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Application of Self-Adaptive Population Rao Algorithms to Optimization of Steel Grillage Structures

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The self-adaptive population Rao algorithms (SAP-Rao) are employed in this study to produce the optimal designs for steel grillage structures. The size variables in the optimization problem consist of the cross-sectional area of the discrete W-shapes of these beams. The LRFD-AISC design code was used to optimize the constrained size of this kind of structure. The solved problem's primary goal is to determine the grillage structure's minimum weight. As constraints, it is decided to use the maximum stress ratio and the maximum displacement at the inner point of the steel grillage structure. The finite element method (FEM) was employed to compute the moment and shear force of each member, as well as the joint displacement. A computer program for the study and design of grillage structures, as well as the optimization technique for SAP-Rao, was created in MATLAB. The outcomes of this study are compared to earlier efforts on grillage structures. The findings demonstrate that the optimal design of grillage structures can be successfully accomplished using the SAP-Rao method described in this paper.

Keywords: minimum weight, grillage structures, layout and size optimization, discrete variables, SAP-Rao algorithms, MATLAB.

1. Introduction

In practice, engineers have always been interested in structurally optimal design. They have placed their emphasis on the geometry of structures and construction costs. Engineers have the responsibility of designing structures that are both highly reliable and cost-effective. Numerous algorithms, both traditional and cutting-edge, have been investigated to accomplish these goals. Over time, numerous metaheuristic approaches have been introduced and employed for structural optimization problems. Some of the most well-known optimization algorithms include genetic algorithm (GA), tabu search (TS), teaching-learning-based optimization (TLBO), harmony search (HS), and artificial bee colony (ABC). Genetic algorithms are rooted in Darwin's theory of evolution [18]. These algorithms start with a randomly generated initial population representing a set of possible solutions related to the problem at hand. In each generation of the optimization process, biological operators are used to create the next population with the hope that it will be better than its predecessor.

HS was first proposed in the dissertation by Geem [13], and then presented in a journal paper by the Geem *et al.* [12]. It draws inspiration from a phenomenon found in music, namely the process of searching for better harmony. The ABC algorithm [19] simulates the intelligent foraging behavior of a honey bee swarm, defining three types of bees: employed bees, unemployed bees, and scout bees. Employed bees search for food around the food source and store the nectar. In a similar manner, musical performances strive to achieve the best state determined by aesthetic estimation, while optimization algorithms aim to achieve the best state determined through objective function evaluation.

The TS algorithm [4] explores the search space by a sequence of moves. To escape the local optimum, the certain moves are recorded in a memory called the forbidden (tabu) search. This algorithm contains several elements: a tabu list, neighborhood, aspiration criterion, termination criterion, and cost function. TLBO [26] consists of two phases: teacher phase and learner phase. In the first phase, the best solution with the minimum objective function is defined as the teacher. By using the mean solution and a teaching factor, the new solutions are generated in the vicinity of the teacher. If a new solution is better than the old one, the new solution replaces the old one. In the learner phase, the solutions obtained from the learner phase are called students.

Grillage systems are widely used in structures to cover large areas. These structures are generally optimized for the minimum weight of the total structures by selecting a discrete set from the available steel profiles. The displacement of the middle point and the stress ratio of all grillage members are taken as constraint in the optimization problem. A grillage structure is a planar structural system composed of longitudinal and transverse beams, loaded perpendicular to the plane. The major considerations for a grillage design are the number of beams in both directions and the cross-sectional diameters of these beams. However, the majority of earlier investigations on grillage optimization solely focused on examining the dimensions of the beams' cross sections (Saka [29], Saka and Erdal [31], Erdal *et al.* [11], Kaveh and Talatahari [20], Kaveh *et al.* [21], Dede [6, 7]).

A few analyses have been conducted on layout optimization for grillage structures. Saka *et al.* [30] developed a grillage system with the best possible spacing configuration. An optimum design method based on the HS algorithm was studied by Erdal and Saka [10]. The size and configuration of grillages were simultaneously optimized using JayaX, a binary variation of the Jaya algorithm developed by Aydin [3]. Lewiński and Telega [22] generalized the Michell theory of plane pseudo-continua to antiplane problems in which the loading is perpendicular to plane of the structure.

