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SINGLE SIDE BAND MODULATION AND BANDPASS SUBSAMPLING

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Paper describes the efficient method of gaining information from beamformer based on band subsampling technique. In active sonar systems, where centre (carrier) frequency is known (neglecting the Doppler shift), it is feasible to use the subsampling techniques, widely used elsewhere. The subsampling techniques can be used for narrow band signals. One of the advantages of this method lays in the use of noiseless heterodyne in mixer circuit, which provides errorless signal shifting to zero frequency. The use of subsampling provides good opportunity for decreasing the amount of digital processing, hardware components and followed by capital expenditure consumed during fabrication of sonars systems. It also allows increasing the spatial resolution of sonar by increasing the number of preformed beams in the sonar array.

INTRODUCTION

The appearance of digital techniques of signal processing allows sonar systems to develop and achieve hundreds of beams as thin as half a degree. Since digital computers took a lead in sonar signal processors system, it become necessary to sample the received analog waveforms at discrete times. For fear of lose of waveform character it is important to sample at a faster rate for rapidly varying waveforms and at lower rate for slowly varying ones. If X(t) is a deterministic signal band limited to $B/2\pi$, thus we have to sample at $f = B/\pi$ samples per second, i.e. at the Nyquist rate. Under this circumstances in case of sonar systems which are bandpass processes with frequencies confined to $\omega_0 \pm \pi\varpi$, it would be necessary to sample at rate $f = 2(f_0 + \varpi/2)$. Nevertheless there are several techniques known which allow sampling the achieved signal at much lower rate than a Nyquist rate.

1. METHODS

There are three methods of real signal complex sampling: the method of quadrature components simultaneously sampling obtained from conventional analog quadratic demodulator (used together with bandshifting method), the method of analytical signal sampling obtained from analog Hilbert filtration and the method of second order sampling which is the particular case of nonlinear sampling method [1]. In this paper only experimental part is presented. For further theory the Knight et al. [3] is strongly recommended. The only method presented in this article is the technique of sampling at the rate of $f_S=\varpi$, so called analytic signal sampling.

Let introduce band-limited signal X(t) that can be a random process of two-sided power spectra density given by :

$$S_{xx}(\omega)=\begin{cases}|\omega-\omega_0|\geq\pi\varpi\\ |\omega+\omega_0|\geq\pi\varpi\end{cases} \tag{1}$$

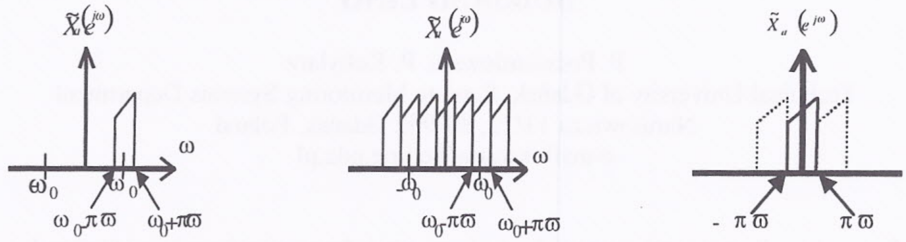

Fig. 1. a) Analytical signal spectra b) Spectra of sampled analytical signal c) Baseband replica of the sampled analytical signal

The analytic signal is obtained from the analog Hilbert transformer, which means that the analytical signal has special features that negative frequency spectra is zero (Fig.1a), and the positive has the same bandwidth as it has before the transformation, i.e. ϖ. The output signal obtained after sampling is as follows:

$$\tilde{X}_a(t)\big|_{t=n\Delta}=\left[X(t)+\breve{X}(t)\right]_{t=n\Delta}=X(n\Delta)+\breve{X}(n\Delta) \tag{2}$$

It consists of two signals, the original signal and his Hilbert transformation $\breve{X}(t)=Hilbert\{X(t)\}$. The spectrum of the analytical signal is indicated in the Fig 1a. Now it is possible to sample this signal at the low rate of $f_S=\varpi$. So the ideal sampling at this rate is a signal $\tilde{X}_{aK}(t)$, which consists of the series of delta functions as follows:

$$\tilde{X}_{aK}(t)=\tilde{X}_a(t)\sum_{n=-\infty}^{n=\infty}\delta(t-n\Delta)=\sum_{n=-\infty}^{n=\infty}\tilde{X}_a(n\Delta)\delta(t-n\Delta) \tag{3}$$

where Δ is the sampling time equal to $\Delta=1/\varpi$. The spectra of this signal is shown in the Fig.1b and is derived as:

$$\tilde{X}_{aK}\left(e^{j\omega}\right)=\tilde{X}_a\left(e^{j\omega}\right)*F\left\{\sum_{n=-\infty}^{n=\infty}\delta(t-n\Delta)\right\}\Big/2\pi=\frac{1}{\varpi}\sum_{n=-\infty}^{n=\infty}\tilde{X}_a\left(e^{j(\omega-n2\pi\varpi)}\right) \tag{4}$$

When assumption that f_0/ϖ=integer is satisfied than one can obtain baseband replica as indicated in the Fig.1c. Otherwise it is necessary to apply the noiseless heterodyne to move the signal near to the frequency $\omega=0$. This can be accomplished by simple multiplying the desired signal by the exp $(-j\omega n\Delta)$, where $\omega/2\pi=f_0$-n ϖ and n is the largest integer $\leq f_0/\varpi$. The similar procedure of gaining the baseband replica can be applied to the dual sideband signal as it is indicated in [1], the sampling frequency cannot be $f_S=\varpi$ any more, but it has to be equal to $f_S=2\ \varpi$. Then the original signal is undersampled but we still sample without subsampling of the envelope, thus the sampling process is conducted without the loss of information. Furthermore, if we want the upper baseband adhere to the y-axis at the zero frequency [1] than $f_S = 2\ \omega_0 / \pi\ (4\ n + 1)$, where n=1,2,....

