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# HIGHER-ORDER FINITE ELEMENT PARTICLE TRACING IN EXTERNAL ELECTROSTATIC FIELD

Simulation of movement of a large number of charged particles in external electrostatic field is of great importance for a correct design of separators whose aim is to select them according to the sign of their charge. The paper deals with modeling of their trajectories in a specific arrangement consisting of a profile vessel equipped with two electrodes and several collecting bins. Electric field between the electrodes is determined using a fully adaptive higher-order finite element method. The movement (velocities and trajectories) of the particles affected by electric field, gravity and aerodynamic resistance is modeled by an adaptive Runge-Kutta-Fehlberg method with an appropriately varying time step. On the other hand, their charges are rather low, so that it is possible to neglect the Coulomb forces acting among them. The methodology is illustrated by a typical example whose results are discussed.

## 1. INTRODUCTION

Tracing of electrically charged particles in external electrostatic field belongs to mapping techniques that is important for correct design of devices used for sorting various materials. Charged particles in such devices affected by the Coulomb force, gravity, centrifugal forces and drag aerodynamic forces then fall down to the places predetermined by their charges and initial velocity.

This principle is widely used, for example, for separation of plastics, because a sufficiently pure recycled material may replace, in a lot of applications, the original one. A mixture of particles of plastics may be separated on the basis of the triboelectric effect. It is known, that when electrically non-conducting particles of two different plastics come into contact with electric charge, one of them become more positive (or negative) with respect to another one. And the trajectory of the particle in the system is then affected by the charge transferred to it.

This kind of triboelectric separation was analyzed by a number of researchers. For example, papers [1–8] are prevailingly aimed at a detailed description of the technology and its practical applications. On the other hand, modeling of the

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particle trajectories in electric field traps and similar devices (with the aim to appropriately design their arrangement) based on classical finite difference and finite element algorithms can be found in [9–10]. But accuracy of the results was not too high because the distribution of nonuniform electric field and motion of the particles were calculated separately.

The presented paper deals with modeling of trajectories of charged particles in a specific arrangement consisting of a profile vessel equipped with two electrodes and several collecting bins. Electric field between the electrodes is determined using a fully adaptive higher-order finite element method. The movement of the particles affected by electric field, gravity and drag aerodynamic resistance is modeled by an adaptive Runge-Kutta-Fehlberg method with a time-varying time step.

## 2. FORMULATION OF TECHNICAL PROBLEM

Consider an arrangement depicted in Fig. 1. The particles of plastics of charge $Q$ and initial velocity $\boldsymbol{v}_0$ get to the space between two electrodes, one of them being grounded. There they are deflected according to their charge and fall down into the recycle bins (as these charges are low, it is possible to neglect the Coulomb forces acting among the particles, so that their movement is affected only by the external electric field). The task is to find their trajectories and evaluate the influence of the above quantities.

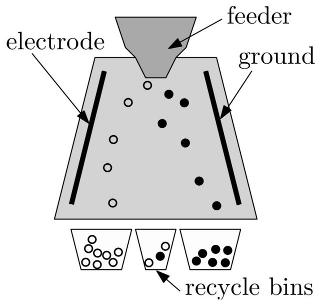


Fig. 1. Basic arrangement of the considered separator

## 3. CONTINUOUS MATHEMATICAL MODEL

Electric field in the domain of the separator is described by the equation [11] for the electric potential $\varphi$

$$\operatorname{div}\left(\varepsilon \operatorname{grad}\varphi\right) = 0, \tag{1}$$

where symbol $\varepsilon$ stands for the dielectric permittivity. The boundary conditions are given by the known values of the electric potential on the electrodes and the Neumann condition along the artificial boundary placed at a sufficiently distance from the device.

The movement of the particle is governed by the equations for velocity $\boldsymbol{v}$ and trajectory $\boldsymbol{s}$ in the forms

$$m\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \boldsymbol{F}_\mathrm{e} + \boldsymbol{F}_\mathrm{a} + \boldsymbol{F}_\mathrm{g}, \qquad \boldsymbol{v} = \frac{\mathrm{d}\boldsymbol{s}}{\mathrm{d}t}, \tag{2}$$

where $\boldsymbol{F}_\mathrm{e}$ denotes the Coulomb force acting on the particle given by the relation

$$\boldsymbol{F}_\mathrm{e} = Q\boldsymbol{E} = -Q\,\mathrm{grad}\,\varphi, \tag{3}$$

$\boldsymbol{E}$ denoting the local value of the electric field strength. Symbol $\boldsymbol{F}_\mathrm{a}$ represents the aerodynamic resistance that is given by formula

$$\boldsymbol{F}_\mathrm{a} = -\boldsymbol{v}\frac{1}{2}\rho c S v, \tag{4}$$

where $c$ is the friction coefficient (depending on geometry of the particle), $\rho$ denotes the density of ambient air, $S$ is the characteristic surface of the particle and $v$ stands for the module of its velocity. Finally,

$$\boldsymbol{F}_\mathrm{g} = m\boldsymbol{g}, \tag{5}$$

where $m$ denotes the mass of the particle and $\boldsymbol{g}$ is the gravitational acceleration.