The Jaya algorithm, a popular and effective metaheuristic optimization technique, was introduced by Rao [25]. In Grzywiński *et al.* [14], braced dome structures were designed by using the Jaya method in the best possible way with regard to natural frequency limitations. Atmaca *et al.* [2] employed the Jaya method to optimize the size of the cables in a cable-stayed bridge. Grzywiński [16] used the Jaya method to demonstrate shape and size optimization of trusses with dynamic constraints. Another study was conducted by Pokusiński and Kamiński [23], where the authors explored lattice dome reliability using perturbation-based approaches vs. a semi-analytical method. Bołbotowski *et al.* [5] presented a novel numerical procedure that takes advantage of the adaptive solution scheme previously developed for truss layout optimization problems, enabling the solution of very large-scale problems.

This paper's major objective is to present an optimization procedure that will reduce the overall weight of grillage structures. Displacements, moments, and shear force limitations are considered during the optimization process according to the American Institute of Steel Construction load and resistance factor design (AISC-LRFD) specification [1]. The optimization procedure for sizing grillage design employs a set of recently developed metaheuristic optimization algorithms known as Rao algorithms. Rao [27] created these simple, metaphor-free optimization methods to determine solutions for constrained and unconstrained optimization problems. These algorithms are built using the best and worst solutions, discovered during optimization phases along with random interactions between possible solutions.

Rao algorithms were used to optimize braced barrel vault by Dede *et al.* [8] and Grzywiński *et al.* [15]. Dede *et al.* [9] used Rao algorithms for the optimization of small, medium and large braced domes, while Grzywiński [17] employed Rao algorithms for optimizing spatial truss towers.

2. Self-adaptative population Rao algorithms

Rao developed several successful optimization algorithms such as TLBO (Rao *et al.* [24]), Jaya (Rao [25]) and Rao-1, Rao-2 and Rao-3 (Rao [27]). In the work by Rao and Keesari [28] a new self-adaptive-population Rao algorithm was presented. In their study, the authors propose a new algorithm named Rao-4. Similar to the previous three Rao algorithms, Rao-4 is characterized by its lack of algorithm-specific parameters and metaphorical explanations:

$$S_{n,m,i}^{\text{new}} = S_{n,m,i} + r_{1,n,i}\left(S_{n,\text{best},i} - S_{n,\text{worst},i}\right), \tag{1}$$

$$\begin{aligned} S_{n,m,i}^{\text{new}} = S_{n,m,i} &+ r_{1,n,i}\left(S_{n,\text{best},i} - S_{n,\text{worst},i}\right) \\ &+ r_{2,n,i}\left(|S_{n,m,i} \text{ or } S_{m,l,i}| - |S_{n,l,i} \text{ or } S_{n,m,i}|\right), \end{aligned} \tag{2}$$

$$\begin{aligned} S_{n,m,i}^{\text{new}} = S_{n,m,i} &+ r_{1,n,i}\left(S_{n,\text{best},i} - S_{n,\text{worst},i}\right) \\ &+ r_{2,n,i}\left(|S_{n,m,i} \text{ or } S_{n,l,i}| - (S_{n,l,i} \text{ or } S_{n,m,i})\right), \end{aligned} \tag{3}$$

$$\begin{aligned} S_{n,m,i}^{\text{new}} = S_{n,m,i} &+ r_{1,n,i}\left(S_{n,\text{best},i} - S_{n,\text{worst},i}\right) \\ &+ 0.5\left\{r_{2,n,i}\left(S_{n,\text{worst},i} - S_{n,m,i}\right) + r_{3,n,i}\left(S_{n,\text{best},i} \text{ or } S_{n,m,i}\right)\right\} \\ &- r_{4,n,i}\left(S_{n,\text{worst},i} - S_{n,m,i}\right), \end{aligned} \tag{4}$$

where n, m and i represent the n-th variable during the i-th iteration, m-th solution, and the iteration number, respectively. $S_{n,m,i}^{\text{new}}$ is the new solution that can be generated using Rao algorithms, and $S_{n,\text{best},i}$ and $S_{n,\text{worst},i}$ are the best and worst solutions in the current population.

