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Identification methods and procedures of climate-weather change process including extreme weather hazards

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Abstract
There are presented the methods of identification of the climate-weather change process. These are the methods and procedures for estimating the unknown basic parameters of the climate-weather change process semi-Markov model and identifying the distributions of the climate-weather change process conditional sojourn times at the climate-weather states.

1. Introduction

There are presented the methods of identification of the climate-weather change process. These are the methods and procedures for estimating the unknown basic parameters of the climate-weather change process semi-Markov model and identifying the distributions of the climate-weather change process conditional sojourn times at the climate-weather states. There are given the formulae estimating the probabilities of the climate-weather change process staying at the particular climate-weather states at the initial moment, the probabilities of the climate-weather change transitions between the climate-weather states and the parameters of the distributions suitable and typical for the description of the climate-weather change process conditional sojourn times at the particular climate-weather states. Namely, the parameters of the uniform distribution, the triangular distribution, the double trapezium distribution, the quasi-trapezium distribution, the exponential distribution, the Weibull distribution, the chimney distribution and the Gamma distribution are estimated using the statistical methods such as the method of moments and the maximum likelihood method. The chi-square goodness-of-fit test is described and proposed to be applied to verify the hypotheses about these distributions choice validity. The general model of the climate-weather change processes is proposed in the project report [EU-CIRCLE Report D2.1-GMU3, 2016]. The safety models of various multistate complex technical systems are considered in Task 3.3 [EU-CIRCLE Reports D3.3-GMU1, 2016]. Consequently, the general joint models linking these system safety models with the model of their climate-weather change processes, allowing us for the safety analysis of the complex technical systems at the variable climate-weather conditions, will be constructed in Task 3.4 [EU-CIRCLE Report D3.3-GMU12, 2016]. To be able to apply these general models practically in the evaluation and prediction of the reliability and safety of real complex technical systems it is necessary to have the statistical methods concerned with determining the unknown parameters of the proposed models [Barbu, Limnios, 2006], [Collet, 1996], [Hryniewicz, 1995], [Kolowrocki, Soszynska, 2009a], [Kolowrocki, Soszynska, 2009b], [Kolowrocki, Soszynska J, 2009d], [Kolowrocki, Soszynska J, 2009e], [Kolowrocki, Soszynska, 2010b], [Soszyńska et al., 2010]. Particularly, concerning the climate-weather change process, the probabilities of the climate-weather change process staying at the particular climate-weather states at the initial moment, the probabilities of the climate-weather change process transitions between the climate-weather states and the distributions of the conditional sojourn times of the climate-weather change process at the particular climate-weather states should be identified [Kolowrocki, 2014], [Kolowrocki, Soszynska, 2009c], [Kołowrocki, Soszyńska, 2009f], [Kolowrocki, Soszynska, 2010a], [Kolowrocki, Soszynska-Budny, 2011]. It is also necessary to have the methods of testing the hypotheses concerned with the climate-weather change process conditional sojourn times at the climate-weather states [Kołowrocki, 2014], [Kołowrocki, Soszyńska, 2009d], [Kolowrocki, Soszynska, 2010b].

2. Identification of climate-weather change process

We assume, as in [EU-CIRCLE Report D2.1-GMU3, 2016], that the climate-weather change process for the critical infrastructure operating area is taking w, $w \in N$, different climate-weather states $c_1, c_2, ..., c_w$. Next, we mark by $C(t)$, $t \in < 0,+\infty >$, the climate-weather change process, that is a function of a continuous variable t, taking discrete values in the set $\{c_1, c_2, ..., c_w\}$ of the climate-weather states. We assume a semi-Markov model [Kołowrocki, 2014], [Kołowrocki, Soszyńska, 2008], [Kołowrocki, Soszyńska-Budny, 2011], [Limnios, Oprisan, 2005], [Limnios et al., 2005], [Macci, 2008], [Mercier, 2008], of the climate-weather change process $C(t)$ and we mark by C_{bl} its random conditional sojourn times at the climate-weather states c_b, when its next climate-weather state is c_l, $b,l = 1,2,\ldots,w$, $b \neq l$.

Under these assumption, the climate-weather change process may be described by the vector $[q_b(0)]_{1xw}$ of probabilities of the climate-weather change process staying at the particular climate-weather states at the initial moment $t = 0$, the matrix $[q_{bl}(t)]_{wxw}$ of the probabilities of the climate-weather change process transitions between the climate-weather states and the matrix $[C_{bl}(t)]_{wxw}$ of the distribution functions of the conditional sojourn times C_{bl} of the climate-weather change process at the climate-weather states or equivalently by the matrix $[c_{bl}(t)]_{wxw}$ of the density functions of the conditional sojourn times C_{bl}, $b,l = 1,2,\ldots,w$, $b \neq l$, of the climate-weather change process at the climate-weather states. These all parameters of the climate-weather change process are unknown and before their use to the prognosis of this process characteristics have to be estimated on the basis of statistical data coming from practice.

2.1. Defining unknown parameters of climate-weather change process and data collection

To make the estimation of the unknown parameters of the climate-weather change process, the experiment delivering the necessary statistical data should be precisely planned.

First, before the experiment, we should perform the following preliminary steps:

i) to analyze the climate-weather change process;
ii) to fix or to define the climate-weather change process following general parameters:
   - the number of the climate-weather states of the climate-weather change process w,
   - the climate-weather states of the climate-weather change process c_1, c_2, …, c_w;
iii) to fix the possible transitions between the climate-weather states;
iv) to fix the set of the unknown parameters of the climate-weather change process semi-Markov model.

Next, to estimate the unknown parameters of the climate-weather change process, based on the experiment, we should collect necessary statistical data performing the following steps:

i) to fix and to collect the following statistical data necessary to evaluate the probabilities $q_b(0)$ of the climate-weather change process staying at the climate-weather states at the initial moment $t = 0$:

- the duration time of the experiment Θ,
- the number of the investigated (observed) realizations of the climate-weather change process $n(0)$,
- the vector of the realizations $q_b(0)$, $b = 1,2,...,w$, of the numbers of staying of the climate-weather change process respectively at the climate-weather states c_1, c_2, …, c_w, at the initial moments $t = 0$ of all $n(0)$ observed realizations of the climate-weather change process

$$[n_b(0)] = [n_1(0), n_2(0), ..., n_w(0)],$$

where

$$n_1(0) = n_2(0) = n_w(0) = n(0);$$

ii) to fix and to collect the following statistical data necessary to evaluate the probabilities q_{bl} of the climate-weather change process transitions between the climate-weather states:

- the matrix of the realizations of the numbers n_{bl}, b, l = 1,2,...,w, $b \neq l$, of the transitions of the climate-weather change process from the climate-weather state c_b into the climate-weather state c_l at all observed realizations of the climate-weather change process

$$[n_{bl}] = \begin{bmatrix} n_{11} \; n_{12} \; \dots \; n_{1w} \\ n_{21} \; n_{22} \; \dots \; n_{2w} \\ \dots \\ n_{w1} \; n_{w2} \; \dots \; n_{ww} \end{bmatrix},$$

where

$n_{bb} = 0$, for b = 1,2,...,w,

- the vector of the realizations of the numbers n_b, b = 1,2,...,w, of departures of the climate-weather change process from the climate-weather states c_b (the sums of the numbers of the b-th rows of the matrix $[n_{bl}]$)

$$[n_b] = [n_1, n_2, ..., n_w],$$

where

$$n_1 = n_{11} + n_{12} + ... + n_{1w},$$
$$n_2 = n_{21} + n_{22} + ... + n_{2w},$$
$$\dots.$$
$$n_w = n_{w1} + n_{w2} + ... + n_{ww};$$

iii) to fix and to collect the following statistical data necessary to evaluate the unknown parameters of the distributions $C_{bl}(t)$ of the conditional sojourn times C_{bl} of the climate-weather change process at the particular climate-weather states:

- the numbers n_{bl}, b, l = 1,2,...,w, $b \neq l$, of realizations of the conditional sojourn times C_{bl}, b, l = 1,2,...,w, $b \neq l$, of the climate-weather change process at the climate-weather state c_b when the next transition is to the climate-weather state c_l during the observation time Θ,
- the realizations C_{bl}^k, k = 1,2,…,n_{bl}, of the conditional sojourn times C_{bl} of the climate-weather change process at the climate-weather state c_b when the next transition is to the climate-weather state c_l during the observation time Θ for each b, l = 1,2,...,w, $b \neq l$.

