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## Sensitivity problems of the coal flotation process control system

### Introduction

One of the primary tasks of the control system is to maintain the value of the controlled variable (process output) at the desired level. An important problem from the point of view of the control quality is also getting the appropriate course of the controlled variable and reduce the influence of disturbances running on the system. Compliance with these requirements may be accomplished by appropriate choice of the structure of the controller and its settings. Many methods of tuning the controller use a dynamic model of a control object (determined on the basis of dynamic characteristics of the controlled process). The description of the properties of the dynamic process by using a mathematical model is helpful in the design of the control system. Knowing the dynamic model and its parameters, we can analytically determine the controller settings for the assumed criterion of the control quality. Different processes with the same dynamic properties can be described by a dynamic model of the same structure which differ only in parameter values. Such an approach makes it possible to apply the same type of controller in the control systems of various industrial processes. This also applies to the processes of coal enrichment.

The dynamic properties of many processes of coal enrichment can be characterized by the inertial model of the first order (described time constant  $T$ ) with time delay  $\tau$ . The transfer function of such a system in the form of the operator  $s$  can be written as:

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$$Y(s) = \frac{ke^{-s\tau}}{sT+1}U(s) \quad (1)$$

- ↪  $Y(s)$  – the output value of the controlled object,  
 $U(s)$  – the input value of the controlled object,  
 $k$  – the gain of the object.

On the basis of several works concerning the coal flotation process (Joostberens 2011; Kalinowski 1991; Kalinowski and Kaula 2000) it can be concluded that the dynamic of the flotation process for the SISO system may be represented by the model of the inertial element with a time delay. In this case, the flow intensity of a flotation reagent is the input signal  $U(s)$ . The ash content in flotation tailings is the output signal  $Y(s)$ . It is assumed that the basic signal transfer functions of the jig may also be described by the first-order inertial element with a time delay (Cierpisz 2012; Cierpisz and Kaula 2013). For example, the dynamic characteristic of the receiving zone of the underflow in the jig can be described by the formula (1) where the output variable  $Y(s)$  is the separation density in the jig while the input variable  $U(s)$  is the flow intensity of the lower product. An analogous description of the system dynamics is used for system of a coal blend of two components: concentrate with a specific content of ash and raw coal. In this system the output variable  $Y(s)$  is the ash content in the coal blend and the input variable  $U(s)$  is the flow rate of the raw coal. From the point of view of the dynamics such technological system (Cierpisz 2003) is a serial connection of the inertial element (properly dosed components of the blend) and transport delay (transport of material on the conveyor belt).

In many industrial processes use the PI (PID) structure of controllers. Extensive use of these controllers is a result a number of advantages, which are characterized. The choice of the structure of the PI or PID controller depends on many factors. The most important are the dynamic properties of the controlled process, the disturbances affecting the process and the requirements for the control system (defined by indicators of control quality) (PN-88/M-4200). Due to a number of disturbances occurring in processes of coal enrichment, the use of the differential elements of the PID controller should be limited, for example, to stabilize the selected parameters of a local control. The use of the structure of the PI controller in the control system seems to be an advisable solution.

## 1. Selected methods of tuning the PI controller

A block diagram of the control system is presented in Figure 1. The basic elements of this system are presented in the Laplace form.

The transfer function  $K(s)$  describes the dynamics of the process, according to formula 1. The PI controller has the structure given by the transfer function  $K_R(s)$ :

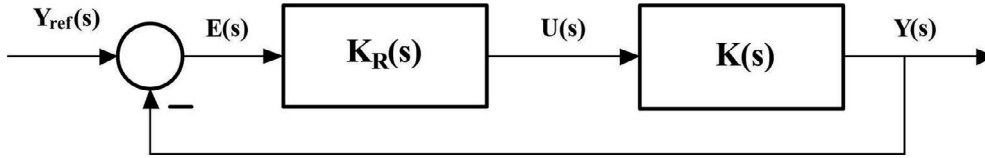


Fig. 1. A block diagram of control system

Rys. 1. Schemat blokowy układu regulacji

$$K_R(s) = k_r \left( 1 + \frac{1}{T_i s} \right) = k_r \left( \frac{T_i s + 1}{T_i s} \right) \quad (2)$$

↳  $k_r$  – proportional gain,  
 $T_i$  – integral time.

