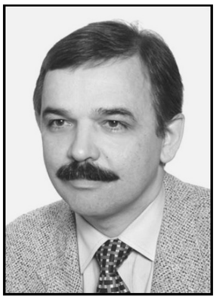
Eligiusz PAWŁOWSKI
POLITECHNIKA LUBELSKA,
ul. Nadbystrzycka 38A, 20-618 Lublin

Simulation tests on errors of the measuring path with a frequency carrier of information

Ph.D., eng. Eligiusz PAWŁOWSKI



He received M.Sc. and Ph.D. degrees in Electrical Engineering from the Lublin University of Technology. He is the author of more than 70 scientific publications. His main research interests are in the field of electrical measurements and instrumentation, virtual instruments, measurement systems and sensors with pulse frequency output. Currently he works at the Department of Automatics and Metrology at the Electrical Engineering and Computer Science Faculty, Lublin University of Technology.

e-mail: e.pawlowski@pollub.pl

Abstract

The paper presents a problem concerning digital processing of signals in measuring systems with sensors of pulse frequency output. Under dynamic conditions instantaneous frequency values represent instantaneous values of a processed quantity and the signal samples are irregularly distributed in time. Moreover, the time between samples may depend on the current and previously values of the processed quantity. Such an irregularly sampled signal can be further analyzed with the application of resampling to extract a regularly sampled signal from the irregular data. The difference between the estimated and the true signal is analyzed as a measuring error.

Keywords: instantaneous frequency, nonuniformly sampling, resampling.

Symulacyjne badania błędów toru pomiarowego z częstotliwościowym nośnikiem informacji

Streszczenie

W pracy rozpatrzono zagadnienie cyfrowego przetwarzania sygnału w torze pomiarowym z czujnikiem posiadającym impulsowe wyjście częstotliwościowe. W stanie statycznym częstotliwość impulsów na wyjściu takiego czujnika jest proporcjonalna do wartości wielkości przetwarzanej w torze pomiarowym. Niezmienną w czasie częstotliwość można wieloma znanymi sposobami przetworzyć na wartość cyfrową ze stosunkowo wysoką dokładnością. W rozdziale 2 przedstawiono problem analizy pracy rozważanego toru pomiarowego w stanach dynamicznych, gdy chwilowe wartości częstotliwości impulsowego sygnału reprezentują chwilowe wartości przetwarzanej wielkości. Ponieważ dla sygnału impulsowego nie jest możliwe wyznaczenie przyrostów fazy mniejszych od jednego okresu, w praktyce nie można również wyznaczyć jego częstotliwości chwilowej. Próbki sygnału częstotliwościowego mogą być wyznaczone nie częściej niż co kolejny jego okres i reprezentują one wartości średnie wielkości wejściowej w tym okresie. Dodatkowym utrudnieniem jest nierównomierne rozmieszczenie tych próbek w czasie. W rozdziale 3 przedstawiono algorytm przetwarzania sygnału częstotliwościowego w celu uzyskania próbek rozmieszczonych równomiernie w czasie, a w rozdziale 4 omówiono metodę symulacyjnego badania błędów w rozpatrywanym torze pomiarowym. Błędy zdefiniowano jako różnicę pomiędzy odtworzonymi wartościami chwilowymi sygnału i wartościami sygnału podanego w postaci analitycznej na wejście algorytmu symulacyjnego. Wyznaczano wartości chwilowe i średniokwadratowe błędów. W rozdziale 5 przedstawiono uzyskane przykładowe wartości błędów kwantowania i uśredniania oraz błędów równomiernego resamplingu, w zależności od parametrów przetwarzanej wielkości: jej częstotliwości oraz stosunku składowej stałej do składowej przemiennej.

Słowa kluczowe: częstotliwość chwilowa, próbkowanie nierównomierne, resampling.

1. Introduction

Digital processing of a signal received from a sensor of frequency output requires the application of adequate algorithms [1]. The conversion of a frequency signal into a digital form does not present a difficult task with the contemporary level of technological development. Many digital methods for frequency measurements are known and practically used. Alas, to convert a frequency signal into a series of samples uniformly spaced in time is not as easy as it is in the case of a voltage signal. The frequency signal is particularly disadvantageous as far as dynamic conditions are concerned [2] - digital measurements of frequency always concern mean values and under dynamic conditions errors related to averaging and to the accepted method of ascribing results to adequate time instants inevitably occur [3]. In order to reduce the dynamic errors, a signal period (inter-pulse time) or its multiple is measured, which yields an irregular distribution of the measurand samples in time [4]. Results are obtained in instants that are spaced by successive periods of the sensor signal, which makes their further processing by the known DSP algorithms impossible, as the algorithms require regular sampling [5]. Hence, it is necessary to apply a resampling algorithm in the measuring path.