The variables $r_{1,n,i}$, $r_{2,n,i}$, $r_{3,n,i}$, and $r_{4,n,i}$ are random numbers generated in the range [0, 1]. These equations are used to create a new solution with the expectation that the new solution will be better than the previous one.

The interaction between the current solution m-th and a randomly selected l-th solution from the current population is shown by the third term on the right side of Eqs. (2) and (3). The S values of the current m-th and l-th randomly selected solutions define these two terms. If the S value of the current solution is superior to that of the randomly selected solution, the third term in Eq. (2) becomes $r_{2,n,i}\left(|S_{n,m,i}| - |S_{n,l,i}|\right)$ and in Eq. (3), it becomes $r_{2,n,i}\left(|S_{n,m,i}| - (S_{n,l,i})\right)$. Similarly, if the randomly selected solution has a superior S value to the current solution, the third term in Eq. (2) becomes $r_{2,n,i}\left(|S_{n,l,i}| - |\,S_{n,m,i}|\right)$ and in Eq. (3), it becomes $r_{2,n,i}\left(|S_{n,l,i}| - (S_{n,m,i})\right)$.

Based on interactions between the current solution and the best, worst, and randomly selected solutions, the Rao algorithms reconfigure the population in the search space. The Rao-1 method only allows the best and worst solutions in the population to interact with the current solution. In the Rao-2, Rao-3, and Rao-4 algorithms, the current solution will interact with the best and worst solutions as well as a randomly chosen solution from the population.

In SAP-Rao algorithms, the following adjustments are applied to the fundamental Rao algorithms:

1. With respect to the quality of the solutions, the proposed SAP-Rao algorithms divide the overall population into four sub-populations. Instead of concentrating on a single area, this approach disperses the solutions across

the search field. As a result, it is anticipated that the proposed algorithms will produce the best results.

2. Based on the quality of fitness values, SAP-Rao algorithms adaptively adjust the size of sub-populations during the search process. This implies that there will be an increase or reduction in the size of sub-populations. With the use of this function, the search process may be aided in finding the best option while also broadening its scope. In order to maintain diversity and improve the exploration process, duplicate solutions are furthermore replaced with newly created solutions.

Readers can refer to the website [32] for additional information regarding Rao algorithms.

3. Finite element analysis of grillage structures

Grillages are widely employed in a variety of structures, including bridge decks, ship hulls, airplane wings, building floors, slabs over water tanks, and particularly in column-free roofs for large spaces. These structures are typically planar and consist of many parallel beams oriented in two directions. The fact that grillages are loaded perpendicular to their planes is another characteristic feature of them. Figure 1 shows a typical steel grillage structure made using steel W-sections. The beams are oriented here both longitudinally and transversely.

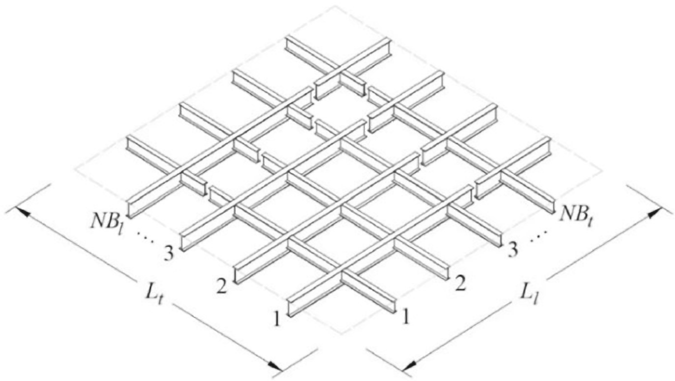

Fig. 1. A typical steel grillage structure.