Thus, the transfer function of the closed system in the Laplace form is as follows:

$$K_{cl}(s) = \frac{K(s)K_R(s)}{1 + K(s)K_R(s)} \quad (3)$$

In order to obtain appropriate process control quality, the tuning of the controller parameters is an important problem. One of the basic methods of tuning of the PI controller is the Ziegler-Nichols identification test method (Ziegler and Nichols 1942). In this method, the dynamics of the process (even higher complex order systems) can be described approximately by means of first order inertial model with time delay. Thus ZN method may be the initial method of tuning for many processes of coal enrichment where the dynamics has the properties of the inertial system of the first order with a time delay.

Many authors dealing with the tuning of PI controllers parameters for processes with dynamics of the first order inertial model propose settings based on the reduction of the time constant (O'Dwyer 2003). In these methods, it is assumed that parameter  $T_i$  of the integral term is equal to the time constant  $T_i = T$  of the system. Differences in the settings are the result of the adopted tuning criteria of parameter  $k_r$ . One of the basic criteria used for tuning the controller settings is related to the transient nature of the controlled variable (Cierpisz and Kaula 2013).

In the paper (Kaula 2015) a new approach for tuning PI controller for an inertial object with a time delay has been proposed. An additional criterion was used in the direct method of controller tuning. The condition on the phase margin from the Nyquist criterion was applied. A comparison of the simulation results (for selected indices of control quality) of the above mentioned methods points to the clear advantages of the direct method with the additional condition on the phase margin.

Table 1 shows the relations between the parameter values of the PI controller and parameter values of the object transfer function in these tuning methods.

Table 1. Settings of PI controller parameters

Tabela 1. Nastawy parametrów regulatora PI

Tuning method	$k_r$	$T_i$
Ziegler-Nichols (ZN)	$0.9 \frac{T}{k\tau}$	$3.33\tau$
Reduction of time constant with condition of transient nature of the controlled variable (R)	$0.34 \frac{T}{k\tau}$	$T$
Direct method with condition of margin phase (B)	$0.52 \frac{T}{k\tau}$	$T$

It should be noted that the identification of the object parameters (including the time constant) in industrial conditions is usually performed during normal operation (with the influence of disturbances). Thus, the determined parameters of the dynamic model may be different from the actual values of the process. The control system with the controller which is tuned on the basis of such the model may not satisfy the assumed requirements of the control quality. The paper analyzes the impact of changes of the object model parameters on the course of the controlled variable. The sensitivity analysis have been used in the studies.

Sensitivity of the controlled variable on changes of the object parameters

One of the important indicators to assess the control quality is a parameter describing the sensitivity of the controlled variable on the change in the object parameters. Changes of two parameters  $\alpha = \{T, \tau\}$  for the object described by formula 1 were analyzed.

The controlled variable in the Laplace form according to the block diagram (Fig. 1) and formula 3 can be written:

$$Y(s) = K_{cI}(s, \alpha) Y_{ref}(s) \quad (4)$$

Thus changes of the controlled variable to the changes of the model parameter  $\alpha = \{T, \tau\}$  are as follows:

$$Y(s, \alpha) + \Delta Y(s, \alpha) \equiv K_{cI}(s, \alpha) Y_{ref}(s) + \Delta K_{cI}(s, \alpha) Y_{ref}(s) \quad (5)$$