The corresponding initial conditions read

$$\boldsymbol{v}(0) = \boldsymbol{v}_0, \quad \boldsymbol{s}(0) = \boldsymbol{s}_0, \tag{6}$$

where $\boldsymbol{s}_0$ is the entry position of the particle in the separator. Equation (2) is strongly nonlinear due to the first and second terms on the right-hand side.

## 4. NUMERICAL SOLUTION

The continuous mathematical model (equations (1) and (2)) is solved numerically. We used our own codes Agros2D [12] and Hermes2D [13]. While Hermes is a library of numerical algorithms for monolithic and fully adaptive solution of systems of generally nonlinear and nonstationary partial differential equations (PDEs) based on the finite element method of higher order of accuracy, Agros is a powerful user’s interface serving for pre-processing and post-processing of the problems solved. Both codes written in C++ are used for monolithic solving complex coupled problems rooting in various domains of physics. They are freely distributable. The most important (and in some cases quite unique) features of the codes follow:

- Solution of the system of PDEs is carried out monolithically, which means that the resultant numerical scheme is characterized by just one stiffness matrix. The PDEs are first rewritten into the weak forms whose numerical integration provides its coefficients. The integration is performed using the Gauss quadrature formulas.
- Fully automatic *hp*-adaptivity. In every iteration step the solution is compared with the reference solution (realized on an approximately twice finer mesh), and the distribution of error is then used for selection of candidates for adaptivity. Based on sophisticated and subtle algorithms the adaptivity is realized either by a subdivision of the candidate element or by its description by a polynomial of a higher order [14].
- Each physical field can be solved on quite a different mesh that best corresponds to its particulars. Special powerful higher-order techniques of mapping are then used to avoid any numerical errors in the process of assembly of the stiffness matrix.
- In nonstationary processes every mesh can change in time, in accordance with the real evolution of the corresponding physical quantities.
- Easy treatment of the hanging nodes [15] appearing on the boundaries of subdomains whose elements have to be refined. Usually, the hanging nodes bring about a considerable increase of the number of the degrees of freedom (DOFs). The code contains higher-order algorithms for respecting these nodes without any need of an additional refinement of the external parts neighboring with the refined subdomain.
- Curved elements able to replace curvilinear parts of any boundary by a system of circular or elliptic arcs. These elements mostly allow reaching highly accurate results near the curvilinear boundaries with very low numbers of the DOFs.

Agros2D also contains a module for the time integration of ordinary differential equations (describing motion of charged particles in an external electric field) based on an adaptive Runge–Kutta–Fehlberg algorithm (the time step is variable and depends on the actual development of the solution).

## 5. ILLUSTRATIVE EXAMPLE

The described way of modeling the behavior of charged particles in an external electrostatic field produced by two electrodes is applied to a separator of granular materials. The samples are made of low-density polyethylene (LDPE) and high-density polyethylene (HDPE). The arrangement is considered planar and the task is to find the appropriate amount of charge $Q$ that has to be transferred to individual particles. The particles are assumed spherical with radius $R$ ranging between 2 mm and 4 mm. The mass density of LDPE $\rho_1 = 925\,\text{kg/m}^3$, the same quantity for HDPE $\rho_2 = 960\,\text{kg/m}^3$. The total mass $m$ and reference area $S$ of the considered particles is shown in Tab. 1. The friction coefficient for a sphere (occurring in equation (4)) is assumed $c = 0.47$ .

Table 1. Basic parameters of the considered particles

| Particle radius | Reference area | LDPE mass | HDPE mass |
|---|---|---|---|
| (mm) | (mm$^2$) | (g) | (g) |
| 2 | 12.566 | 0.0310 | 0.0321 |
| 3 | 28.274 | 0.1046 | 0.1086 |
| 4 | 50.265 | 0.2479 | 0.2573 |

The basic geometry of the separating system is shown in Fig. 2. The device consists of two electrodes, feeder, and three recycle bins (see also Fig. 1). The left electrode carries potential $\varphi_1 = 30\,\text{kV}$, the right one is grounded. The artificial boundary was put in a sufficient distance from the system (three times exceeding its dimensions) and we carefully checked its position with respect to how it affects the results of computations.