2.2. Estimating basic parameters of climate-weather change process

After collecting the statistical data, it is possible to estimate the unknown parameters of the climate-weather change process performing the following steps:

i) to determine the vector

$$[q(0)] = [q_1(0), q_2(0), \dots, q_w(0)], \tag{1}$$

of the realizations of the probabilities $q_b(0)$, b = 1,2,...,w, of the climate-weather change process staying at the climate-weather states at the initial moment t = 0, according to the formula

$$q_b(0) = \frac{n_b(0)}{n(0)} \text{ for } b = 1,2,...,w, \tag{2}$$

where

$$n(0) = \sum_{b=1}^{w} n_b(0), \tag{3}$$

is the number of the realizations of the climate-weather change process starting at the initial moment t = 0;

ii) to determine the matrix

$$[q_{bl}] = \begin{bmatrix} q_{11} \; q_{12} \; \dots \; q_{1w} \\ q_{21} \; q_{22} \; \dots \; q_{2w} \\ \dots \\ q_{w1} \; q_{w2} \; \dots \; q_{vv} \end{bmatrix}, \tag{4}$$

of the realizations of the probabilities q_{bl}, b, l = 1,2,...,w, of the climate-weather change process transitions from the climate-weather state c_b to the climate-weather state c_l according to the formula

$$q_{bl} = \frac{n_{bl}}{n_b}, \text{ for } b, l = 1,2,...,w, \, b \neq l, \quad q_{bb} = 0, \tag{5}$$

for $b = 1,2,...,w$,

where

$$n_b = \sum_{b \neq l}^{w} n_{bl}, \; b = 1,2,...,w, \tag{6}$$

is the realization of the total number of the climate-weather change process departures from the climate-weather state c_b, $b = 1,2,...,w$, during the experiment time Θ.

2.3. Estimating parameters of distributions of climate-weather change process conditional sojourn times at climate-weather states

Prior to estimating the parameters of the distributions of the climate-weather change process conditional sojourn times at the particular climate-weather states, we have to determine the following empirical characteristics of the realizations of the conditional sojourn time of the climate-weather change process at the particular climate-weather states:

- the realizations of the empirical mean values $\bar{C}_{bl}$ of the conditional sojourn times C_{bl} of the climate-weather change process at the climate-weather state c_b, $b = 1,2,...,w$, when the next transition is to the climate-weather state c_l, $l = 1,2,...,w$, according to the formula

$$\bar{C}_{bl} = \frac{1}{n_{bl}} \sum_{k=1}^{n_{bl}} C_{bl}^k, \; b, l = 1,2,...,w, \; b \neq l, \quad (7)$$

- the number $\bar{r}_{bl}$ of the disjoint intervals $I_j = < a_{bl}^j, b_{bl}^j)$, $j = 1,2,...,\bar{r}_{bl}$, that include the realizations C_{bl}^k, $k = 1,2,...,n_{bl}$, of the conditional sojourn times C_{bl} at the climate-weather state c_b when the next transition is to the climate-weather state c_l, according to the formula

$$\bar{r}_{bl} \cong \sqrt{n_{bl}},$$

- the length d_{bl} of the intervals $I_j = < a_{bl}^j, b_{bl}^j),$, $j = 1,2,...,\bar{r}_{bl}$, according to the formula

$$d_{bl} = \frac{\bar{R}_{bl}}{\bar{r}_{bl} - 1},$$

where

$$\bar{R}_{bl} = \max_{1 \le k \le n_{bl}} \theta_{bl}^k - \min_{1 \le k \le n_{bl}} \theta_{bl}^k,$$

- the ends a_{bl}^j, b_{bl}^j, of the intervals $I_j = < a_{bl}^j, b_{bl}^j)$, $j = 1,2,...,\bar{r}_{bl}$, according to the formulae

$$a_{bl}^1 = \max\{\min_{1 \le k \le n_{bl}} \theta_{bl}^k - \frac{d_{bl}}{2}, 0\},$$

$$b_{bl}^j = a_{bl}^1 + j d_{bl}, \; j = 1,2,...,\bar{r}_{bl},$$

$$a_{bl}^j = b_{bl}^{j-1}, \; j = 2,3...,\bar{r}_{bl},$$

in such a way that

$$I_1 \cup I_2 \cup ... \cup I_{\bar{r}_{bl}} = < a_{bl}^1, b_{bl}^{\bar{r}_{bl}})$$

and

$$I_i \cap I_j = \emptyset \text{ for all } i \neq j, \; i, j \in \{1,2,...,\bar{r}_{bl}\},$$

- the numbers n_{bl}^j of the realizations C_{bl}^k in the intervals $I_j = < a_{bl}^j, b_{bl}^j)$, $j = 1,2,...,\bar{r}_{bl}$, according to the formula

$$n_{bl}^j = \#\{k : C_{bl}^k \in I_j, k \in \{1,2,...,n_{bl}\}\}, \; j = 1,2,...,\bar{r}_{bl},$$

where

$$\sum_{j=1}^{\bar{r}_{bl}} n_{bl}^j = n_{bl},$$

whereas the symbol # means the number of elements of the set.

To estimate the parameters of the distributions of the conditional sojourn times of the climate-weather change process at the particular climate-weather states distinguished in [EU-CIRCLE Report D2.1-GMU3, 2016] we proceed respectively in the following way:

- for the uniform distribution with the density function given by (4.5) [EU-CIRCLE Report D2.1-GMU3, 2016], the estimates of the unknown parameters are:

$$x_{bl} = a_{bl}^1 \; y_{bl} = x_{bl} + \bar{r}_{bl} d_{bl}; \quad (8)$$

- for the triangular distribution with the density function given by (4.6) [EU-CIRCLE Report D2.1-GMU3, 2016] the estimates of the unknown parameters are:

$$x_{bl} = a_{bl}^1, \; y_{bl} = x_{bl} + \bar{r}_{bl} d_{bl}, \; z_{bl} = \bar{C}_{bl}; \quad (9)$$

- for the double trapezium distribution with the density function given by (4.7) [EU-CIRCLE Report D2.1-GMU3, 2016], the estimates of the unknown parameters are:

$$x_{bl} = a_{bl}^1,\ y_{bl} = x_{bl} + \bar{r}_{bl} d_{bl},\ q_{bl} = \frac{n_{bl}^1}{n_{bl} d_{bl}},$$
$$w_{bl} = \frac{n_{bl}^{\bar{r}_{bl}}}{n_{bl} d_{bl}},\ z_{bl} = \bar{C}_{bl}; \tag{10}$$

- for the quasi-trapezium distribution with the density function given by (4.8) [EU-CIRCLE Report D2.1-GMU3, 2016], the estimates of the unknown parameters are:

$$x_{bl} = a_{bl}^1,\ y_{bl} = x_{bl} + \bar{r}_{bl} d_{bl},\ q_{bl} = \frac{n_{bl}^1}{n_{bl} d_{bl}},$$
$$w_{bl} = \frac{n_{bl}^{\bar{r}_{bl}}}{n_{bl} d_{bl}},\ z_{bl}^1 = \bar{C}_{bl}^1,\ z_{bl}^2 = \bar{C}_{bl}^2, \tag{11}$$

where

$$\bar{C}_{bl}^1 = \frac{1}{n_{(me)}} \sum_{k=1}^{n(me)} C_{bl}^k,\ \bar{C}_{bl}^2 = \frac{1}{n_{bl} - n_{(me)}} \sum_{k=n(me)+1}^{n_{bl}} C_{bl}^k,$$
$$n_{(me)} = \left[\frac{n_{bl}+1}{2}\right], \tag{12}$$

and $[x]$ denotes the entire part of x;