2. Digital processing of frequency signal

Figure 1 presents a block diagram of the considered measuring path. The sensor X/F converts an input value $x(t)$ into a frequency signal $f(t)$, which gets further converted in a section for digital measurements of frequency F/D into a number sequence $\{T(n)\}$ of successive inter-pulse time values that are irregularly distributed in time. Generator G is a source of the reference frequency f_{ref} for digital measurement of frequency $f(t)$. Samples $x_R(l)$ that represent the input signal $x(t)$ in time instants regularly distributed in time are determined in a digital-to-digital processing section D/D.

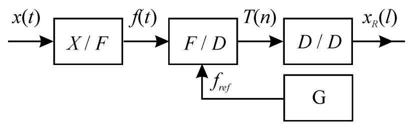


Fig. 1. Block diagram of measuring path with a frequency carrier of information
Rys. 1. Schemat blokowy toru pomiarowego z częstotliwościowym nośnikiem informacji

The block diagram of a corresponding measurements circuit is shown in figure 2. It contains the sensor X/F, counter C, generator G, buffer register R and the processing section D/D. The output of the sensor is connected to the latch input of the buffer register R. In this circuit at each edge of the pulse signal $f(t)$ the current value K of the counter C is written to the buffer register R.

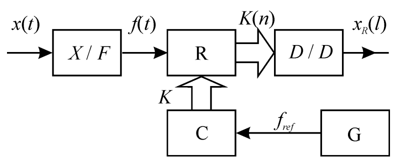


Fig. 2. Block diagram of the considered measurements circuit
Rys. 2. Schemat blokowy rozpatrywanego układu pomiarowego

Inter-pulse time values $T(n)$ are given as differences between two successive values obtained form the buffer register:

$$T(n) = \frac{K(n) - K(n-1)}{f_w} \; . \tag{1}$$

Figure 3 illustrates successive stages of the digital-to-digital processing. The recorded signal is represented by a number sequence of N elements (2), whose successive values $T(n)$ are equal to the measured time intervals. The $T(n)$ values are not regularly distributed in time (Fig. 3), as they are obtained in successive time instants $t(n)$ that are defined by the initial instant t_0 when the measurement gets started and an adequate sum of $T(n)$ values. The total duration of the measurement and recording of N values of the $T(n)$ time is equal to T_M (3).

$$\{T(n)\}, \quad n = 1, \dots, N \tag{2}$$

$$t(n) = t_0 + \sum_{i=1}^{n} T(i) \ , \ T_M = \sum_{i=1}^{N} T(i) \tag{3}$$

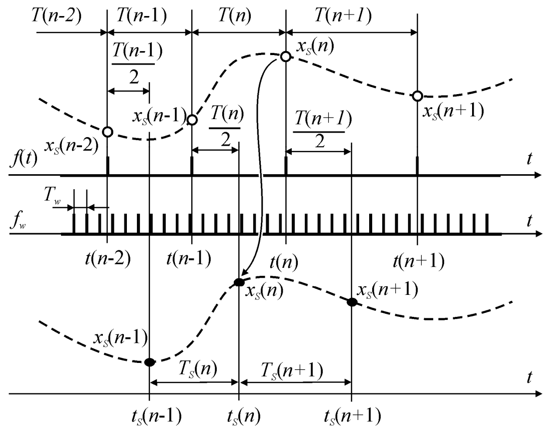


Fig. 3. Successive stages of frequency signal processing: irregular sampling
Rys. 3. Kolejne etapy przetwarzania sygnału częstotliwościowego: nierównomierne próbkowanie

For the X/F converter of a conversion constant equal to c_p the successive values of the inter-pulse time $T(n)$ represent the successive sampled values $x_S(n)$ of the input signal:

$$x_S(n) = c_p \cdot f(n) = c_p \cdot \frac{1}{T(n)} \ . \tag{4}$$

In order to minimize dynamic errors related to averaging during the frequency measurement, values $x_S(n)$ should be ascribed to sampling time instants $t_S(n)$ that are located in the middle of the inter-pulse time $T(n)$, from $t(n-1)$ to $t(n)$:

$$t_S(n) = \frac{t(n-1) + t(n)}{2} = t_0 + \sum_{i=1}^{n-1} T(i) + \frac{1}{2} T(n) \ . \tag{5}$$

Successive values $x_S(n)$ that are presented in figure 3 are not equally spaced from one another in time. Two successive values $x_S(n\text{-}1)$ and $x_S(n)$ are spaced from each other by the time $T_S(n)$:

$$T_S(n) = \frac{T(n-1) + T(n)}{2} \ . \tag{6}$$

3. Realisation of resampling

In order to obtain values that are regularly distributed in time, there is a need to perform a resampling operation [6], which on the basis of value sequences $x_S(n)$ and $t_S(n)$ can determine a L-element sequence of values $x_R(l)$ for time instants $t_R(l)$:

$$\{x_R(l)\}, \quad l = 1, \dots, L \ , \ t_R(l) = t_S(1) + (l-1) T_R \ . \tag{7}$$

