^{-1} \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&\min_{\mathbf{d}_{\mathbf{X}_{\mathrm{i+1}}}} \Phi^{\mathrm{D-R}}\left(\mathbf{d}_{\mathbf{X}_{\mathrm{i+1}}}\right) = \Phi^{\mathrm{D-R}}\left(\hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{i+1}}}\right) = \\
&= \Phi_{\boldsymbol{x}}^{\mathrm{D-R}}\left(\hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{i+1}}}\right) + \Phi_{\mathrm{DGPS}}^{\mathrm{D-R}}\left(\hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{i+1}}}\right) + \Phi_{\mathrm{P_i}}^{\mathrm{D-R}}\left(\hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{i+1}}}\right) + \Phi_{\mathrm{R_i}}^{\mathrm{D-R}}\left(\hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{i+1}}}\right)
\end{aligned}\right\} \qquad (11)$$

with equivalent covariance matrices $\tilde{\tilde{\mathbf{C}}} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}^{-1}$ which have replaced the original matrices $\mathbf{C} = \sigma_0^2\mathbf{P}^{-1}$. When we introduce these denotations:

$$\mathbf{V}_{i+1} = \begin{bmatrix} \mathbf{V}_{x_{i+1}} \\ \mathbf{V}_{\mathbf{X}_{i+1}^{DGPS}} \\ \mathbf{V}_{\hat{\mathbf{X}}_{P_i}} \\ \mathbf{V}_{\hat{\mathbf{X}}_{R_i}} \end{bmatrix},$$

$$\mathbf{A}_{i+1} = \begin{bmatrix} \mathbf{A}_{P_{i+1}} & \mathbf{A}_{R_{i+1}} & \mathbf{A}_{R_i}^{(i+1)} & \mathbf{A}_{P_i}^{(i+1)} \\ \mathbf{I}_{(2)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{I}_{(2)} \\ 0 & 0 & \mathbf{I}_{(2n_{R_i})} & 0 \end{bmatrix},$$

$$\mathbf{L}_i = \begin{bmatrix} \mathbf{L}_{x_{i+1}} \\ \mathbf{X}_{P_{i+1}}^0 - \mathbf{X}_{P_{i+1}}^{DGPS} \\ -\hat{\mathbf{d}}_{X_{P_i}}^{(i)} \\ -\hat{\mathbf{d}}_{X_{R_i}}^{(i)} \end{bmatrix}$$

$$\tilde{\tilde{\mathbf{C}}}_{i+1} = Diag\left(\tilde{\tilde{\mathbf{C}}}_{x_{i+1}}, \tilde{\tilde{\mathbf{C}}}_{\mathbf{X}_{i+1}^{DGPS}}, \tilde{\tilde{\mathbf{C}}}_{\hat{\mathbf{X}}_{P_i}^{(i)}}, \tilde{\tilde{\mathbf{C}}}_{\hat{\mathbf{X}}_{R_i}^{(i)}}\right),$$

$$\tilde{\tilde{\mathbf{P}}}_{i+1} = Diag\left(\tilde{\tilde{\mathbf{P}}}_{x_{i+1}}, \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_{i+1}^{DGPS}}, \tilde{\tilde{\mathbf{P}}}_{\hat{\mathbf{X}}_{P_i}^{(i)}}, \tilde{\tilde{\mathbf{P}}}_{\hat{\mathbf{X}}_{R_i}^{(i)}}\right)$$

we can write problem (11) in the following form:

$$\begin{aligned} &\mathbf{V}_{\mathrm{i+1}} = \mathbf{A}_{\mathrm{i+1}}\hat{\mathbf{d}}_{\mathrm{X_{i+1}}} + \mathbf{L}_{\mathrm{i+1}} \\ &\tilde{\tilde{\mathbf{C}}}_{\mathrm{i+1}} = \sigma_{\mathrm{o}}^2\tilde{\tilde{\mathbf{P}}}_{\mathrm{i+1}}^{-1} \\ &\min_{\mathbf{d}_{\mathbf{X}_{\mathrm{i+1}}}} \Phi^{\mathrm{D-R}}\left(\mathbf{d}_{\mathbf{X}_{\mathrm{i+1}}}\right) = \Phi^{\mathrm{D-R}}\left(\hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{i+1}}}\right) \end{aligned} \qquad (12)$$