- for the exponential distribution with the density function given by (4.9) [EU-CIRCLE Report D2.1-GMU3, 2016], the estimates of the unknown parameters are:

$$x_{bl} = a_{bl}^1,\ \alpha_{bl} = \frac{1}{\bar{C}_{bl} - x_{bl}}; \tag{13}$$

- for the Weibull distribution with the density function given by (4.10) [EU-CIRCLE Report D2.1-GMU3, 2016] the estimates of the unknown parameters are (the expressions for estimates of parameters α_{bl} and β_{bl} are not explicit):

$$x_{bl} = a_{bl}^1,\ \alpha_{bl} = \frac{n_{bl}}{\sum_{k=1}^{n_{bl}} (C_{bl}^k)^{\beta_{bl}}},$$

$$\alpha_{bl} = \frac{\frac{n_{bl}}{\beta_{bl}} + \sum_{k=1}^{n_{bl}} \ln(C_{bl}^k - x_{bl})}{\sum_{k=1}^{n_{bl}} (C_{bl}^k)^{\beta_{bl}} \ln(C_{bl}^k - x_{bl})}; \tag{14}$$

- for the chimney distribution with the density function given by (4.11) [EU-CIRCLE Report D2.1-GMU3, 2016], the estimates of the unknown parameters are:

$$x_{bl} = a_{bl}^1,\ y_{bl} = x_{bl} + \bar{r}_{bl} d_{bl}, \tag{15}$$

and moreover, if

$$\hat{n}_{bl} = \max_{1 \le j \le \bar{r}_{bl}} \{n_{bl}^j\} \tag{16}$$

and i, $i \in \{1,2,...,\bar{r}_{bl}\}$, is the number of the interval including the largest number of realizations i.e. such as that

$$n_{bl}^i = \hat{n}_{bl}, \tag{17}$$

then:

- for $i = 1$

either

$$z_{bl}^1 = x_{bl} + (i-1)d_{bl},\ z_{bl}^2 = x_{bl} + i d_{bl},\ A_{bl} = 0,$$
$$K_{bl} = \frac{n_{bl}^i}{n_{bl}},\ D_{bl} = \frac{n_{bl}^{i+1} + \ldots + n_{bl}^{\bar{r}_{bl}}}{n_{bl}}, \tag{18}$$

while

$$n_{bl}^{i+1} = 0 \quad \text{or} \quad n_{bl}^{i+1} \neq 0 \quad \text{and} \quad \frac{n_{bl}^i}{n_{bl}^{i+1}} \ge 3, \tag{19}$$

or

$$z_{bl}^1 = x_{bl} + (i-1)d_{bl},\ z_{bl}^2 = x_{bl} + (i+1)d_{bl},$$
$$A_{bl} = 0, \tag{20}$$

$$K_{bl} = \frac{n_{bl}^i + n_{bl}^{i+1}}{n_{bl}},\ D_{bl} = \frac{n_{bl}^{i+2} + \ldots + n_{bl}^{\bar{r}_{bl}}}{n_{bl}}, \tag{21}$$

while

$$n_{bl}^{i+1} \neq 0 \text{ and } \frac{n_{bl}^{i}}{n_{bl}^{i+1}} < 3; \quad (22)$$

- for $i = 2,3,...,\bar{r}_{bl} - 1$

either

$$z_{bl}^{1} = x_{bl} + (i-1)d_{bl}, \; z_{bl}^{2} = x_{bl} + id_{bl},$$
$$A_{bl} = \frac{n_{bl}^{1} + ... + n_{bl}^{i-1}}{n_{bl}}, \; K_{bl} = \frac{n_{bl}^{i}}{n_{bl}}, \quad (23)$$

$$D_{bl} = \frac{n_{bl}^{i+1} + ... + n_{bl}^{\bar{r}bl}}{n_{bl}}, \quad (24)$$

while

$$n_{bl}^{i-1} = 0 \text{ or } n_{bl}^{i-1} \neq 0 \text{ and } \frac{n_{bl}^{i}}{n_{bl}^{i-1}} \geq 3 \quad (25)$$

and while

$$n_{bl}^{i+1} = 0 \text{ or } n_{bl}^{i+1} \neq 0 \text{ and } \frac{n_{bl}^{i}}{n_{bl}^{i+1}} \geq 3, \quad (26)$$

or

$$z_{bl}^{1} = x_{bl} + (i-1)d_{bl}, \; z_{bl}^{2} = x_{bl} + (i+1)d,$$
$$A_{bl} = \frac{n_{bl}^{1} + ... + n_{bl}^{i-1}}{n_{bl}}, \quad (27)$$

$$K_{bl} = \frac{n_{bl}^{i} + n_{bl}^{i+1}}{n_{bl}}, \; D_{bl} = \frac{n_{bl}^{i+2} + ... + n_{bl}^{\bar{r}bl}}{n_{bl}}, \quad (28)$$

while

$$n_{bl}^{i-1} = 0 \quad \text{or} \quad n_{bl}^{i-1} \neq 0 \quad \text{and} \quad \frac{n_{bl}^{i}}{n_{bl}^{i-1}} \geq 3$$

(29)

and while

$$n_{bl}^{i+1} \neq 0 \quad \text{and} \quad \frac{n_{bl}^{i}}{n_{bl}^{i+1}} < 3,$$

(30)

or

$$z_{bl}^{1} = x_{bl} + (i-2)d_{bl}, \; z_{bl}^{2} = x_{bl} + id_{bl},$$

$$A_{bl} = \frac{n_{bl}^{1} + ... + n_{bl}^{i-2}}{n_{bl}}, \; K_{bl} = \frac{n_{bl}^{i-1} + n_{bl}^{i}}{n_{bl}}, \quad (31)$$

$$D_{bl} = \frac{n_{bl}^{i+1} + ... + n_{bl}^{\bar{r}bl}}{n_{bl}}, \quad (32)$$

while

$$n_{bl}^{i-1} \neq 0 \text{ and } \frac{n_{bl}^{i}}{n_{bl}^{i-1}} < 3 \quad (33)$$

and while

$$n_{bl}^{i+1} = 0 \text{ or } n_{bl}^{i+1} \neq 0 \text{ and } \frac{n_{bl}^{i}}{n_{bl}^{i+1}} \geq 3, \quad (34)$$

or

$$z_{bl}^{1} = x_{bl} + (i-2)d_{bl}, \; z_{bl}^{2} = x_{bl} + (i+1)d_{bl},$$
$$A_{bl} = \frac{n_{bl}^{1} + ... + n_{bl}^{i-2}}{n_{bl}}, \quad (35)$$

$$K_{bl} = \frac{n_{bl}^{i-1} + n_{bl}^{i} + n_{bl}^{i+1}}{n_{bl}},$$
$$D_{bl} = \frac{n_{bl}^{i+2} + ... + n_{bl}^{\bar{r}bl}}{n_{bl}}, \quad (36)$$

while

$$n_{bl}^{i-1} \neq 0 \text{ and } \frac{n_{bl}^{i}}{n_{bl}^{i-1}} < 3 \quad (37)$$

and while

$$n_{bl}^{i+1} \neq 0 \text{ and } \frac{n_{bl}^{i}}{n_{bl}^{i+1}} < 3; \quad (38)$$