Relative sensitivity of the transfer function of the closed system in according to the parameter  $\alpha = \{T, \tau\}$  can be expressed by the formula:

$$S_{K_{cl}(\alpha)} = \frac{\frac{\Delta K_{cl}(s, \alpha)}{K_{cl}(s, \alpha)}}{\frac{\Delta \alpha}{\alpha}} \cong \frac{\alpha}{K_{cl}(s, \alpha)} \frac{\partial K_{cl}(s, \alpha)}{\partial \alpha} \quad (6)$$

Calculating the partial derivative we obtain the relationship:

$$\frac{\partial K_{cl}(s, \alpha)}{\partial \alpha} = \frac{\partial \left( \frac{K(s, \alpha) K_R(s)}{1 + K(s, \alpha) K_R(s)} \right)}{\partial \alpha} = \frac{\frac{\partial K(s, \alpha)}{\partial \alpha} K_R(s)}{(1 + K(s, \alpha) K_R(s))^2} \quad (7)$$

Relative sensitivity of the transfer function of the object model in according to the parameter  $\alpha = \{T, \tau\}$ :

$$S_{K(\alpha)} = \frac{\frac{\Delta K(s, \alpha)}{K(s, \alpha)}}{\frac{\Delta \alpha}{\alpha}} = \frac{\alpha}{K(s, \alpha)} \frac{\Delta K(s, \alpha)}{\Delta \alpha} \cong \frac{\alpha}{K(s, \alpha)} \frac{\partial K(s, \alpha)}{\partial \alpha} \quad (8)$$

Thus the relative sensitivity of the transfer function of the closed loop system can be written as a function of the sensitivity of the object model:

$$S_{K_{cl}(\alpha)} = \frac{\alpha}{K_{cl}(s, \alpha)} \frac{\frac{\partial K(s, \alpha)}{\partial \alpha} K_R(s)}{(1 + K(s, \alpha) K_R(s))^2} = S_{K(\alpha)} \frac{1}{(1 + K(s, \alpha) K_R(s))} \quad (9)$$

The following are obtained on the basis of formulas 8 and 9:

$$\frac{\frac{\Delta K_{cl}(s, \alpha)}{K_{cl}(s, \alpha)}}{\frac{\Delta \alpha}{\alpha}} = \frac{\frac{\Delta K(s, \alpha)}{K(s, \alpha)}}{\frac{\Delta \alpha}{\alpha}} \frac{1}{(1 + K(s, \alpha) K_R(s))} \quad (10)$$

After simplification:

$$\Delta K_{cl}(s, \alpha) = \frac{K_R(s)}{(1 + K(s, \alpha) K_R(s))^2} \Delta K(s, \alpha) \quad (11)$$

The following are obtained on the basis of formulas 5 and 11:

$$\Delta Y = \left[ \frac{K_R(s)}{(1 + K(s, \alpha)K_R(s))^2} \Delta K(s, \alpha) \right] Y_{ref}(s) \quad (12)$$

$$\Delta K(s, \alpha_T) = \frac{k}{T_2 s + 1} e^{-\tau_2 s} - \frac{k}{T_1 s + 1} e^{-\tau_1 s}$$

$$\Delta K(s, \alpha_\tau) = \frac{k}{T s + 1} e^{-\tau_2 s} - \frac{k}{T s + 1} e^{-\tau_1 s}$$

$T_1, \tau_1$  – parameter values for which the controller parameters have been determined,  
 $T_2, \tau_2$  – actual values of the system parameters.

### 3. The sensitivity analysis of the control system in selected coal enrichment processes

The sensitivity analysis of the control system to changes in the object parameters was carried out for the dynamic characteristics of the coal flotation process (dynamic model). The parameters of the dynamic model were identified on the basis of industrial research of the flotation circuit. Data for the analysis from the work (Joostberens 2011) were taken.

In this work the dynamic model parameters of the coal flotation process using various methods of identification were determined. The input variable was assumed as the flow rate of the flotation reagent and the output variable was assumed as the ash content in the flotation tailings. Depending on the chosen method of identification and the set point of the system, different values of system parameters (time constant  $T$  and the delay  $\tau$ ) were obtained. The time constant  $T$  in the range of values (200–800) s, while the time delay  $\tau$  in the range of values (200–400) s have been identified.