The structure is analyzed with the matrix displacement technique. Three degrees of freedom are taken at each point. Figure 2 depicts the end forces and end deformations for a grillage component in the local coordinate system.

Figure 2 shows the shear force as Q_z, the torsional and bending moments as M_x and M_y, respectively, the linear displacement as delta δ_z, and the angular

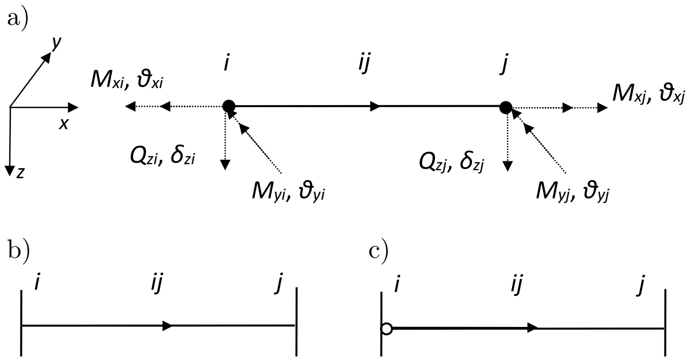

FIG. 2. A grillage member: a) degrees of freedom, b) connect fix-fix, c) connect pic-fix.

displacements as theta x and theta y. The relationship between end forces (f) and end deformations (u) for a grillage member (k) based on the element stiffness matrix is given as:

$$\{f\} = [K]\{u\}, \tag{5}$$

where the vectors and matrix are given:

$$\{f\} = \left\{ M_{xi} \;\; M_{yi} \;\; Q_{zi} \;\; M_{xj} \;\; M_{yj} \;\; Q_{zj} \right\}^T, \tag{6}$$

$$[K] = \begin{bmatrix} \frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & \frac{4EI}{L} & \frac{6EI}{L^2} & 0 & \frac{2EI}{L} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & -\frac{12EI}{L^3} \\ -\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & \frac{2EI}{L} & \frac{6EI}{L^2} & 0 & \frac{4EI}{L} & -\frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & -\frac{12EI}{L^3} & 0 & -\frac{6EI}{L^2} & \frac{12EI}{L^3} \end{bmatrix}, \tag{7}$$

$$\{u\} = \left\{ \theta_{xi} \;\; \theta_{yi} \;\; \delta_{zi} \;\; \theta_{xj} \;\; \theta_{yj} \;\; \delta_{zj} \right\}^T, \tag{8}$$

where G is the shear modulus, J is the torsional constant, L is the length of grillage member, E is the modulus of elasticity, and I is the moment of inertia.

4. Grillage structure optimization

The overall structural weight is one of the most critical aspects in structural design. There are cross-sectional areas defined in the AISC-LRFD [1] standards that are used to minimize the weight of the structure and fulfill the behavioral and performance limits outlined in AISC-LRFD. The grillage structures' objective function to achieve this goal is as follows:

$$\text{minimize} \quad W = \sum_{k=1}^{ng} A_k \sum_{i=1}^{nk} \rho \cdot L_i, \tag{9}$$

where W is the objective function, ρ is the density of materials, A is the cross-section area of the each member, nk is the number of member belonging to group k ($k = 1/ng$) in grillage structures, and ng is the number of groups. The optimization also considers

$$\delta_i \leq \delta_u, \qquad c_{\delta,i} = \left|\frac{\delta_i}{\delta_u}\right| - 1 \leq 0, \qquad i = 1, 2, ..., np, \tag{10}$$

$$M_{u,j} \leq (\varphi M_{n,j}), \qquad c_{m,j} = \left|\frac{M_{u,j}}{\varphi M_{n,j}}\right| - 1 \leq 0, \qquad j = 1, 2, ..., nm, \tag{11}$$

$$V_{u,j} \leq (\varphi V_{n,j}), \qquad c_{v,j} = \left|\frac{V_{u,j}}{\varphi V_{n,j}}\right| - 1 \leq 0, \qquad j = 1, 2, ..., nm, \tag{12}$$

where δ_i and δ_u are the computed and permitted displacement, respectively, np is the total number of points with restricted displacements, $M_{u,j}$ and $V_{u,j}$ are the factored service load moment and the factored service load shear for member j, with resistant factor φ equal to 0.9, and nm being the total number of members in the grillage, respectively.