- for $i = \bar{r}_{bl}$

either

$$z_{bl}^{1} = x_{bl} + (i-1)d_{bl}, \; z_{bl}^{2} = x_{bl} + id_{bl},$$
$$A_{bl} = \frac{n_{bl}^{1} + ... + n_{bl}^{i-1}}{n_{bl}}, \quad (39)$$

$$K_{bl} = \frac{n_{bl}^{i}}{n_{bl}},\ D_{bl} = 0, \quad (40)$$

while

$$n_{bl}^{i-1} = 0 \quad \text{or} \quad n_{bl}^{i-1} \neq 0 \quad \text{and} \quad \frac{n_{bl}^{i}}{n_{bl}^{i-1}} \geq 3, \quad (41)$$

or

$$z_{bl}^{1} = x_{bl} + (i-2)d_{bl},\ z_{bl}^{2} = x_{bl} + id_{bl},$$
$$A_{bl} = \frac{n_{bl}^{1} + \ldots + n_{bl}^{i-2}}{n_{bl}}, \quad (42)$$

$$K_{bl} = \frac{n_{bl}^{i-1} + n_{bl}^{i}}{n_{bl}},\ D_{bl} = 0, \quad (43)$$

while

$$n_{bl}^{i-1} \neq 0 \text{ and } \frac{n_{bl}^{i}}{n_{bl}^{i-1}} < 3. \quad (44)$$

- for the Gamma distribution with the density function given by (4.12) [EU-CIRCLE Report D2.1-GMU3, 2016] the estimates of the unknown parameters are given as follows

$$x_{bl} = a_{bl}^{1},\ \alpha_{bl} = \frac{(\bar{C}_{bl} - x_{bl})^2}{(S_{bl}^{*})^2},$$

$$\beta_{bl} = \frac{(S_{bl}^{*})^2}{\bar{C}_{bl} - x_{bl}}, \quad (45)$$

where $(S_{bl}^{*})^2$ is the variance of the conditional sojourn times C_{bl}, given by

$$(S_{bl}^{*})^2 = \frac{1}{n_{bl}-1}\sum_{k=1}^{n_{bl}}(C_{bl}^{k} - \bar{C}_{bl})^2. \quad (46)$$

2.4. Identification of distribution functions of climate-weather change process conditional sojourn times at climate-weather states

To formulate and next to verify the non-parametric hypothesis concerning the form of the distribution of the climate-weather change process conditional sojourn time C_{bl} at the climate-weather state c_b when the next transition is to the climate-weather state c_l, on the basis of at least 30 its realizations C_{bl}^{k}, $k = 1,2,\ldots,n_{bl}$, it is due to proceed according to the following scheme:
- to construct and to plot the realization of the histogram of the climate-weather change process conditional sojourn time C_{bl} at the climate-weather state c_b, defined by the following formula

$$\bar{h}_{n_{bl}}(t) = \frac{n_{bl}^{j}}{n_{bl}} \text{ for } t \in I_j, \quad (47)$$

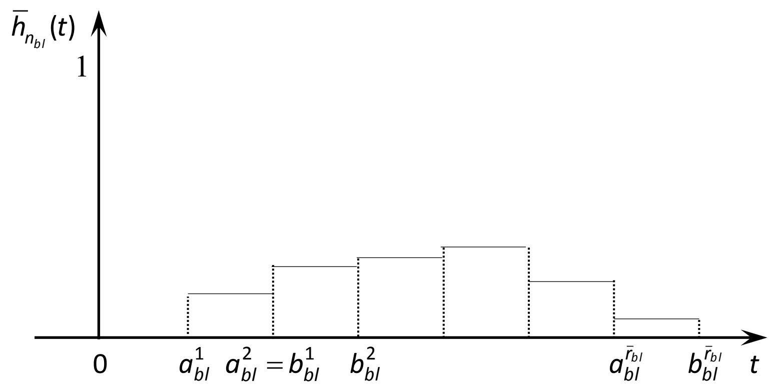


Figure 1. The graph of the realization of the histogram of the climate-weather change process conditional sojourn time C_{bl} at the climate-weather state c_b

- to analyze the realization of the histogram $\bar{h}_{n_{bl}}(t)$, comparing it with the graphs of the density functions $c_{bl}(t)$ of the previously distinguished in [EU-CIRCLE Report D2.1-GMU3, 2016] distributions, to select one of them and to formulate the null hypothesis H_0, concerning the unknown form of the distribution of the conditional sojourn time C_{bl} in the following form:

H_0: The climate-weather change process conditional sojourn time C_{bl} at the climate-weather state c_b when the next transition is to the climate-weather state c_l, has the distribution with the density function $c_{bl}(t)$

- to join each of the intervals I_j that has the number n_{bl}^j of realizations less than 4 either with the neighbour interval I_{j+1} or with the neighbour interval I_{j-1} this way that the numbers of realizations in all intervals are not less than 4;
- to fix a new number of intervals $\bar{\bar{r}}_{bl}$;
- to determine new intervals

$$\bar{I}_j = < \bar{a}_{bl}^j, \bar{b}_{bl}^j), \ \ j = 1,2,..,\bar{\bar{r}}_{bl};$$

- to fix the numbers $\bar{n}_{bl}^j$ of realizations in new intervals $\bar{I}_j$, $j = 1,2,..,\bar{\bar{r}}_{bl}$;
- to calculate the hypothetical probabilities that the variable C_{bl} takes values from the interval $\bar{I}_j$, under the assumption that the hypothesis H_0 is true, i.e. the

$$p_j = P(C_{bl} \in \bar{I}_j) = P(\bar{a}_{bl}^j \le C_{bl} < \bar{b}_{bl}^j)$$
$$= C_{bl}(\bar{b}_{bl}^j) - C_{bl}(\bar{a}_{bl}^j), \ j = 1,2,..,\bar{\bar{r}}_{bl}, \tag{48}$$

where $C_{bl}(\bar{b}_{bl}^j)$ and $C_{bl}(\bar{a}_{bl}^j)$ are the values of the distribution function $C_{bl}(t)$ of the random variable C_{bl} corresponding to the density function $c_{bl}(t)$ assumed in the null hypothesis H_0;

- to calculate the realization of the χ^2 (chi-square)-Pearson's statistics $U_{n_{bl}}$, according to the formula

$$u_{n_{bl}} = \sum_{j=1}^{\bar{\bar{r}}_{bl}} \frac{(\bar{n}_{bl}^j - n_{bl} p_j)^2}{n_{bl} p_j}; \tag{49}$$

- to assume the significance level α (α = 0.01, α =0.02, α = 0.05 or α = 0.10) of the test;
- to fix the number $\bar{\bar{r}}_{bl} - l - 1$ of degrees of freedom, substituting for I for the distinguished in Task 2.1 [EU-CIRCLE Report D2.1-GMU3, 2016] distributions respectively the following values: l = 0, for the uniform, triangular, double trapezium, quasi-trapezium and chimney distributions, l = 1, for the exponential distribution and l = 2, for the Weibull distribution and Gamma distribution;
- to read from the Tables of the χ^2 Pearson's distribution the value u_α for the fixed values of the significance level α and the number of degrees of freedom $\bar{\bar{r}}_{bl} - l - 1$ such that the following equality holds

$$P(U_{n_{bl}} > u_\alpha) = \alpha, \tag{50}$$

and next to determine the critical domain in the form of the interval $(u_\alpha, +\infty)$ and the acceptance domain in the form of the interval $< 0, u_\alpha >$,

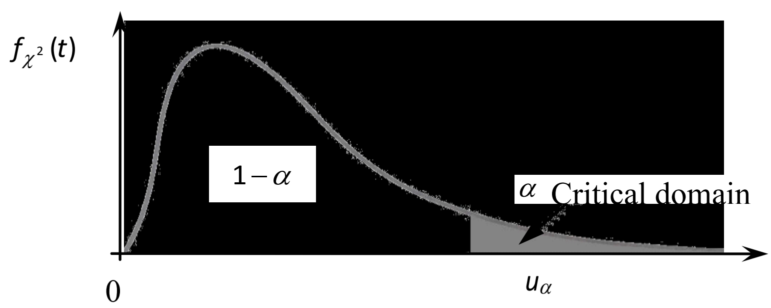


Figure 8.2. The graphical interpretation of the critical interval and the acceptance interval for the chi-square goodness-of-fit test

- to compare the obtained value $u_{n_{bl}}$ of the realization of the statistics $U_{n_{bl}}$ with the read from the Tables critical value u_α of the chi-square random variable and to decide on the previously formulated null hypothesis H_0 in the following way: if the value $u_{n_{bl}}$ does not belong these to the critical domain, i.e. when $u_{n_{bl}} \le u_\alpha$, then we do not reject the hypothesis H_0, otherwise if the value $u_{n_{bl}}$ belongs to the critical domain, i.e. when $u_{n_{bl}} > u_{\alpha}$, then we reject the hypothesis H_0.