It is therefore advisable to analyze the control system for the process with parameters that differ even tens of percent from the parameters used in the controller settings.

The paper presents the sensitivity analysis for the three methods of tuning PI controller. Based on the relationships shown in Table 1 the values of the PI controller parameter settings (depending on the parameters  $k, T, \tau$  of the dynamics model) have been calculated. The simulation studies of the response of the control system on the step change of the set point were carried out. In each considered case a constant value gain  $k = 1$  of the object was assumed. The controller settings for  $T = 500$ s and  $\tau = 250$ s were calculated. The study on the system with parameters  $T \pm 0.5T$  and  $\tau \pm 0.5\tau$  were carried out. Changes in one and both parameters simultaneously were assumed. The results are presented in the form of a table and graph. To assess the control quality the criterion of the integral of the squared control error (in relative terms) and parameters: rise time  $t_n$  and settling time  $t_u$  are used.

The results are shown in Table 2.

The responses of the control system at a step excitation for the considered cases of the analysis in Figure 2 to Figure 9 are shown. The legend on the graphs are as follows: the response of the control system marked as  $y$ , the response of the control system with changes of the parameters of the dynamic model marked as  $y + dy$ .

Table 2. Indices of the control quality

Tabela 2. Wskaźniki jakości regulacji

No.	$\Delta T$	$\Delta \tau$	$\frac{T \pm \Delta T}{\tau \pm \Delta \tau}$	Integral criterion in the relative terms ( $I_2$ )/( $I_{2min}$ ), %			Rise time $t_n$ , s (Settling time $t_u$ , s)		
				ZN	R	B	ZN	R	B
1	+0.5T	0	3	190	295	237	343 (1200)	1015 (2861)	557 (2400)
2	-0.5T	0	1	267	208	170	136 (5200)	1324 (2100)	667 (1378)
3	0	+0.5 $\tau$	4/3	318	303	278	214 (4800)	686 (2640)	400 (2520)
4	0	-0.5 $\tau$	4	117	211	151	786 (1757)	1324 (1862)	760 (1120)
5	+0.5T	+0.5 $\tau$	2	278	352	316	331 (3062)	920 (3280)	480 (2400)
6	+0.5T	-0.5 $\tau$	6	139	250	184	643 (1114)	1311 (1714)	827 (2193)
7	-0.5T	+0.5 $\tau$	2/3	unstable system	257	271	–	386 (2143)	240 (3680)
8	-0.5T	-0.5 $\tau$	2	100	171	119	124 (1945)	1600 (2400)	1000 (1680)