It is necessary to modify the objective function to include limitations. A penalty function that calculates the value of constraint violation is established with this objective in mind. The objective function is converted to a constrained function using this function.

The penalty function is defined as:

$$C = \sum_{i=1}^{np} c_{\delta,i} + \sum_{j=1}^{nm} c_{m,j} + \sum_{j=1}^{nm} c_{v,j}. \tag{13}$$

By including the penalty function, the objective function becomes a penalized objective function (PF):

$$PF = W[1 + P \cdot C], \tag{14}$$

where P is a constant in the positive range, which acts as a variable in each problem. The user can choose this constant, while considering the limitations. In this study, P is set to 10. The total number of violated constraints must be zero at the end of the optimization procedure. Consequently, the penalized objective function might then be set to match the overall weight of the structure. In other words, the algorithm seeks the optimal solution while complying with the limitations.

4.1. The nominal flexural strength

The nominal flexural strength is computed according to AISC-LRFD [1] as follows:

$$M_n = \begin{cases} M_p = F_y Z \leq 1.5 F_y S & \text{for } \lambda \leq \lambda_p, \\ M_p - (M_p - M_r)\left(\dfrac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right) & \text{for } \lambda_p < \lambda \leq \lambda_r, \\ M_{cr} = F_{cr} S & \text{for } \lambda > \lambda_r, \end{cases} \tag{15}$$

where M_p is plastic moment, F_y is the specified minimum yield strength, Z is the plastic section modulus, S is the section modulus, M_{cr} is the buckling moment, and M_r is the limiting buckling moment, namely:

$$M_r = \begin{cases} F_L S_x & \text{for FLB}, \\ R_e F_{yf} S_x & \text{for WLB}, \end{cases} \tag{16}$$

where F_L is the min$[(F_{yf} - F_r)$ or $F_{yw}]$, FLB is flange local buckling, WLB is web local buckling, F_r is the compressive residual stress in the flange (10 ksi or 69 N/mm^2) for rolled shapes, F_{yf} is the yield strength of the flange, F_{yw} is the yield strength of the web, and R_e is the hybrid girder factor (1.0 for non-hybrid girders). The critical stress F_{cr} is defined as:

$$F_{cr} = \frac{0.69E}{\lambda^2}, \tag{17}$$

where λ is a slenderness parameter computed as follows:

$$\lambda = \begin{cases} b_f/(2t_f) & \text{for flange}, \\ h/t_w & \text{for web}, \end{cases} \tag{18}$$

where b_f and t_f are the width and thickness of the flange, respectively, t_w is the thickness of the web, and h is the clear distance between flanges less the fillet or corner radius. The values of $b_f/(2t_f)$ and h/t_w can be determined from the tables for W-shapes:

$$\lambda_p = \begin{cases} 0.38\sqrt{\dfrac{E}{F_y}} & \text{for compressive flange,} \\ 3.76\sqrt{\dfrac{E}{F_y}} & \text{for web,} \end{cases} \tag{19}$$

$$\lambda_r = \begin{cases} 0.83\sqrt{\dfrac{E}{F_y}} & \text{for compressive flange,} \\ 5.70\sqrt{\dfrac{E}{F_y}} & \text{for web,} \end{cases} \tag{20}$$

where λ_p is the largest value of λ for which $M_n = M_p$, and λ_r is the largest value of λ for which buckling is inelastic.

As the nominal moment strength for the section under consideration, the minimum value of M_n calculated for the flange or web in accordance with the values of λ is chosen.