2.5. Testing uniformity of statistical data of climate-weather change process

The statistical data that are needed in Section 8.2.4 for estimating the unknown parameters of the climate-weather change process very often are coming from different experiments of the same climate-weather change process and they are collected into separate data sets. Before joining them into one set of data in order to do the unknown parameters evaluation with the methods and procedures described in Section 2.4, we have to make the uniformity testing these statistical data sets.

2.5.1. Procedure of critical infrastructure climate-weather change process data collection

To make the uniformity testing of the statistical data collected in two separate data sets coming from the same climate-weather change process realizations in two different experiments, we should collect necessary statistical data performing the following steps:
i) to fix two independent experiments of the climate-weather change process data collection and their following basic parameters:
- the duration times of the experiments Θ_1, Θ_2,
- the climate-weather change processes single realizations,
- the numbers of the investigated (observed) realizations of the climate-weather change process $n_1(0)$, $n_2(0)$;
ii) to fix and to collect the following statistical data concerned with the empirical distributions of the conditional sojourn times C^1_{bl} and C^2_{bl}, $b,l \in \{1,2,\ldots,w\}$, $b \neq l$, of the climate-weather change process at the particular climate-weather states, respectively in the first experiment and in the second experiment:
- the number of realizations

n^1_{bl}, $b,l \in \{1,2,\ldots,w\}$, $b \neq l$,

of the sojourn time C^1_{bl}, $b,l \in \{1,2,\ldots,w\}$, in the first experiment,
- the sample of non-decreasing ordered realizations

$$C^{1k}_{bl},\ k = 1,2,\ldots,n^1_{bl},\ b \neq l, \tag{51}$$

of the sojourn time C^1_{bl}, $b,l \in \{1,2,\ldots,w\}$, in the first experiment,

- the number of realizations

n^2_{bl}, $b,l \in \{1,2,\ldots,w\}$, $b \neq l$,

of the sojourn time C^2_{bl}, $b,l \in \{1,2,\ldots,w\}$, in the second experiment,
- the sample of non-decreasing ordered realizations

$$C^{2k}_{bl},\quad k = 1,2,\ldots,n^2_{bl},\quad b \neq l, \tag{52}$$

of the sojourn time C^2_{bl}

2.5.2. Procedure of testing uniformity of distributions of climate-weather change process conditional sojourn times at climate-weather states

We consider test λ based on Kolmogorov-Smirnov theorem [Kolowrocki, Soszynska, 2010c] that can be used for testing whether two independent samples of realizations of the conditional sojourn time C_{bl}, $b,l \in \{1,2,\ldots,w\}$, $b \neq l$, at the particular climate-weather states of the climate-weather change process are drawn from the population with the same distribution.
We assume that we have defined in previous section two independent samples of non-decreasing ordered realizations (51) and (52) of the sojourn times C^1_{bl} and C^2_{bl}, $b,l \in \{1,2,\ldots,w\}$, $b \neq l$, coming from two different experiments, respectively composed of n^1_{bl} and n^2_{bl} realizations and we define their corresponding empirical distribution functions

$$C^1_{bl}(t) = \frac{1}{n^1_{bl}} \#\{k : C^{1k}_{bl} < t,\ k \in \{1,2,\ldots,n^1_{bl}\}\},$$
$$t \geq 0,\quad b,l \in \{1,2,\ldots,w\},\quad b \neq l, \tag{53}$$

and

$$C^2_{bl}(t) = \frac{1}{n^2_{bl}} \#\{k : C^{2k}_{bl} < t,\ k \in \{1,2,\ldots,n^2_{bl}\}\}, \tag{54}$$
$$t \geq 0,\ b,l \in \{1,2,\ldots,w\},\ b \neq l.$$

Then, according to Kolmogorov-Smirnov theorem [Kolowrocki, Soszynska-Budny, 2011], the sequence of distribution functions given by the equation

$$Q_{n_1 n_2}(\lambda) = P(D_{n_1 n_2} < \frac{\lambda}{\sqrt{n}}) \tag{55}$$

defined for $\lambda > 0$, where

$$n_1 = n_{bl}^1,\ n_2 = n_{bl}^2,\ n = \frac{n_1 n_2}{n_1 + n_2}, \tag{56}$$

and

$$D_{n_1 n_2} = \max_{-\infty < t < +\infty} \left| C_{bl}^1(t) - C_{bl}^2(t) \right|, \tag{57}$$

is convergent, as $n \to \infty$, to the limit distribution function

$$Q(\lambda) = \sum_{k=-\infty}^{+\infty} (-1)^k e^{-2k^2\lambda^2}, \qquad \lambda > 0.$$
(58)

The distribution function $Q(\lambda)$ given by (58) is called λ distribution and its Tables of values are available.

The convergence of the sequence $Q_{n_1 n_2}(\lambda)$ to the λ distribution $Q(\lambda)$ means that for sufficiently large n_1 and n_2 we may use the following approximate formula

$$Q_{n_1 n_2}(\lambda) \cong Q(\lambda). \tag{59}$$

Hence, it follows that if we define the statistic

$$U_n = D_{n_1 n_2} \sqrt{n}, \tag{60}$$

where $D_{n_1 n_2}$ is defined by (57), then by (55) and (59), we have

$$P(U_n < \lambda) = P(D_{n_1 n_2} \sqrt{n} < \lambda)$$
$$= P(D_{n_1 n_2} < \frac{\lambda}{\sqrt{n}})$$
$$= Q_{n_1 n_2}(\lambda) \cong Q(\lambda), \text{ for } \lambda > 0. \tag{61}$$

This result means that in order to formulate and next to verify the hypothesis that the two independent samples of the realizations of the climate-weather change process conditional sojourn times C_{bl}^1 and C_{bl}^2, $b, l \in \{1,2,\ldots,w\}$, $b \neq l$, at the climate-weather state c_b when the next transition is to the climate-weather state c_l are coming from the population with the same distribution, it is necessary to proceed according to the following scheme:

- to fix the numbers of realizations n_{bl}^1 and n_{bl}^2 in the samples,
- to collect the realizations (51) and (52) of the conditional sojourn times C_{bl}^1 and C_{bl}^2 of the climate-weather change process in the samples,
- to find the realization of the empirical distribution functions $C_{bl}^1(t)$ and $C_{bl}^2(t)$ defined by (53) and (54) respectively, in the following forms:
- to find the realization of the empirical distribution functions $C_{bl}^1(t)$ and $C_{bl}^2(t)$ defined by (53) and (54) respectively, in the following forms:

$$C_{bl}^1(t) = \begin{cases} \dfrac{n_{bl}^{11}}{n_{bl}^1} = 0, & t \le C_{bl}^{11} \\ \dfrac{n_{bl}^{12}}{n_{bl}^1}, & C_{bl}^{11} < t \le C_{bl}^{12} \\ \dfrac{n_{bl}^{13}}{n_{bl}^1}, & C_{bl}^{12} < t \le C_{bl}^{13} \\ \cdot\ \cdot\ \cdot \\ \dfrac{n_{bl}^{1k}}{n_{bl}^1}, & C_{bl}^{1k-1} < t \le C_{bl}^{1k} \\ \cdot\ \cdot\ \cdot \\ \dfrac{n_{bl}^{1n_{bl}^1}}{n_{bl}^1}, & C_{bl}^{1n_{bl}^1 - 1} < t \le C_{bl}^{1n_{bl}^1} \\ \dfrac{n_{bl}^{1n_{bl}^1 + 1}}{n_{bl}^1} = 1, & t \ge C_{bl}^{1n_{bl}^1} \end{cases}, \tag{62}$$