This estimator is the solution to the problem:

$$\hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{i+1}}} = -\left(\mathbf{A}_{\mathrm{i+1}}^{\mathrm{T}}\tilde{\tilde{\mathbf{P}}}_{\mathrm{i+1}}\mathbf{A}_{\mathrm{i+1}}\right)^{-1}\mathbf{A}_{\mathrm{i+1}}^{\mathrm{T}}\tilde{\tilde{\mathbf{P}}}_{\mathrm{i+1}}\mathbf{L}_{\mathrm{i+1}} \qquad (13)$$

Moreover,

$$\hat{\mathbf{C}}_{\hat{\mathbf{X}}_{\mathrm{i+1}}} = \hat{\mathbf{C}}_{\hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{i+1}}}} = \hat{\sigma}_{\mathrm{o}}^2\left(\mathbf{A}_{\mathrm{i+1}}^{\mathrm{T}}\tilde{\tilde{\mathbf{P}}}_{\mathrm{i+1}}\mathbf{A}_{\mathrm{i+1}}\right)^{-1} \qquad (14)$$

and

$$\hat{\sigma}_{\mathrm{o}}^2 = \frac{1}{f_{\mathrm{i+1}}}\mathbf{V}_{\mathrm{i+1}}^{\mathrm{T}}\tilde{\tilde{\mathbf{P}}}_{\mathrm{i+1}}\mathbf{V}_{\mathrm{i+1}} \qquad (15)$$

Then the vector of all of the adjusted coordinates takes the following form:

$$\hat{\mathbf{X}}_{\mathrm{i+1}} = \mathbf{X}_{\mathrm{i+1}}^0 + \hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{i+1}}} = \begin{bmatrix} \mathbf{X}_{\mathrm{P_{i+1}}}^0 \\ \mathbf{X}_{\mathrm{R_{i+1}}}^0 \\ \mathbf{X}_{\mathrm{R_i}}^0 \\ \mathbf{X}_{\mathrm{P_i}}^0 \end{bmatrix} + \begin{bmatrix} \hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{P_{i+1}}}}^{(\mathrm{i+1})} \\ \hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{R_{i+1}}}}^{(\mathrm{i+1})} \\ \hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{R_i}}}^{(\mathrm{i+1})} \\ \hat{\mathbf{d}}_{\mathbf{X}_{\mathrm{P_i}}}^{(\mathrm{i+1})} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{X}}_{\mathrm{P_{i+1}}}^{(\mathrm{i+1})} \\ \hat{\mathbf{X}}_{\mathrm{R_{i+1}}}^{(\mathrm{i+1})} \\ \hat{\mathbf{X}}_{\mathrm{R_i}}^{(\mathrm{i+1})} \\ \hat{\mathbf{X}}_{\mathrm{P_i}}^{(\mathrm{i+1})} \end{bmatrix} \qquad (16)$$

5 NUMERICAL TEST

In order to illustrate the above theoretical analyses, a numerical test will be presented. This test will demonstrate the possibility of utilising *IANS* while sailing along the coast by using geo-navigation. Let us assume that a vessel is manoeuvring in the waters of the Szczecin Lagoon and the navigator determines the vessel's position by using the DGPS and observing navigation marks on shore. The way in which the vessel manoeuvres is presented in Figure 5.