In the mentioned subsampling technique was assumed that the all filters were ideal. Certainly, since we use any real filter one has to sample at more that the Nyquist rate. Usually, depending on the filter length and type, this frequency is about 2.5 times the highest frequency [3].

Now we try to apply the intentional undersampling technique. Single side band amplitude modulated signal with suppressed carrier will act as the signal which is sampled at much lower rate that is apparently originate from the Nyquist Law for the same signal, but not smaller than Nyquist rate for modulating signal. We experimentally check that during this kind of processing the signal is totally reproducible, thus that there is no loss of information after demodulation. The tests were conducted in Matlab environment, with use of the Simulink package.

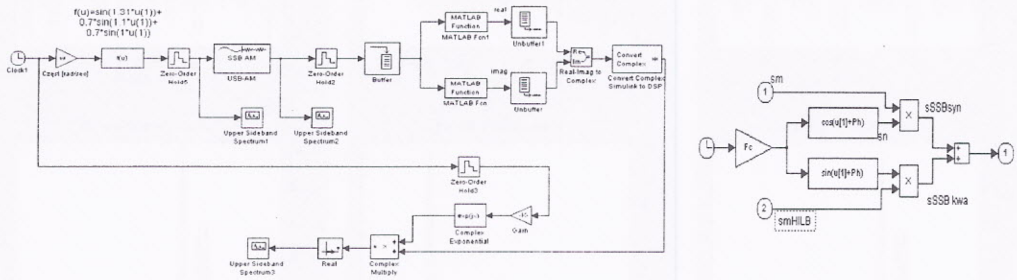

Fig. 2 a) Block diagram of the system b) Modulator

Fig. 2a presents a block diagram of the signal processing system. A clock generator, whose step is selected to overprobe the modulating signal, stimulates the unit. The modulating signal is formed by user defined function. Then the signal is prepared for modulation and modulated single-band by Hilbert transform. Hilbert transform operates on blocks of signals, and this causes delay in signal flow. After the Hilbert transform a quadrature modulation is performed (Fig. 2b), where: ***sm*** – modulating signal, ***snmHilb*** – Hilbert transform of the ***sm*** signal, multiplied by ±1, depending on whether we take the lower or the higher band, F_C – carrier frequency. After the SSB-SC modulation, the signal is underprobed with the following speed:

$$\mathrm{Fsh}=\frac{4\cdot \mathrm{Fc}}{4\cdot k+1}$$

where: Fc is the carrier frequency and ***k*** is a natural number.
In the following experiment k was equal to 3 and 4, which gives Fsh=14.1 Hz. The next several blocks serves as QMI modulators, i.e. to make the signal analytic. The last part of the scheme moves signal spectrum in the right place, according to the QMI-QDD (modulator-demodulator) coupling.

2. RESULTS

We have tested the program with many sets of modulating signals and several values of the ***k*** parameter. The results achieved show the correspondence of simulation results with theory. It can be seen, that for some signals (i.e. amplitude-modulated signals) it is possible to probe them with lower frequency than twice the highest frequency of the signal. In fact, the minimum sampling frequency must be twice as high as the width of the band. So we can underprobe the signal only if there are no other signals outside the specified band.

The results of three sinusoids gained together are presented in Fig. 3.

function $f(u)=\sin(1.31 \cdot u)+0.7 \cdot \sin(1.1 \cdot u)+0.7 \cdot \sin(1 \cdot u)$

modulating signal — modulated signal — undersampled signal

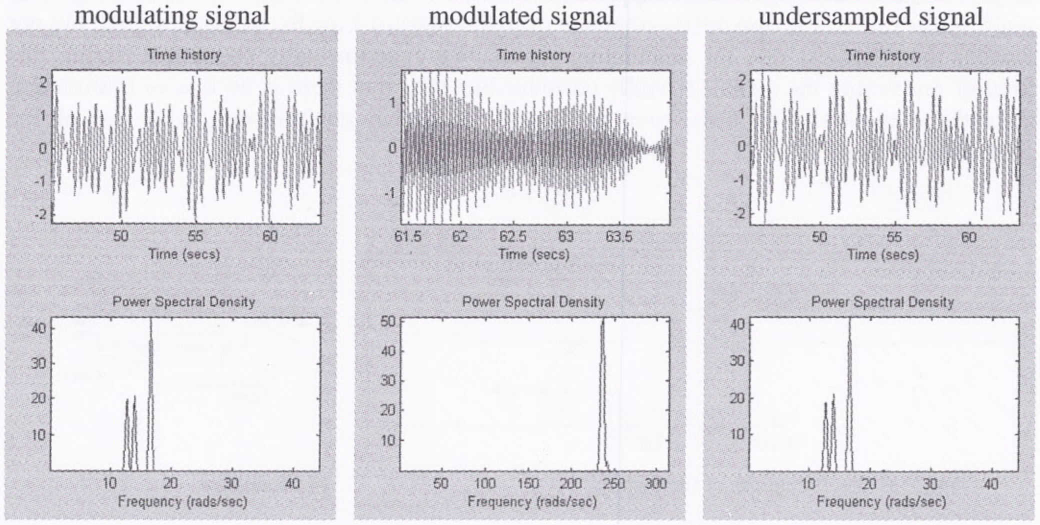

Fig. 3. Sample simulation resuts (upper figures present time waveforms, lower ones – its power spectral density)

Apart from the above signals we tested many signals with different frequencies and amplitudes, and every time we got satisfactory results. The experiments show that input signal can have any shape and it will be recovered without loss of information.

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