$$C_{bl}^2(t) = \begin{cases} \frac{n_{bl}^{21}}{n_{bl}^2} = 0, & t \le C_{bl}^{21} \\ \frac{n_{bl}^{22}}{n_{bl}^2}, & C_{bl}^{21} < t \le C_{bl}^{22} \\ \frac{n_{bl}^{23}}{n_{bl}^2}, & C_{bl}^{22} < t \le C_{bl}^{23} \\ \cdot \quad \cdot \quad \cdot \\ \frac{n_{bl}^{2k}}{n_{bl}^2}, & C_{bl}^{2k-1} < t \le C_{bl}^{2k} \\ \cdot \quad \cdot \quad \cdot \\ \frac{n_{bl}^{2n_{bl}^2}}{n_{bl}^2}, & C_{bl}^{2n_{bl}^2-1} < t \le C_{bl}^{2n_{bl}^1} \\ \frac{n_{bl}^{2n_{bl}^2+1}}{n_{bl}^2} = 1, & t \ge C_{bl}^{2n_{bl}^2} \end{cases}, \quad (63)$$

where

$$n_{bl}^{11} = 0, \; n_{bl}^{1\,n_{bl}^1+1} = n_{bl}^1, \quad (64)$$

and

$$n_{bl}^{1k} = \#\{j : C_{bl}^{1j} < C_{bl}^{1k}, \; j \in \{1,2,\ldots,n_{bl}^1\}\}, \quad (65)$$
$$k = 2,3,\ldots,n_{bl}^1,$$

is the number of the sojourn time C_{bl}^1 realizations less than its realization C_{bl}^{1k}, $k = 2,3,\ldots,n_{bl}^1$, and respectively

$$n_{bl}^{21} = 0, \qquad n_{bl}^{2\,n_{bl}^2+1} = n_{bl}^2, \quad (66)$$

and

$$n_{bl}^{2k} = \#\{j : C_{bl}^{2j} < C_{bl}^{2k}, \; j \in \{1,2,\ldots,n_{bl}^2\}\}, \quad (67)$$
$$k = 2,3,\ldots,n_{bl}^2,$$

is the number of the sojourn time C_{bl}^2 realizations less than its realization C_{bl}^{2k}, $k = 2,3,\ldots,n_{bl}^2$,

- to calculate the realization of the statistic u_n defined by (60) according to the formula

$$u_n = d_{n_{bl}^1 n_{bl}^2} \sqrt{n}, \quad (68)$$

where

$$d_{n_{bl}^1 n_{bl}^2} = \max\{ d^1_{n_{bl}^1 n_{bl}^2}, \; d^2_{n_{bl}^1 n_{bl}^2} \}, \quad (69)$$

$$d^1_{n_{bl}^1 n_{bl}^2} = \max\{ |C_{bl}^1(C_{bl}^{1k}) - C_{bl}^2(C_{bl}^{1k})|, \; k \in \{1,2,\ldots,n_{bl}^1\}\}, \quad (70)$$

$$d^2_{n_{bl}^1 n_{bl}^2} = \max\{ |C_{bl}^1(C_{bl}^{2k}) - C_{bl}^2(C_{bl}^{2k})|, \; k \in \{1,2,\ldots,n_{bl}^2\}\}, \quad (71)$$

$$n = \frac{n_{bl}^1 n_{bl}^2}{n_{bl}^1 + n_{bl}^2}, \quad (72)$$

- to formulate the null hypothesis H_0 in the following form:

H_0: The samples of realizations (51) and (52) are coming from the populations with the same distributions,

- to fix the significance level α ($\alpha = 0.01$, $\alpha = 0.02$, $\alpha = 0.05$ or $\alpha = 0.10$) of the test,
- to read from the Tables of λ distribution, corresponding to $1-\alpha$, the value λ_0 such that the following equality holds

$$P(U_n < \lambda_0) = Q(\lambda_0) = 1 - \alpha, \quad (73)$$

- to determine the critical domain in the form of the interval $(\lambda_0, +\infty)$ and the acceptance domain in the form of the interval $(0, \lambda_0 >$,

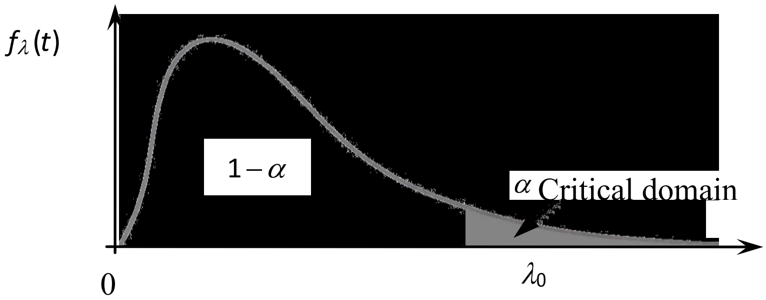


Figure 3. The graphical interpretation of the critical domain and the acceptance domain for the two-sample Smirnov-Kolmogorov test

- to compare the obtained value u_n of the realization of the statistics U_n with the read from the Tables critical value λ_0,
- to decide on the previously formulated null hypothesis H_0 in the following way:

if the value u_n does not belong to the critical domain, i.e. when $u_n \le \lambda_0$ then we do not reject the hypothesis H_0, otherwise if the value u_n belongs to the critical domain, i.e. when $u_n > \lambda_0$, then we reject the hypothesis H_0.

In the case when the null hypothesis H_0 is not rejected we may join the statistical data from the considered two separate sets into one new set of data and if there are no other sets of statistical data including the realizations of the sojourn time C_{bl}, we proceed with the data of this new set in the way described in Sections 8.2.1-8.2.4. Otherwise, if there are other sets of statistical data including the realizations of the sojourn time C_{bl}, we select the next one of them and perform the procedure of this section for data from this set and data from the previously formed new set. We continue this procedure up to the moment when the store of the statistical data sets including the realizations of the sojourn time C_{bl}, is exhausted.

3. Procedures of climate-weather change process identification in case study 2

3.1. Methodology of defining the unknown parameters of climate-weather change process of the critical infrastructure

To make the estimation of the unknown parameters of the climate-weather change process, the experiment delivering the necessary statistical data should be precisely planned. To model the climate-weather change process for the critical infrastructure we should perform the following preliminary steps:

i) to analyze the climate-weather change process;
ii) to fix or to define the climate-weather change process following general parameters:
  - the number of the climate-weather states of the climate-weather change process w,
  - the climate-weather states of the climate-weather change process $c_1, c_2, \ldots, c_w$;
iii) to fix the possible transitions between the climate-weather states;
iv) to fix the set of the unknown parameters of the climate-weather change process semi-Markov model.