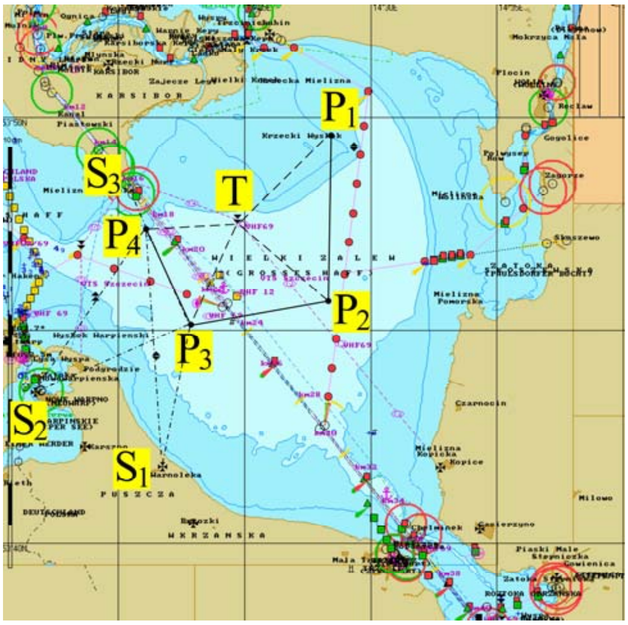

Figure 5. The vessel's manoeuvring path in the waters of the Szczecin Lagoon (own work prepared by using Navi-Sailor 4000 ECS).

During the test, the vessel's positions were determined by using the DGPS (P_1, P_2) and they were also calculated analytically by utilising observations of navigation marks on land.

Table 1. Vessel's positions at sea during the test.

Vessel's positions	Geographic coordinates φ [°]	λ[°]	Gauss-Krüger coordinates X[m]	Y[m]
P_1	53°49.56′N	014°28.14′E	5966784.67	465033.79
P_2	53°46.67′N	014°27.47′E	5961425.53	464255.31
P_3	53°45.33′N	014°23.13′E	5958982.34	459467.49
P_4	53°47.40′N	014°21.60′E	5962837.07	457820.13

The visual navigation marks that were used in the test are presented in Table 2.

Table 2. Selected navigation marks in the waters of the Szczecin Lagoon.

Fixed navigation marks	Geographic coordinates φ [°] λ[°] Gauss-Krüger coordinates X[m] Y[m]
Church in Warnołęka (S_1)	53°41.8′N 014°23.10′E 5952434.41 459377.85
Church in Nowe Warpno (S_2)	53°43.50′N 014°23.10′E 5955587.98 459405.13
Left light at Fairway Gate no. 1 (S_3)	53°48.40′N 014°20.30′E 5964704.43 456409.44

Table 3 presents the navigational observations that were made by observing the navigation marks listed in Table 2.

Table 3. Optical bearings to the landmarks.

Vessel's positions	Landmarks	True bearings [°]
P_3	Church in Warnołęka (S_1)	181.0
	Church in Nowe Warpno (S_2)	245.0
	Left light at Fairway Gate no. 1 (S_3)	332.0
P_4	Church in Warnołęka (S_1)	171.5
	Church in Nowe Warpno (S_2)	257.5
	Left light at Fairway Gate no. 1 (S_3)	323.0

As part of this test, the navigator had to determine the coordinates of object T (Fig. 5) apart from determining the vessel's positions (P_i). With this end in view, the optical bearings to the object which was being observed were determined, one for each position. Table 4 presents the values of the bearings that were measured.

Table 4. True bearings to mark T.

Vessel's position	True bearing [°]
P_1	226.2°
P_2	300.1°
P_3	028.0°
P_4	084.5°

The calculations are presented below. The navigator determined the vessel's position at P1 as well as the first bearing to object T. Then, after moving to position P2 and changing course, the navigator determined the vessel's position as well as another bearing to object T. Next, the navigator determined the approximate coordinates of object T (Table 5) by using the coordinates of the vessel's positions (P1 and P2) as well as the observations that had been made relative to the object.

Table 5. Approximate coordinates of object T that were determined at position P2.

Geographic coordinate system	φ = 53° 47.6′ N λ = 014° 24.7′ E
Rectangular coordinate system	X = 5963178.25 Y = 461228.31

The vessel was on course 243° when moving from position P2 to position P3. The distance travelled between these positions was 2.9 nautical miles (5375 m). At position P3 the navigator took bearings to three navigation marks (Table 3) and to object T (Table 4). Therefore, it became possible to determine the vessel's position by using the observations that had been made as well as the first estimate of the coordinates of object T.