All values $x_R(l)$ are spaced from each other by a constant time interval T_R, as shown in figure 4. The number L and the regular sampling period T_R have to be selected so that values $x_S(n)$ and $x_R(l)$ can describe the processed signal for the same measurement time interval of sampling algorithm T_{MS}:

$$T_R = \frac{T_{MS}}{L-1} \ , \ T_{MS} = \sum_{i=1}^{N-1} T_S(i) \ . \tag{8}$$

The values of samples $x_R(l)$ for time instants $t_R(l)$ can be determined in a few ways (Fig. 4): when the S&H (*Sample & Hold*) method is applied, a sample $x_R(l)$ is ascribed a value of the directly preceding sample $x_S(n)$, while within the NNR (*Nearest Neighbor Resampling*) method the ascribed value belongs to the closest located sample. With the application of the linear interpolation or the polynomial method it is possible to determine the $x_R(m)$ sample value based on the values of two or more $x_S(n)$ samples. Within the discussed research, the signal approximation by a straight line in intervals from $t_S(n\text{-}1)$ to $t_S(n)$ has been applied [7].

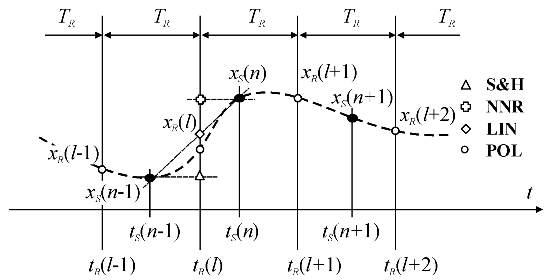


Fig. 4. Successive stages of frequency signal processing: resampling algorithm
Rys. 4. Kolejne etapy przetwarzania sygnału częstotliwościowego: algorytm resamplingu

4. Simulation tests

The measuring path that is presented in figure 1 and the discussed algorithm of data processing have been subjected to simulation tests using as an example a rotary-pulse converter coupled with a motor [1] whose rotational speed ω_0 sinusoidally changes along with the variations of amplitude Ω and frequency ω_1 :

$$\omega(t) = \frac{d\varphi}{dt} = \omega_0 + \Omega \cdot \sin(\omega_1 t) \ . \tag{9}$$

After having (9) integrated the (10) dependence for an instantaneous value of the shaft rotation angle $\varphi(t)$ is obtained.

$$\varphi(t) = \omega_0 t - \frac{\Omega}{\omega_1} \cdot \cos(\omega_1 t) + \frac{\Omega}{\omega_1}, \varphi(0) = 0 \tag{10}$$

The dependence makes it possible to digitally simulate the converter input signal i.e. to determine time instants $t(n)$, when the φ angle reaches the value of a total multiple of the angle φ between holes in the disk (10) - at those instants its successive output pulses occur. At the assumption that the number of disk holes is k the equation (11) can be iteratively solved, which eventually makes it possible to determine values (12) of the inter-pulse time $T(n)$, which make input values (2) to the considered algorithm of data processing.

$$\varphi(t(n)) = n\alpha \, , \alpha = \frac{2\pi}{k} \, , n = 1, 2, \dots, N \tag{11}$$

$$T(n) = t(n) - t(n-1), t(0) = 0 \tag{12}$$

The iteration step accepted at the search for the (11) equation solutions is equivalent to the quantization error at period measurements in the real circuit of figures 1 and 2.

The (2) - (8) dependences that describe the frequency-signal processing algorithm and the simulated generation of a standard signal on the basis of the (9) - (12) formulae make it possible to independently determine errors at each stage of the processing. The instantaneous error values - of a dynamic error $\Delta\omega_D$ that is related to averaging and ascribing results to selected time instants (with the quantization error considered) and of a total error $\Delta\omega_R$ at the resampling algorithm output - have been determined in the testing course (13). The block diagram of the corresponding calculation algorithm is shown in figure 5. The errors $\Delta\omega_D$ and $\Delta\omega_R$ are calculated as the differences between the estimated ($\omega_S(t)$ or $\omega_R(t)$ respectively) and the true values of signal $\omega(t)$.

$$\Delta\omega_D(n)=\omega(t_S(n))-\omega_S(n)\ ,\ \Delta\omega_R(l)=\omega(t_R(l))-\omega_R(l) \quad (13)$$