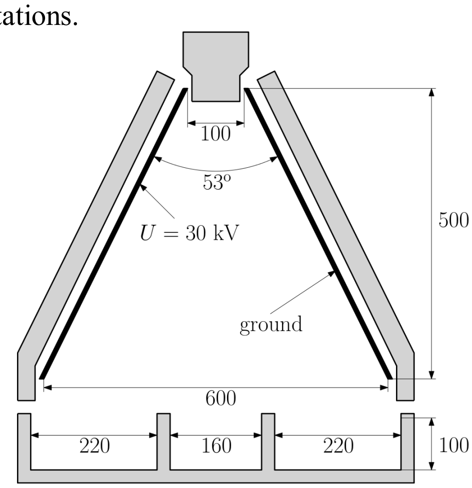


Fig. 2. The main dimensions (in mm) of the separator for granular materials

The computations of electric field in the system were carried out using the *hp*-adaptivity with prescribed tolerance of 1 %. The process of adaptivity required 8 iteration steps starting from the rough mesh with polynomial order $p = 1$. After finishing the adaptive process the maximum order of polynomials is 8 and the number of the degrees of freedom (DOFs) is 3966. The final mesh is shown in Fig. 3; the rectangles on the right side contain the orders of polynomials in particular elements of the mesh. The convergence curve of the adaptive process is shown in Fig. 4. The black circles on it show the situation after particular iteration steps.

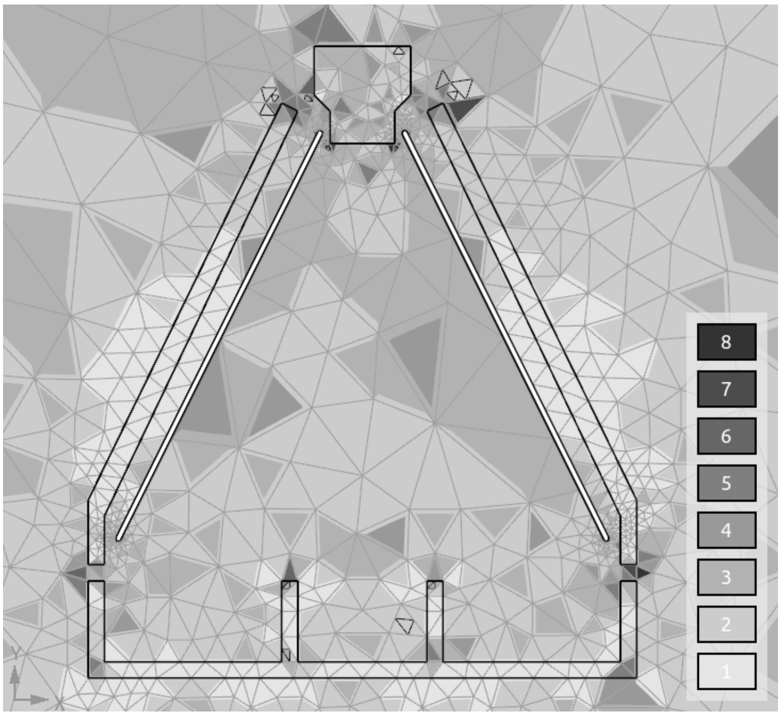


Fig. 3. Final mesh after the adaptive process (relative error 0.95 %, 3966 DOFs, 8 steps)

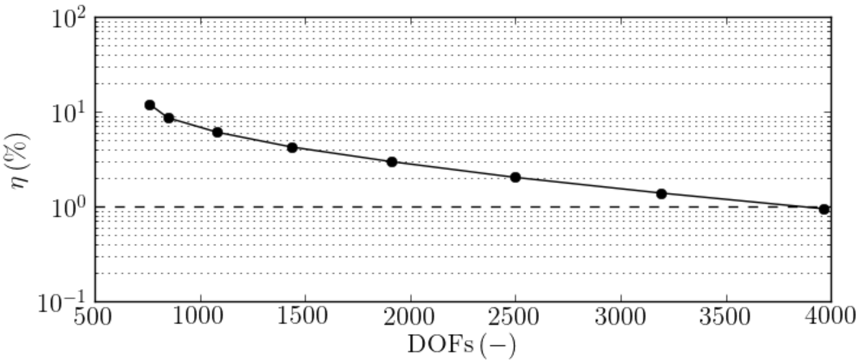


Fig. 4. Dependence of relative error on number of DOF during the adaptive process

Figure 5 shows the equipotential lines and vectors of the electric field. Its module varies almost linearly with height in the range from 200 kV/m (feeder) and 45 kV/m (recycle bins).