For the purposes of the test that is described here, systems of observation equations are formulated according to the following model:

$$\left.\begin{aligned} NR_{i,j} &= FNR_{i,j}\left(\mathbf{X}_{P_j}, \mathbf{X}_T\right) \\ KR_{j-1,j} &= FNR_{j-1,j}\left(\mathbf{X}_{P_j}, \mathbf{X}_T\right) \\ d_{j-1,j} &= Fd_{j-1,j}\left(\mathbf{X}_{P_j}, \mathbf{X}_T\right) \\ NR_{T_j} &= FNR_{T_j}\left(\mathbf{X}_{P_j}, \mathbf{X}_T\right) \end{aligned}\right\} \Leftrightarrow \mathbf{F}_j\left(\mathbf{X}_{P_j}, \mathbf{X}_T\right) = \mathbf{F}_j(\mathbf{X})$$

where:

$NR_{i,j}$ – i-th bearing to the n-th navigation mark at position P_j

$KR_{j-1,j}$ – vessel's course between positions P_{j-1} and P_j

$d_{j-1,j}$ – distance between positions P_{j-1} and P_j

NR_{T_j} – bearing to object T at position P_j

$X_{P_j} = \begin{bmatrix} x_{P_j} \\ y_{P_j} \end{bmatrix}$ – coordinates of the vessel at position „j"

$X_T = \begin{bmatrix} x_T \\ y_T \end{bmatrix}$ – coordinates of object T

Later in the paper only the determinations of adjustments to the observations that were made will be presented, as well as the estimated coordinates of the vessel's and object T's positions.

After formulating the system of observation equations and the adjustment problem, one should establish, according to the principles of robust adjustment that were described in the first part of this paper, whether the observations that were made are fraught with gross errors, which may influence the results of the adjustment. In order to do this, an adjustment covariance matrix is determined:

$$\mathbf{C}_{\mathbf{V}_i} = m_o^2 \mathbf{Q}_{\mathbf{V}_i} \tag{17}$$

where:

$m_o^2 = \dfrac{\mathbf{V}_i^T \tilde{\tilde{\mathbf{P}}}_i \mathbf{V}_i}{n-4}$ – coefficient of variance

$\mathbf{Q}_{\mathbf{V}_i} = \tilde{\tilde{\mathbf{P}}}_i^{-1} - \mathbf{A}_i \left(\mathbf{A}_i^T \tilde{\tilde{\mathbf{P}}}_i \mathbf{A}_i \right)^{-1} \mathbf{A}_i^T$ – cofactor matrix of the vector of adjustments **V**

Moreover, the acceptable range of standardised adjustments is determined based on the formula:

$$\Delta \mathrm{v_i} = \left\langle -\mathrm{k}\sigma_{\mathrm{v_i}}; \mathrm{k}\sigma_{\mathrm{v_i}} \right\rangle \tag{18}$$

Standardised adjustments make it possible to determine the value of the attenuation function which is an element of the decision and attenuation matrix described by formula (3). The attenuation function serves the purpose of obtaining an equivalent weight matrix from formula (2). The range of acceptable adjustments depends on the adopted confidence level γ. Given that adjustments (v_i) assume random values, but in accordance with normal distribution, we can write the following:

$$\gamma = \mathrm{P}\left(-\mathrm{k}\sigma_{\mathrm{v_i}} < \mathrm{v_i} < \mathrm{k}\sigma_{\mathrm{v_i}}\right) = \mathrm{P}\left(-\mathrm{k} < \bar{\mathrm{v}}_\mathrm{i} < \mathrm{k}\right) = \\ = \int_{-\mathrm{k}}^{\mathrm{k}} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{\bar{\mathrm{v}}_\mathrm{i}^2}{2} \right] d\bar{\mathrm{v}}_\mathrm{i} \tag{19}$$

where:

$\sigma_{v_i} = \sqrt{\left[C_{v_i}\right]_{ii}}$ – standard deviation for the i-th adjustment

$\bar{v}_i = \dfrac{v_i}{\sigma_{v_i}}$ – standardised adjustment within the acceptable range $\Delta \bar{v} = \langle -k, k \rangle$

In navigational practice, it is assumed that coefficient $k = 2$ for confidence level $\gamma = 0{,}95$.