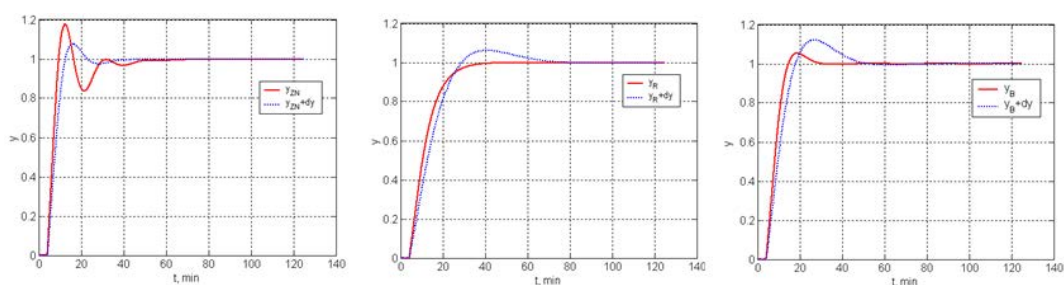


Fig. 2. The response of the control system at the step excitation for case 1

Rys. 2. Odpowiedź układu regulacji na skok jednostkowy dla przypadku 1

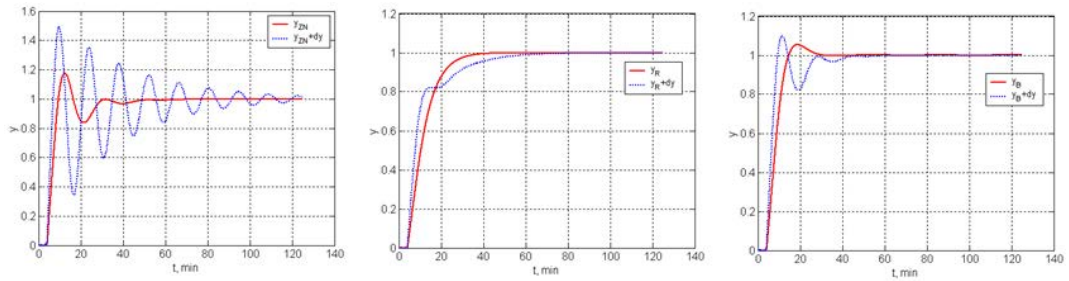


Fig. 3. The response of the control system at the step excitation for case 2

Rys. 3. Odpowiedź układu regulacji na skok jednostkowy dla przypadku 2

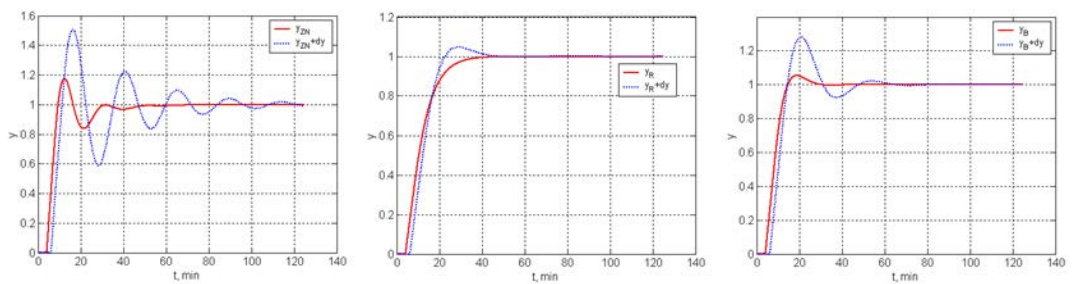


Fig. 4. The response of the control system at the step excitation for case 3

Rys. 4. Odpowiedź układu regulacji na skok jednostkowy dla przypadku 3

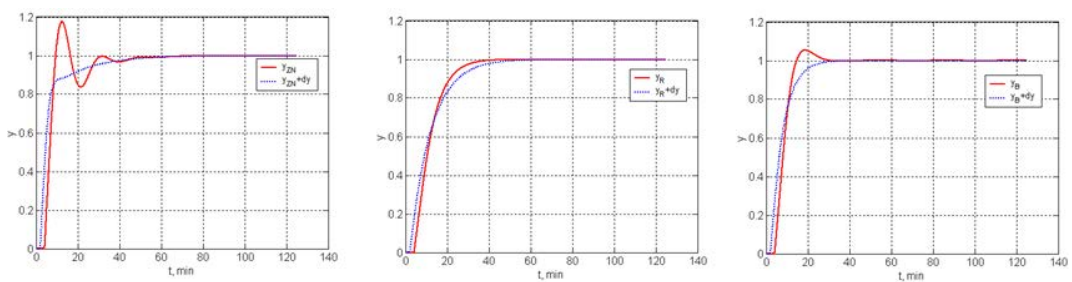


Fig. 5. The response of the control system at the step excitation for case 4

Rys. 5. Odpowiedź układu regulacji na skok jednostkowy dla przypadku 4



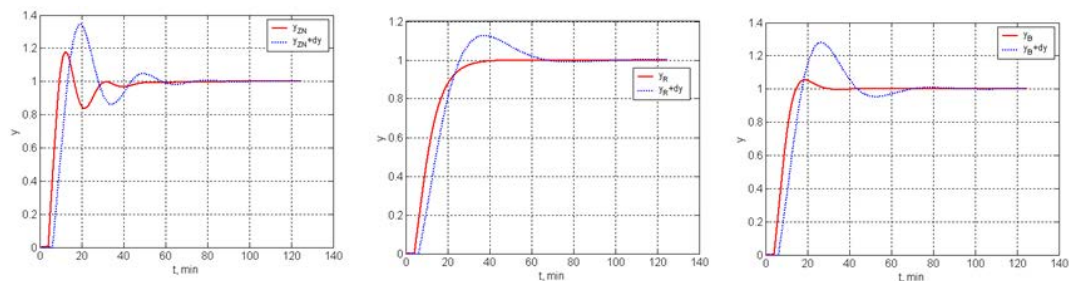


Fig. 6. The response of the control system at the step excitation for case 5

Rys. 6. Odpowiedź układu regulacji na skok jednostkowy dla przypadku 5

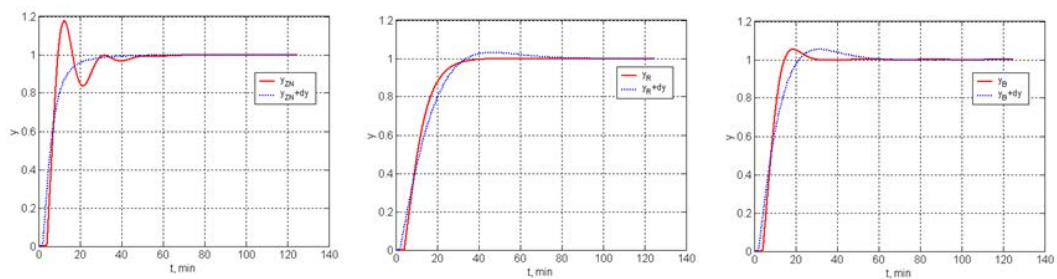


Fig. 7. The response of the control system at the step excitation for case 6

Rys. 7. Odpowiedź układu regulacji na skok jednostkowy dla przypadku 6

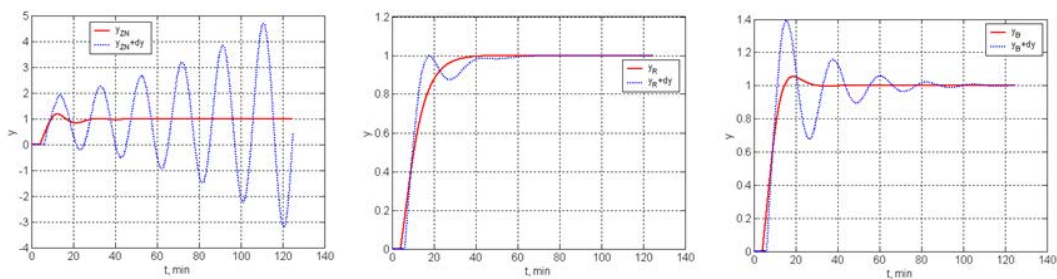


Fig. 8. The response of the control system at the step excitation for case 7

Rys. 8. Odpowiedź układu regulacji na skok jednostkowy dla przypadku 7

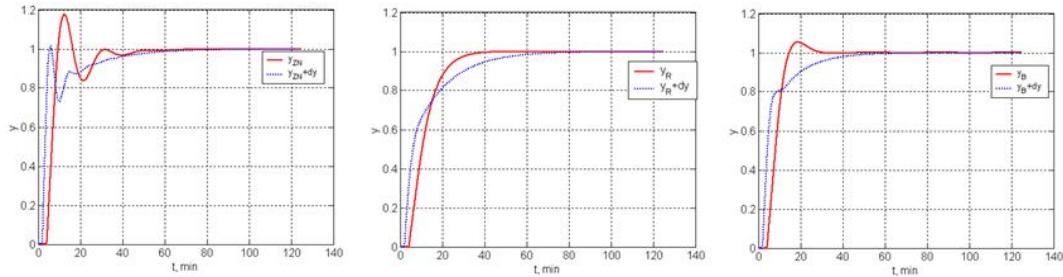


Fig. 9. The response of the control system at the step excitation for case 8

Rys. 9. Odpowiedź układu regulacji na skok jednostkowy dla przypadku 8

## Conclusions

The impact of changes of the parameters of the dynamic model of the system on the course of the controlled variable were analyzed. The sensitivity analysis has been used in the studies. The sensitivity analysis for three methods of the tuning PI controller in the control systems of coal preparation processes characterized