4.2. The nominal shear strength

The nominal shear strength is computed as:

$$V_n = \begin{cases} 0.6F_{yw}A_w & \text{for } \dfrac{h}{t_w} \le 2.45\sqrt{\dfrac{E}{F_{yw}}}, \\ 0.6F_{yw}A_w\left(2.45\sqrt{\dfrac{E}{F_{yw}}}\right)\Bigg/\dfrac{h}{t_w} & \text{for } 2.45\sqrt{\dfrac{E}{F_{yw}}} < \dfrac{h}{t_w} \le 3.07\sqrt{\dfrac{E}{F_{yw}}}, \\ A_w\,(4.52E)\Bigg/\left(\dfrac{h}{t_w}\right)^2 & \text{for } 3.07\sqrt{\dfrac{E}{F_{yw}}} < \dfrac{h}{t_w} < 260, \end{cases} \tag{21}$$

where A_w is the cross-sectional area of the web.

5. Numerical examples

Two grillage structures from the literature are taken into consideration to illustrate the effect of the presented algorithm. The material has the following characteristics: shear modulus of 81 kN/mm^2, yield stress of 250 MPa, and elasticity modulus of 205 kN/mm^2. A family of 272 W-sections is used to optimize these examples as a discrete set of available steel profiles. The allowable displacement for middle joints is 25 mm for all examples. The grillage area is uniformly loaded with a pressure of 15 $\mathrm{kN/m^2}$.

5.1. The 144 square meters grillage with pin supports

The grillage structures under consideration have dimensions of 12 m in both the transverse and longitudinal directions (L_x, L_y). Figure 3 depicts the overall model of such a grillage. It is estimated that all unsupported points add up to a total external load of 2160 kN. The example is conducted in two separate calculations: one with two design groups (Fig. 3), and another with four groups (Fig. 4). The size of initial populations is 20, and the maximum number of cycles is 100.

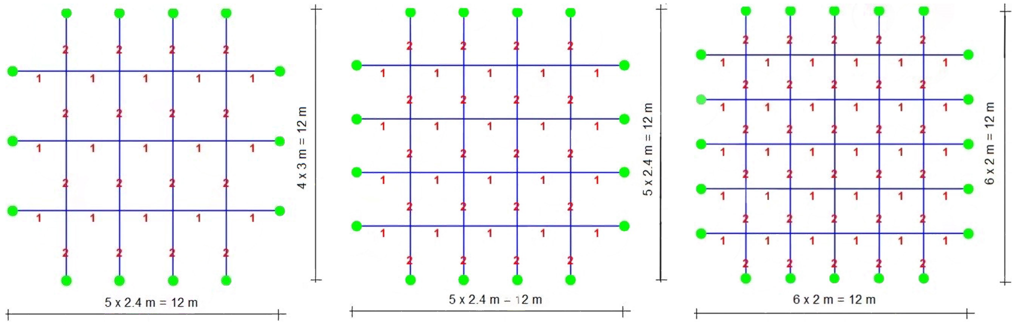

Fig. 3. Schematic of a model with two groups variables: case 1 (left), case 2 (middle), case 3 (right) grillage structure.

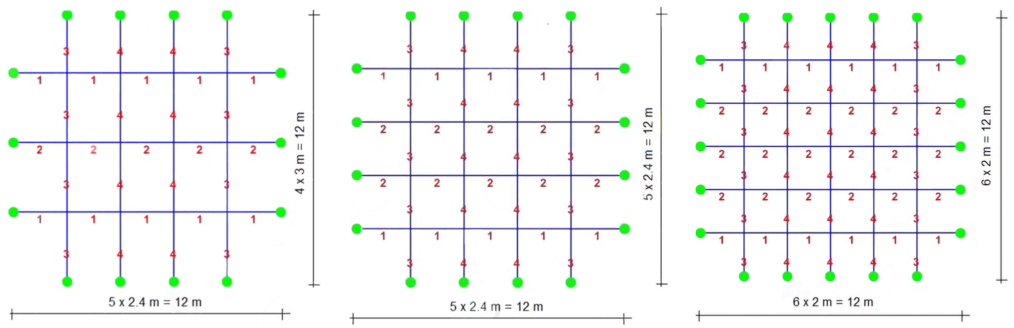

Fig. 4. Schematic of a model with four groups variables: case 1 (left), case 2 (middle), case 3 (right) grillage structure.