In this numerical test, analyses were limited to attenuation function $\mathbf{T}(\mathbf{V}_{\mathbf{x}_i}) = Diag\{t(v_1), t(v_2), \ldots, t(v_n)\}$, whereas decision function $\Im(\mathbf{x}_i) = Diag\left(\Im(\mathbf{x}_{Z_i}), \Im(\mathbf{x}_R^{(i)})\right)$ was omitted. For the purpose of navigation tasks, the Danish attenuation function is used most often; it is described in detail in papers written by Hampel et al. (1986), Wiśniewski (2009), and Czaplewski (2004). This function can be expressed as the formula:

$$\mathrm{t}\left(\bar{\mathrm{v}}_\mathrm{i}\right) = \begin{cases} 1 & \text{dla} \quad \bar{\mathrm{v}}_\mathrm{i} \in \langle -\mathrm{k}, \mathrm{k} \rangle \\ \exp\left\{-\mathrm{l}\left(\left|\bar{\mathrm{v}}_\mathrm{i}\right| - \mathrm{k}\right)^\mathrm{g}\right\} & \text{dla} \quad \bar{\mathrm{v}}_\mathrm{i} \notin \langle -\mathrm{k}, \mathrm{k} \rangle \end{cases} \tag{20}$$

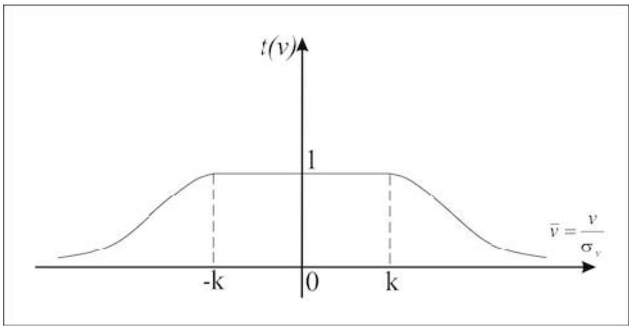

Figure 6. Danish attenuation function.

The attenuation function is used to determine equivalent weights, in accordance with the following formula:

$$\hat{\mathrm{p}}_\mathrm{i} = \mathrm{t}\left(\bar{\mathrm{v}}_\mathrm{i}\right)\mathrm{p}_\mathrm{i} = \begin{cases} \mathrm{p}_\mathrm{i} & \text{dla} \quad \bar{\mathrm{v}}_\mathrm{i} \in \langle -\mathrm{k}, \mathrm{k} \rangle \\ \exp\left\{-\mathrm{l}\left(\left|\bar{\mathrm{v}}_\mathrm{i}\right| - \mathrm{k}\right)^\mathrm{g}\right\}\mathrm{p}_\mathrm{i} & \text{dla} \quad \bar{\mathrm{v}}_\mathrm{i} \notin \langle -\mathrm{k}, \mathrm{k} \rangle \end{cases} \tag{21}$$

The values of parameters l and g are selected experimentally so that the number of iterations is not too large. It is usually assumed that parameters $l = 0.01$ and $g = 2$. In accordance with assumptions (2) and (3), an equivalent weight matrix is constructed:

$$\hat{\mathbf{P}}_{\mathbf{x}_i} = \mathbf{T}(\bar{\mathbf{V}}_{\mathbf{x}_i})\mathbf{P}_{\mathbf{x}_i} = \begin{bmatrix} \mathrm{t}(\bar{\mathrm{v}}_1)\mathrm{p}_1 & 0 & \vdots & 0 \\ 0 & \mathrm{t}(\bar{\mathrm{v}}_2)\mathrm{p}_2 & \vdots & 0 \\ \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & \vdots & \mathrm{t}(\bar{\mathrm{v}}_\mathrm{i})\mathrm{p}_\mathrm{i} \end{bmatrix} \tag{22}$$