by the dynamic properties of the inertial system with time delay was carried out. The controller parameters settings by the parameter  $T$  and  $t$  were tuned. The studies of the control system at the object parameters which differed to a certain range from  $T$  and  $\tau$  (Table 2) were performed.

Responses of the control system (where the controller settings were designated in accordance with the formulas in Table 1) for a step change in the set point were obtained. Based on the courses (Fig. 2 to Fig. 9) it can be noted that the controlled variable takes a steady state value for all considered cases (except one) regardless of the choice of the method of the parameter settings of the PI controller. This shows a large robustness of the PI controller on the disturbances connected with the change of the dynamic properties of the object.

The system is unstable only at controller settings according to the ZN criterion for case 7 (Table 2, Fig. 8). In this case the ratio  $((T - \Delta T)/(\tau - \Delta\tau)) = 2/3$  is less than the ratio  $T/\tau = 2$  for which settings of the PI controller were tuned. The situation in which the ratio of  $((T - \Delta T)/(\tau - \Delta\tau))$  is less than the  $T/\tau$  ratio also refers to cases 2 and 3. This ratio is 1 and  $4/3$  adequately. In these cases we may notice the presence of the larger oscillations of the controlled variable (Fig. 3 and Fig. 4, adequately).

The best results of the control (the control quality) from the point of view of the integral index for cases 4 and 8 were obtained. This occurs when the parameter of the time delay  $\tau$  (accepted for the settings) is larger than the actual time delay of the object. Analyzing the transient nature of the controlled variable and the quality parameters, it should be noted that