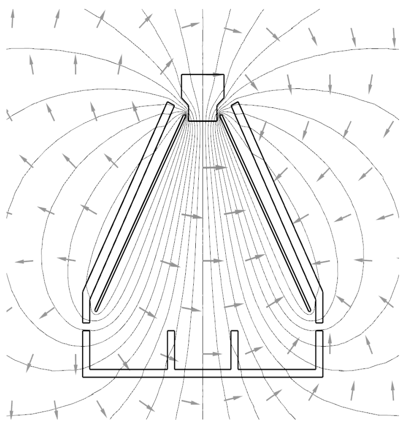

Fig. 5. Equipotential lines and vectors of the electric field in the system

The following figures show the most important results for LDPE particles. Figure 6 shows the trajectories and impact sites of particles with radii $R = 3$ mm charged by different charges $Q$ ranging between 1 nC and 5 nC. The particles enter the chamber with initial velocity $v_0 = 0$ . It is clear that for reaching the required impact site the charge $Q$ must be higher than 1 nC. Figure 7 shows the appropriate charge of the traced particles securing their impact site in the rightmost bin.

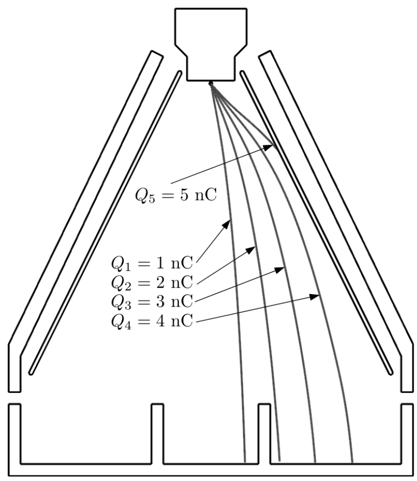


Fig. 6. Impact sites of charged particles with radius $R = 3$ mm

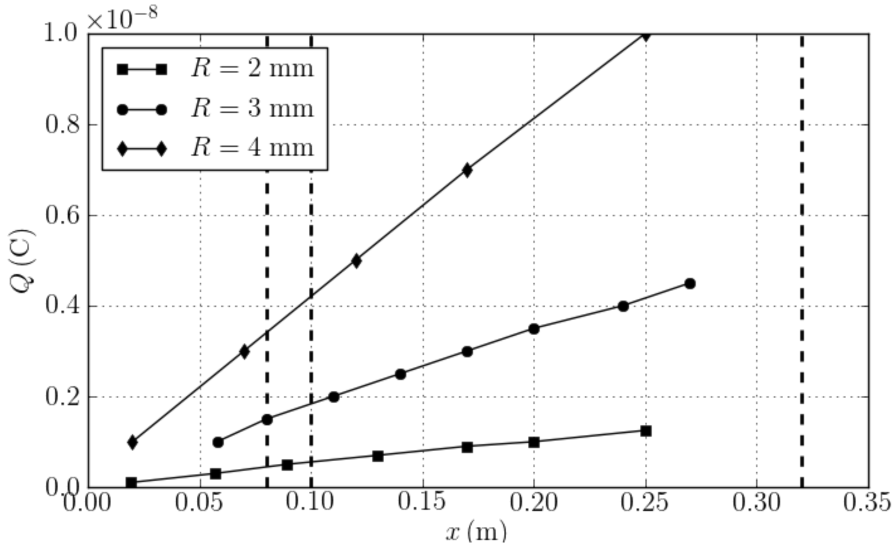


Fig. 7. Dependence of charge $Q$ of the particle on impact site (dashed lines indicate the location of containers)

## 6. CONCLUSION

For given parameters of the system and particles to be separated it is relatively easy to find the charge they must carry for falling down to the correct bin. The next step of this work will be to propose an appropriate mechanism how to transfer the required charge to the processed particles. Another problem to be solved is connected with capturing of small particles on the electrode (see the trajectory in Fig. 6 for $Q = 5$ nF) and influence of this effect on correct operation of the device.

The presented results, therefore, are only preliminary; however, they will serve for the design of an experimental stand.

**ACKNOWLEDGMENT**

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