After adopting the range of standardised adjustments $\Delta \bar{v}_i = \langle -2; 2 \rangle$ and confidence level $\gamma = 95\%$, the following values of standardised adjustments were obtained at the initial (identification) stage:

$$\bar{v}_1 = \frac{v_1}{\sigma_{v_1}} = -6.8136 \quad \notin \Delta\bar{v},$$

$$\bar{v}_2 = \frac{v_2}{\sigma_{v_2}} = 41.5704 \quad \notin \Delta\bar{v},$$

$$\bar{v}_3 = \frac{v_3}{\sigma_{v_3}} = 0.8955 \quad \in \Delta\bar{v},$$

$$\bar{v}_4 = \frac{v_4}{\sigma_{v_4}} = -17.3476 \quad \notin \Delta\bar{v},$$

$$\bar{v}_5 = \frac{v_5}{\sigma_{v_5}} = -25.0979 \quad \notin \Delta\bar{v},$$

$$\bar{v}_6 = \frac{v_6}{\sigma_{v_6}} = 15.4231 \quad \notin \Delta\bar{v},$$

One can easily notice that all of the observations, except for the third bearing, are fraught with gross error, which results from the assumed range of standardised adjustments $(\Delta\bar{v})$. Therefore, we used the procedure of robust adjustment, which is iterative. In order to simplify the presentation of results, the tables below show the results of robust adjustment for position P_3.

For the purpose of the example that is described here, the bearing to the church in Nowe Warpno (S_2) was fraught with gross error. This is confirmed by the results presented in Tables 6 and 7. The bearing which was adopted for the calculations and which was fraught with error was 245.0°, and it should be 247.0°. The robust adjustment process allowed us to obtain:

1 Estimated coordinates of the vessel's position:

$$\hat{\mathbf{X}}_{P_3} = X^o_{P_3} + \hat{\mathbf{dx}}_{P_3} = \begin{bmatrix} 5958982.34 \\ 459467.49 \end{bmatrix} + \begin{bmatrix} 8.87 \\ -4.38 \end{bmatrix} = \begin{bmatrix} 5958991.21 \\ 459463.11 \end{bmatrix}$$

with an error in determining the position $M_{P_3} = 43.9m$.

2 Estimated coordinates of object T's position:

$$\hat{\mathbf{X}}^{P_3}_T = X^{P_2}_T + \hat{\mathbf{dx}}^{P_3}_T = \begin{bmatrix} 5963178.25 \\ 461228.31 \end{bmatrix} + \begin{bmatrix} -0.05 \\ 0.12 \end{bmatrix} = \begin{bmatrix} 5963178.20 \\ 461228.43 \end{bmatrix}$$

with an error in determining the position $M^{P_3}_T = 17.3m$.

The vessel was on course $KR = 337.2^\circ$ when moving from position P_3 to position P_4. The distance between these positions was 2.26 nautical miles (4182 m). At position P_4 the navigator made observations relative to the same navigation marks that were used at position P_3 (Table 3) and took the fourth bearing to object T (Table 4). Like at the previous position, it is possible to determine the coordinates of the vessel's position and to once again adjust the coordinates of object T's position. As at the previous stage, we will now start identifying observations that are fraught with gross errors. The accepted range of standardised adjustments and the confidence level did not change. The values of the standardised adjustments to observations made at position P_4 are as follows:

$$\bar{v}_1 = \frac{v_1}{\sigma_{v_1}} = -0.11278 \quad \in \Delta\bar{v},$$

$$\bar{v}_2 = \frac{v_2}{\sigma_{v_2}} = 0.2361 \quad \in \Delta\bar{v},$$

$$\bar{v}_3 = \frac{v_3}{\sigma_{v_3}} = 0.0235 \quad \in \Delta\bar{v},$$

$$\bar{v}_4 = \frac{v_4}{\sigma_{v_4}} = -0.0985 \quad \in \Delta\bar{v},$$

$$\bar{v}_5 = \frac{v_5}{\sigma_{v_5}} = -0.4352 \quad \in \Delta\bar{v},$$

$$\bar{v}_6 = \frac{v_6}{\sigma_{v_6}} = -0.4467 \quad \in \Delta\bar{v},$$

Table 6. Values of the attenuation function for particular observations at position P_3.

Iteration Step	Parameters of the attenuation function l	g	Values of the attenuation function for particular observations $t(\bar{v}_1)$	$t(\bar{v}_2)$	$t(\bar{v}_3)$	$t(\bar{v}_4)$	$t(\bar{v}_5)$	$t(\bar{v}_6)$
1	0.003	2.0	1	**0.0091**	1	1	1	1
2	0.03	2.0	1	**0.8186**	1	1	1	1
3	0.1	2.0	1	**0.6305**	1	1	1	1
4	0.8	2.0	1	1	1	1	1	1

Table 7. Values of the standardised adjustments for observations made at position P_3.