in these cases better control quality was achieved than at the control system in which the controller settings for the actual object parameters were determined.

This is due to the fact that the time delay of the system has a negative effect on the dynamic properties of the control system and thus the control quality. Comparing indicators of the control quality for the analyzed methods of the tuning of the controller settings you will notice the largest variability for the ZN method and the smallest for the method of reduction (R). The transients of the controlled variable for the tuning method of ZN in most cases are oscillatory. The oscillations are not present in the transients of the controlled variable for the method of the reduction (R). This is achieved at the expense of significant deterioration of other quality parameters. The transients have the longest settling time and rise time for the settings according to the criterion of the reduction (R).

The value of the integral index is the smallest in five cases for the method of ZN and in two cases for the direct method (B). The integral index for the direct method (B) and method of the reduction (R) is comparable at the case 7 where system is unstable for the method ZN. In most cases the shortest settling time of the transient in the direct method (B) was obtained.

The assumption of a larger time delay in the controller settings relative to the real delay of the system (identified parameter larger than actual) improved the quality of the control.

The obtained results (Fig. 4, 6, 8) in identifying of the parameters of the system model and a new approach to the tuning of the PI controller can be very helpful. The research results for another cases should to be verified. Thus the further research should analyze the impact of changes in the object model parameters on the course of controlled variable for the ratio  $T/\tau$  different from the value 2.