The results presented in Table 1 can be compared for the same number of beams, which are 4×4 and 5×5. For case 2, the weight is 10 297 kg, which is the same as Dede [6]. For case 3, the weight is 11 365 kg, which is better than

TABLE 1. Optimal results for the 144 square meters grillage structure with two groups variables.

Search method	HS Erdal and Saka [10]	CSS Kaveh and Talatahari [20]	TLBO Dede [6]	Jaya Dede [7]	SAP-Rao case 1	SAP-Rao case 2	SAP-Rao case 3
Beam numbers	5×5	5×5	4×4	3×3	3×4	4×4	5×5
Group 1	W200 × 22.5	W150 × 13.5	W150 × 13.5	W150 × 13.5	W150 × 13.5	W150 × 13.5	W150 × 13.5
Group 2	W610 × 217	W840 × 176	W920 × 201	W1000 × 249	W1000 × 222	W920 × 201	W840 × 176
$\delta_{\max}$ [mm]	25.0	24.2	21.5	21.2	22.9	21.5	23.5
Max strength ratio	0.48	0.54	0.53	0.58	0.60	0.53	0.53
Weight [kg]	14 384	11 358	10 297	9447	8649	10 297	11 365

TABLE 2. Results for the 144 square meters grillage structure with four groups variables.

Search method	CSS Kaveh and Talatahari [20]	TLBO Dede [6]	CSS Kaveh and Talatahari [20]	TLBO Dede [6]	SAP-Rao case 1	SAP-Rao case 2	SAP-Rao case 3
Beam numbers	7×7	7×7	5×5	5×5	3×4	4×4	5×5
Group 1	W150 × 13.5	W150 × 13.5	W150 × 13.5	W150 × 13.5	W150 × 13.5	W150 × 13.5	W150 × 13.5
Group 2	W740 × 176	W760 × 161	W920 × 201	W920 × 201	W1000 × 393	W530 × 66	W920 × 201
Group 3	W150 × 13.5	W150 × 13.5	W300 × 21	W310 × 23.8	W410 × 46.1	W310 × 44.5	W250 × 25.3
Group 4	W300 × 21	W250 × 17.9	W300 × 32.5	W310 × 23.8	W460 × 68	W1000 × 222	W310 × 28.3
$\delta_{\max}$ [mm]	24.4	23.4	24.3	24.2	24.3	24.5	24.2
Max strength ratio	0.75	0.92	0.99	0.99	0.97	0.82	0.99
Weight [kg]	11 548	11 329	9251	9153	7790	8306	9189

Erdal and Saka [10] at 14 384 kg, but worse than Kaveh and Talatahari [20] at 11 358 kg. The advantage of the current solution is the adoption of a different number of beams, namely 3×4, for which the results are better than before at 8649 kg.

The results in Table 2 can only be compared for the same number of beams, that is, for 5×5. For case 3, the weight is 9189 kg, which is better than the result of Kaveh and Talatahari [20] at 9251 kg, but unfortunately worse than Dede [6] at 9153 kg. The advantage of the current solution is the adoption of a different number of beams, i.e., 4×4 and 3×4, for which the results are better than the previous ones at 8306 kg and 7790 kg, respectively.

5.2. The 225 square meters grillage with fix supports

The grillage structures under study have dimensions of 15 m in both the longitudinal and transverse directions (L_x, L_y). The total force that all unsupported points share is 3375 kN. The example is calculated with four groups variables (Fig. 5). The initial population size is 20, and the maximum cycle number is 100.