Iteration Step	Parameters of the attenuation function l	g	Values of the standardised adjustments for particular observations $\lvert\bar{v}_1\rvert$	$\lvert\bar{v}_2\rvert$	$\lvert\bar{v}_3\rvert$	$\lvert\bar{v}_4\rvert$	$\lvert\bar{v}_5\rvert$	$\lvert\bar{v}_6\rvert$
1	0.003	2.0	0.4380	**4.5830**	0.2288	0.0026	0.3165	0.2425
2	0.03	2.0	0.4236	**4.1477**	0.2313	0.0452	0.2474	0.2061
3	0.1	2.0	0.3996	**3.2949**	0.2355	0.1163	0.1319	0.1453
4	0.8	2.0	0.3694	1.6858	0.2407	0.2063	0.0144	0.0686

As one can see above, all of the adjustments are within the accepted range $\Delta\bar{v}_i = \langle -2;2 \rangle$. Therefore, it is not necessary to use the procedure of robust adjustment and decision and attenuation function $\mathbf{T}(\mathbf{x}_i, \mathbf{V}_{\mathbf{x}_i})$. Now, the coordinates of position P_4 and of object T's position can be determined. The determinations of adjustments at the last stage of calculations in this test are as follows:

1 Estimated coordinates of the vessel's position:

$$\hat{\mathbf{X}}_{P_4} = X^o_{P_4} + \hat{\mathbf{d}}\mathbf{x}_{P_4} = \begin{bmatrix} 5962837.06 \\ 457820.13 \end{bmatrix} + \begin{bmatrix} 1.39 \\ 6.90 \end{bmatrix} = \begin{bmatrix} 5962838.45 \\ 457827.03 \end{bmatrix}$$

with an error in determining the position

$M_{P_4} = 24.98m$.

2 Estimated coordinates of object T's position:

$$\hat{\mathbf{X}}^{P_4}_T = X^{P_3}_T + \hat{\mathbf{d}}\mathbf{x}^{P_4}_T = \begin{bmatrix} 5963178.20 \\ 461228.43 \end{bmatrix} + \begin{bmatrix} 1.92 \\ -0.05 \end{bmatrix} = \begin{bmatrix} 5963180.12 \\ 461228.38 \end{bmatrix}$$

with an error in determining the position

$M^{P_4}_T = 16.72m$.

6 CONCLUSIONS

The mathematical methods that are proposed in this paper allow one to create Interactive Navigational Structures (*IANS*), which can be helpful in performing many typical navigational tasks. *IANS* can be an invaluable tool that aids in determining the coordinates of a wreck's position when the depth of water makes it impossible for measurement teams to come close to it (Czaplewski, 2004). Moreover, the methodology for creating and developing *IANS* was successfully used in radar navigation as a tool that made the determinations carried out by artificial neural networks more precise (Czaplewski and Wąż, 2004).

The development of *IANS* can act as a supplementary, analytical method of determining a vessel's position at sea. Let us assume that a vessel is moving through waters where the typical navigation marks are unavailable, but where one can observe other stationary objects. At the first stage, the navigator determines the vessel's positions by using the available systems of navigation marks and at the same time makes observations relative to stationary objects. Observations that are made in accordance with the principles that are described in this paper will allow one to determine the coordinates of such objects and then further use them in navigation as alternative navigation marks. It should be remembered that the quality of determinations will not be as high as when the coordinates of typical navigation marks are used. However, since one needs to have continuous information about the position of one's vessel when there are no other possibilities of determining this position, the proposal presented in this paper is a proper alternative.

Currently, the author and his research team are carrying out studies that focus on the use of Interactive Navigational Structures in the VTS (Vessel Traffic Service) system. A vessel traffic controller using the VTS system can easily specify the position of a vessel which has stated its position while being in waters that are covered by this system if the controller has the appropriate software which uses the mathematical methods of creating and developing *IANS*.

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