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#### SENSITIVITY PROBLEMS OF THE COAL FLOTATION PROCESS CONTROL SYSTEM

##### Keywords

coal enrichment, dynamic model, control system of coal enrichment processes, sensitivity of control system

##### Abstract

Control of the technological processes of coal enrichment takes place in the presence of wide disturbances. Thus, one of the basic tasks of the coal enrichment process control systems is the stabilization of coal quality parameters at a preset level. An important problem is the choice of the controller which is robust for a variety of disturbances. The tuning of the controller parameters is no less important in the control process. Many methods of tuning the controller use the dynamic characteristics of the controlled process (dynamic model of the controlled object). Based on many studies it was found that the dynamics of many processes of coal enrichment can be represented by a dynamic model with properties of the inertial element with a time delay. The identification of object parameters (including the time constant) in industrial conditions is usually performed during normal operation (with the influence of disturbances) from this reason, determined parameters of the dynamic model may differ from the parameters of the actual process. The control system with controller parameters tuned on the basis of such a model may not satisfy the assumed control quality requirements.

In the paper, the analysis of the influence of changes in object model parameters in the course of the controlled value has been carried out. Research on the controller settings calculated according to parameters  $T$  and  $\tau$  were carried out on objects with other parameter values. In the studies, a sensitivity analysis method was used. The sensitivity analysis for the three methods of tuning the PI controller for the coal enrichment processes control systems characterized by dynamic properties of the inertial element with time delay has been presented. Considerations are performed at various parameters of the object on the basis of the response of the control system for a constant value of set point. The assessment of considered tuning methods based on selected indices of control quality have been implemented.

## ZAGADNIENIA WRAŻLIWOŚCI UKŁADU REGULACJI PROCESU FLOTACJI WĘGLA

## Słowa kluczowe

wzbogacanie węgla, dynamika obiektu, układy regulacji procesów wzbogacania węgla, wrażliwość układu regulacji

## Streszczenie

Sterowanie procesów technologicznych wzbogacania węgla odbywa się w obecności licznych zakłóceń. Zatem jednym z podstawowych zadań układów regulacji procesów wzbogacania węgla jest stabilizacja parametrów jakościowych na zadanym poziomie. Istotnym problemem jest wybór regulatora odpornego na różnorodne zakłócenia. Niemniej ważnym zagadnieniem w regulacji procesu jest dobór nastaw regulatora. W wielu metodach doboru nastaw regulatora wykorzystuje się charakterystyki dynamiczne sterowanego procesu (model dynamiczny obiektu sterowania). Na podstawie badań stwierdzono, że dynamika wielu procesów wzbogacania węgla może być przedstawiona za pomocą modelu o właściwościach elementu inercyjnego z opóźnieniem czasowym. Identyfikacja parametrów obiektu (w tym stałej czasowej) w warunkach przemysłowych realizowana jest zwykle w trakcie normalnej eksploatacji (z oddziaływaniem zakłóceń), tym samym wyznaczone parametry modelu dynamicznego mogą się różnić od wartości rzeczywistych procesu. Układ regulacji z nastawami regulatora dobranymi na podstawie takiego modelu może nie spełniać założonych wymagań jakości regulacji.

W artykule przeprowadzono analizę wpływu zmian parametrów modelu obiektu na przebieg wielkości regulowanej. Dla nastaw regulatora wyznaczonych według parametrów  $T$  i  $\tau$  przeprowadzono badania na obiekcie o parametrach różniących się w pewnym zakresie. W badaniach zastosowano analizę wrażliwości. Przedstawiono analizę wrażliwości dla trzech metod doboru nastaw regulatora PI układów regulacji procesów wzbogacania węgla charakteryzujących się właściwościami dynamicznymi obiektu inercyjnego z opóźnieniem. Rozważania przeprowadzono dla różnych parametrów obiektu, na podstawie odpowiedzi układu regulacji dla stałej wartości wielkości zadanej. W podsumowaniu dokonano oceny rozpatrywanych metod doboru w odniesieniu do wybranych wskaźników jakości regulacji.