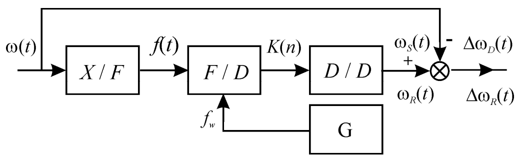


Fig. 5. The block diagram of the error calculation algorithm
Rys. 5. Schemat blokowy algorytmu obliczania błędów

5. Test results

Figures 6 and 7 present the example results obtained from the the realised simulation. The instantaneous values of errors $\Delta\omega_D$ and $\Delta\omega_R$ described by relations (13) are shown in figure 6 as a percentage from the amplitude Ω. The parameters of the simulated signal $\omega(t)$ have been as follow: mean value of rotational speed ω_0=50 rad/s, frequency of its changes ω_1=1256 rad/s (i.e. 200 Hz) and amplitude Ω=10 rad/s. As it can be seen in figure 6, the instantaneous values of errors $\Delta\omega_D$ and $\Delta\omega_R$ are the smallest at the instants, when the signal $\omega(t)$ passes the mean value level ω_0 and reach higher values in the other time instants.

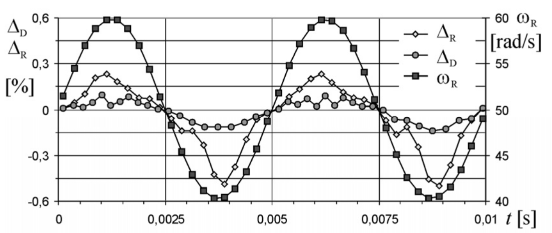


Fig. 6. Instantaneous values of resampled rotational speed ω_R, dynamic $\Delta\omega_D$ and resampling $\Delta\omega_R$ errors vs. time for ω_0=50 rad/s, ω_1=1256 rad/s, Ω=10 rad/s
Rys. 6. Wartości chwilowe prędkości obrotowej ω_R, błędów dynamicznych $\Delta\omega_D$ i błędów resamplingu $\Delta\omega_R$ względem czasu dla ω_0=50 rad/s, ω_1=1256 rad/s, Ω=10 rad/s

In order to compare the processing results for various values of simulation parameters, the mean-square values of errors Δ^{σ}_D and Δ^{σ}_R defined by (14) have been applied. Figure 7 presents the obtained simulation results for several values of ω_0 (20, 50, and 100 rad/s) vs. a ratio of amplitude Ω to frequency ω_0 (from 0,1 to 0,8). The frequency of its changes is equal to ω_1=1256 rad/s, just as previously.

$$\Delta^{\sigma}_D(n)=\sqrt{\frac{1}{N}\sum_{i=1}^{N}\Delta^2_D(n)}\ ,\ \Delta^{\sigma}_R(m)=\sqrt{\frac{1}{M}\sum_{i=1}^{M}\Delta^2_R(m)} \quad (14)$$

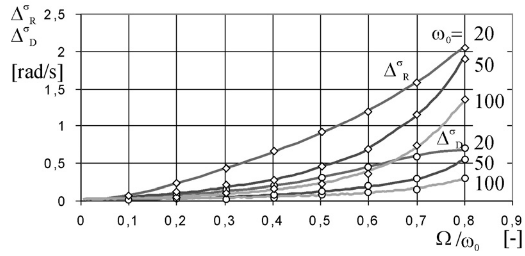


Fig. 7. Root-mean-square values of dynamic Δ^{σ}_D and resampling Δ^{σ}_R errors vs. ratio Ω/ω_0 for ω_0=20, 50, 100 rad/s, Ω/ω_0=0,1 .. 0,8 and ω_1=1256 rad/s
Rys. 7. Wartości średniokwadratowe błędów dynamicznych Δ^{σ}_D i resamplingu Δ^{σ}_R względem stosunku Ω/ω_0 dla ω_0=20, 50, 100 rad/s, Ω/ω_0=0,1 .. 0,8, oraz ω_1=1256 rad/s

6. Conclusions

The performed simulation tests of a measuring path with a sensor of frequency output have shown, first of all, the strong dependence of the dynamic error and the error introduced by the resampling algorithm on the ratio of the amplitude Ω of signal frequency changes to its mean value ω_0. The applied resampling algorithm increases the conversion errors by two - three times depending on the signal parameters. The instantaneous values of the errors are smallest at the instants, when the signal passes the mean value level. It is also possible to perform simulation for other resampling methods (Fig. 4) as well as for each input signal that is recorded in the form of a time function and with the primary function known as it is in the case of (9) and (10).

The presented simulating algorithm can be applied in all the cases when the frequency of a pulse signal carries information about the instantaneous values of a physical quantity processed in the measuring system. The following tests commonly applied in practice make good examples of such situations: testing the angular motion in rotating machines with the application of an incremental encoder, testing the converters with frequency outputs in dynamic states, testing the voltage-controlled oscillators (VCO), stability testing of power generators or frequency in power grids etc.

7. References

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