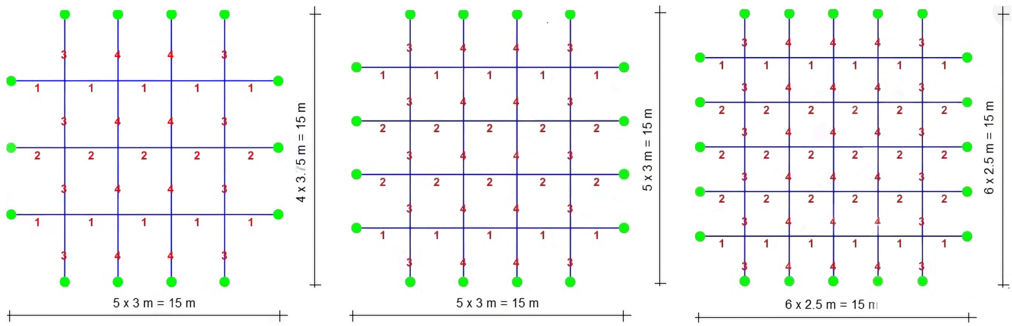

FIG. 5. Schematic of a model with four groups variables:
case 1 (left), case 2 (middle), case 3 (right) grillage structure (note: all supports are fixed).

The results in Table 3 can only be compared for the same number of beams, that is, for 4×4 and 3×4. For case 2, the weight is 9969 kg, which is better than the results of Kaveh *et al.* [21] at 10 508 kg. For case 1, it is 9279 kg, which is worse than the results of Aydin [3] at 9216 kg.

In this study, the proposed algorithm was executed on a personal computer with an Intel(R) Core(TM) i7-4510 CPU @2.60 GHz 8.00 GB RAM. The CPU time for algorithms and the number of function evaluations (NFE) are presented in Table 4.

TABLE 3. Results of the 225 square meters grillage structure with four groups variables.

Search method	CBO Kaveh *et al.* [21]	ECBO Kaveh *et al.* [21]	2D-CBO Kaveh *et al.* [21]	JayaX Aydin [3]	SAP-Rao case 1	SAP-Rao case 2	SAP-Rao case 3
Beam numbers	4×4	4×4	4×4	4×3	3×4	4×4	5×5
Group 1	W460 × 52	W360 × 57.8	W460 × 60	W460 × 52	W150 × 13.5	W150 × 13.5	W200 × 15
Group 2	W530 × 85	W530 × 74	W530 × 72	W610 × 82	W1000 × 296	W920 × 201	W760 × 147
Group 3	W150 × 22.5	W100 × 19.3	W150 × 13	W310 × 23.8	W530 × 66	W460 × 52	W310 × 32.7
Group 4	W840 × 176	W840 × 193	W840 × 193	W1000 × 296	W460 × 82	W530 × 66	W360 × 64
δ_{max} [mm]	19.4	21.0	19.4	15.4	16.2	16.1	23.9
Max strength ratio	0.97	0.99	0.97	0.99	0.96	0.98	0.97
Weight [kg]	10 508	10 775	10 675	9216	9279	9969	10 915

TABLE 4. The CPU time and NFE for SAP-Rao algorithms.

Numerical example	SAP-Rao case 1		SAP-Rao case 2		SAP-Rao case 3	
Beam numbers	3×4		4×4		5×5	
	Time [s]	NFE	Time [s]	NFE	Time [s]	NFE
144 m^2 area grillage 2 groups	34.32	2335	36.90	2121	54.37	2127
144 m^2 area grillage 4 groups	46.16	3104	94.58	4821	81.55	3372
225 m^2 area grillage 4 groups	76.87	5329	75.56	3708	86.81	3141

6. Conclusions

In order to optimize grillages, discrete sizes of W-shapes were used as design variables for beams, while strength and displacement limitations were used as constraints. The design outcomes were compared with the outcomes reported in the literature. This comparison unequivocally demonstrates that the proposed method, known as SAP-Rao, can be successfully employed in the design of grillage structures. An innovative and effective method, named SAP-Rao, was implemented in MATLAB to improve grillage structures. Like the other population-based optimization algorithms, the SAP-Rao algorithms use an initial population.

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