3.2. Procedure of the climate-weather change process of critical infrastructure statistical data collection

To estimate the unknown parameters of the climate-weather change process, during the experiment, we should collect necessary statistical data performing the following steps:

i) to fix and to collect the following statistical data necessary to evaluate the probabilities of the initial states of the climate-weather change process
- the duration time of the experiment Θ,
- the number of the investigated (observed) realizations of the climate-weather change process $n(0)$,
- the numbers of staying of the climate-weather change process at the climate-weather states c_1, c_2, …, c_w, at the initial moment $t = 0$ of all $n(0)$ observed realizations of the climate-weather change process

$$n_1(0),\ n_2(0),\ \ldots,\ n_w(0),$$

where

$$n_1(0) = n_2(0) = n_w(0) = n(0);$$

- the vector of the realizations of the numbers of staying of the climate-weather change process at the climate-weather states at the initial moment

$$[n_b(0)] = [n_1(0), n_2(0), ..., n_w(0)],$$

ii) to fix and to collect the following statistical data necessary to evaluate the probabilities of the climate-weather change process transitions between the climate-weather states:
- the numbers n_{bl}, $b, l = 1,2,...,w$, $b \neq l$, of the transitions of the climate-weather change process from the climate-weather state c_b into the climate-weather state c_l at all observed realizations of the climate-weather change process

$$n_{11} = 0, n_{12}, n_{13}, ..., n_{1w},$$
$$n_{21}, n_{22} = 0, n_{23}, ..., n_{2w},$$
$$...$$
$$n_{w1}, n_{w2}, n_{w3}, ..., n_{ww} = 0,$$

- the matrix of the realizations of the transitions' numbers of the climate-weather change process between the climate-weather states

$$[n_{bl}] = \begin{bmatrix} n_{11} \ n_{12} \ \dots \ n_{1w} \\ n_{21} \ n_{22} \ \dots \ n_{2w} \\ \dots \\ n_{w1} \ n_{w2} \ \dots \ n_{ww} \end{bmatrix},$$

- the numbers n_b, $b = 1,2,...,w$, of departures of the climate-weather change process from the climate-weather states c_b (the sums of the numbers of the b-th rows of the matrix $[n_{bl}]$)

$$n_1 = n_{11} + n_{12} + ... + n_{1w},$$
$$n_2 = n_{21} + n_{22} + ... + n_{2w},$$
$$....$$
$$n_w = n_{w1} + n_{w2} + ... + n_{ww};$$

- the vector of the realizations of the numbers of departures of the climate-weather change process from the climate-weather states

$$[n_b] = [n_1, n_2, ..., n_w],$$

iii) to fix and to collect the following statistical data necessary to evaluate the unknown parameters of the distributions of the conditional sojourn times of the climate-weather change process at the particular climate-weather states:

- the numbers n_{bl}, $b, l = 1,2,...,w$, $b \neq l$, of realizations of the conditional sojourn times C_{bl}, $b, l = 1,2,...,w$, $b \neq l$, of the climate-weather change process at the climate-weather state c_b when the next transition is to the climate-weather state c_l during the observation time Θ,
- the realizations C_{bl}^k, $k = 1,2,\dots,n_{bl}$, of the conditional sojourn times C_{bl} of the climate-weather change process at the climate-weather state c_b when the next transition is to the climate-weather state c_l during the observation time Θ for each $b, l = 1,2,...,w$, $b \neq l$.

3.3. Procedure of estimating the basic parameters of the climate-weather change process of critical infrastructure

After collecting the statistical data, it is possible to estimate the unknown parameters of the climate-weather change process performing the following steps:
i) to determine the vector

$$[q(0)] = [q_1(0), q_2(0), \dots, q_w(0)], \tag{74}$$

of the realizations of the probabilities $q_b(0)$, $b = 1,2,...,w$, of the climate-weather change process staying at the climate-weather states at the initial moment $t = 0$, according to the formula

$$q_b(0) = \frac{n_b(0)}{n(0)} \text{ for } b = 1,2,...,w, \tag{75}$$

where

$$n(0) = \sum_{b=1}^{w} n_b(0), \tag{76}$$

is the number of the realizations of the climate-weather change process starting at the initial moment $t = 0$;
ii) to determine the matrix

$$[q_{bl}] = \begin{bmatrix} q_{11} \ q_{12} \ \dots \ q_{1w} \\ q_{21} \ q_{22} \ \dots \ q_{2w} \\ \dots \\ q_{w1} \ q_{w2} \ \dots \ q_{vv} \end{bmatrix}, \tag{77}$$

of the realizations of the probabilities q_{bl}, b, l = 1,2,...,w, of the climate-weather change process transitions from the climate-weather state c_b to the climate-weather state c_l according to the formula

$$q_{bl} = \frac{n_{bl}}{n_b}, \text{ for } b, l = 1,2,...,w, b \neq l, \quad (78)$$

$$q_{bb} = 0, \text{ for } b = 1,2,...,w,$$

where

$$n_b = \sum_{b \neq l}^{w} n_{bl}, \ b = 1,2,...,w, \quad (79)$$

is the realization of the total number of the climate-weather change process departures from the climate-weather state c_b, b = 1,2,...,w, during the experiment time Θ.

3.4. Procedure of estimating the parameters of distributions of the climate-weather change process conditional sojourn times at climate-weather states for critical infrastructure

Prior to estimating the parameters of the distributions of the climate-weather change process conditional sojourn times at the particular climate-weather states, we have to perform the following steps:

i) to determine the following empirical characteristics of the realizations of the conditional sojourn time of the climate-weather change process at the particular climate-weather states:

- the realizations of the empirical mean values $\bar{C}_{bl}$ of the conditional sojourn times C_{bl} of the climate-weather change process at the climate-weather state c_b, b = 1,2,...,w, when the next transition is to the climate-weather state c_l, l = 1,2,...,w, according to the formula

$$\bar{C}_{bl} = \frac{1}{n_{bl}} \sum_{k=1}^{n_{bl}} C_{bl}^k, \ b, l = 1,2,...,w, b \neq l, \quad (80)$$

- the number $\bar{r}_{bl}$ of the disjoint intervals $I_j = < a_{bl}^j, b_{bl}^j)$, $j = 1,2,...,\bar{r}_{bl}$, that include the realizations C_{bl}^k, $k = 1,2,...,n_{bl}$, of the conditional sojourn times C_{bl} at the climate-weather state c_b when the next transition is to the climate-weather state c_l, according to the formula

$$\bar{r}_{bl} \cong \sqrt{n_{bl}},$$

- the length d_{bl} of the intervals $I_j = < a_{bl}^j, b_{bl}^j)$, $j = 1,2,...,\bar{r}_{bl}$, according to the formula

$$d_{bl} = \frac{\bar{R}_{bl}}{\bar{r}_{bl} - 1},$$

where

$$\bar{R}_{bl} = \max_{1 \le k \le n_{bl}} \theta_{bl}^k - \min_{1 \le k \le n_{bl}} \theta_{bl}^k,$$

- the ends a_{bl}^j, b_{bl}^j, of the intervals $I_j = < a_{bl}^j, b_{bl}^j)$, $j = 1,2,...,\bar{r}_{bl}$, according to the formulae

$$a_{bl}^1 = \max\{\min_{1 \le k \le n_{bl}} \theta_{bl}^k - \frac{d_{bl}}{2}, 0\},$$

$$b_{bl}^j = a_{bl}^1 + j d_{bl}, \ j = 1,2,...,\bar{r}_{bl},$$

$$a_{bl}^j = b_{bl}^{j-1}, \ j = 2,3...,\bar{r}_{bl},$$

in such a way that

$$I_1 \cup I_2 \cup ... \cup I_{\bar{r}_{bl}} = < a_{bl}^1, b_{bl}^{\bar{r}_{bl}})$$

and

$$I_i \cap I_j = \emptyset \text{ for all } i \neq j, \ i, j \in \{1,2,...,\bar{r}_{bl}\},$$

- the numbers n_{bl}^j of the realizations C_{bl}^k in the intervals $I_j = < a_{bl}^j, b_{bl}^j)$, $j = 1,2,...,\bar{r}_{bl}$, according to the formula

$$n_{bl}^j = \#\{k : C_{bl}^k \in I_j, k \in \{1,2,...,n_{bl}\}\}, \ j = 1,2,...,\bar{r}_{bl},$$

where

$$\sum_{j=1}^{\bar{r}_{bl}} n_{bl}^j = n_{bl},$$

whereas the symbol # means the number of elements of the set.

ii) to estimate the parameters of the distributions of the conditional sojourn times of the climate-weather change process at the particular climate-weather states for the following distinguished distributions respectively in the following way:

- for the uniform distribution with the density function

$$c_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \dfrac{1}{y_{bl} - x_{bl}}, & x_{bl} \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases} \tag{81}$$

where $0 \le x_{bl} < y_{bl} < +\infty$;

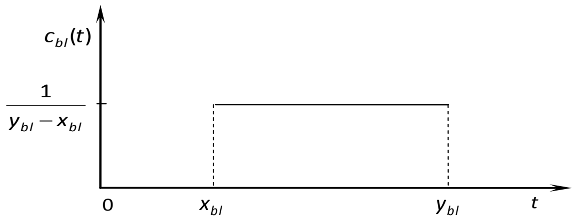


Figure.4. The graph of the uniform distribution's density function

the estimates of the unknown parameters of this distribution are:

$$x_{bl} = a_{bl}^{1} \;\; y_{bl} = x_{bl} + \bar{r}_{bl} d_{bl}; \tag{82}$$

- the triangular distribution with the density function

$$c_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \dfrac{2}{y_{bl} - x_{bl}} \dfrac{t - x_{bl}}{z_{bl} - x_{bl}}, & x_{bl} \le t \le z_{bl} \\ \dfrac{2}{y_{bl} - x_{bl}} \dfrac{y_{bl} - t}{y_{bl} - z_{bl}}, & z_{bl} \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases} \tag{83}$$

where

$0 \le x_{bl} \le z_{bl} \le y_{bl} < +\infty, y_{bl} \ne 0$;

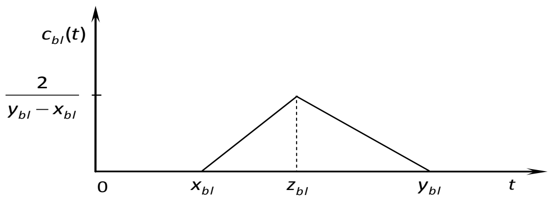


Figure 5. The graph of the triangular distribution's density function

the estimates of the unknown parameters of this distribution are:

$x_{bl} = a_{bl}^{1}$, $\quad y_{bl} = x_{bl} + \bar{r}_{bl} d_{bl}$, $\quad z_{bl} = \overline{C}_{bl}$; (84)

- the double trapezium distribution with the density function

$$c_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ q_{bl} + [\dfrac{2 - q_{bl}(z_{bl} - x_{bl}) - w_{bl}(y_{bl} - z_{bl})}{y_{bl} - x_{bl}} \\ - q_{bl}] \dfrac{t - x_{bl}}{z_{bl} - x_{bl}}, & x_{bl} \le t \le z_{bl} \\ w_{bl} + [\dfrac{2 - q_{bl}(z_{bl} - x_{bl}) - w_{bl}(y_{bl} - z_{bl})}{y_{bl} - x_{bl}} \\ - w_{bl}] \dfrac{y_{bl} - t}{y_{bl} - z_{bl}}, & z_{bl} \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases} \tag{85}$$

where

$0 \le x_{bl} \le z_{bl} \le y_{bl} < +\infty, y_{bl} \ne 0;\ 0 \le q_{bl} < +\infty,$
$0 \le w_{bl} < +\infty,\ 0 \le q_{bl}(z_{bl} - x_{bl}) + w_{bl}(y_{bl} - z_{bl}) \le 2;$

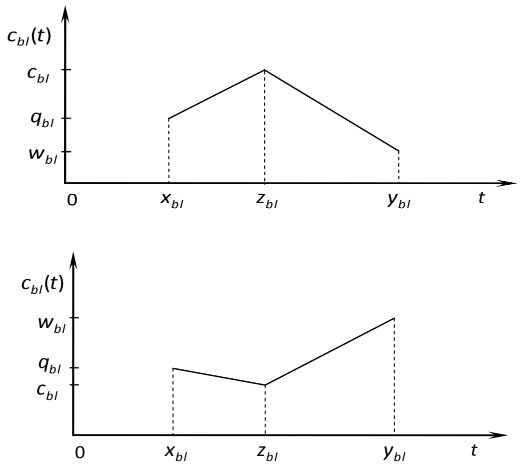


Figure 6. The graphs of the double trapezium distribution's density functions

the estimates of the unknown parameters of this distribution are:

$$x_{bl} = a_{bl}^{1},\ y_{bl} = x_{bl} + \bar{r}_{bl} d_{bl},\ q_{bl} = \frac{n_{bl}^{1}}{n_{bl} d_{bl}},$$

$$w_{bl} = \frac{n_{bl}^{\bar{r}_{bl}}}{n_{bl} d_{bl}},\ z_{bl} = \overline{C}_{bl}; \tag{86}$$

- the quasi-trapezium distribution with the density function

$$c_{bl}(t)=\begin{cases}0, & t<x_{bl}\\ q_{bl}+\dfrac{A_{bl}-q_{bl}}{z_{bl}^{1}-x_{bl}}(t-x_{bl}), & x_{bl}\le t\le z_{bl}^{1}\\ A_{bl}, & z_{bl}^{1}\le t\le z_{bl}^{2}\\ w_{bl}+\dfrac{A_{bl}-w_{bl}}{y_{bl}-z_{bl}^{2}}(y_{bl}-t), & z_{bl}^{2}\le t\le y_{bl}\\ 0, & t>y_{bl},\end{cases} \tag{87}$$

where

$$A_{bl}=\frac{2-q_{bl}(z_{bl}^{1}-x_{bl})-w_{bl}(y_{bl}-z_{bl}^{2})}{z_{bl}^{2}-z_{bl}^{1}+y_{bl}-x_{bl}},$$
$$0\le x_{bl}\le z_{bl}^{1}\le z_{bl}^{2}\le y_{bl}<+\infty, y_{bl}\ne 0,$$
$$0\le q_{bl}<+\infty,\ 0\le w_{bl}<+\infty;$$

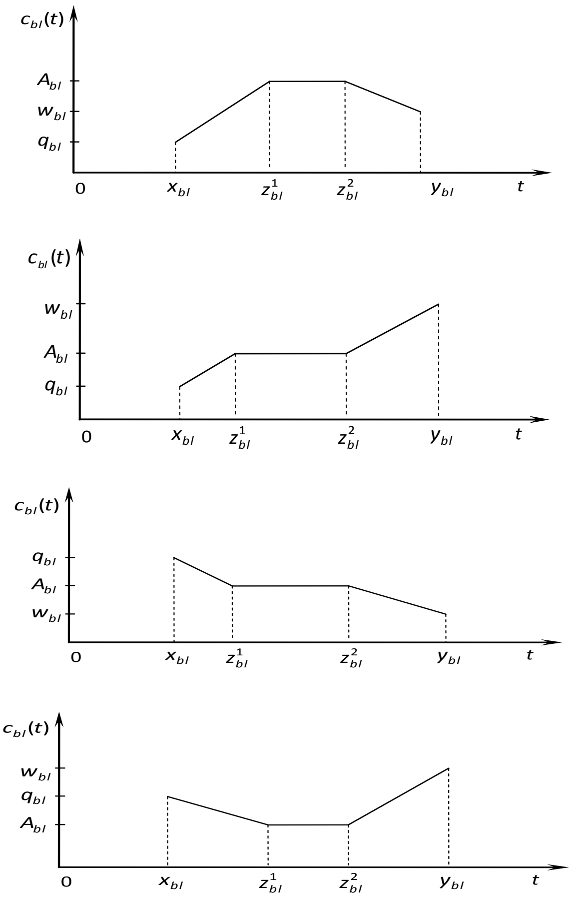


Figure 7. The graphs of the quasi-trapezium distribution's density functions

the estimates of the unknown parameters of this distribution are:

$$x_{bl}=a_{bl}^{1},\ y_{bl}=x_{bl}+\bar{r}_{bl}d_{bl},\ q_{bl}=\frac{n_{bl}^{1}}{n_{bl}d_{bl}},$$

$$w_{bl}=\frac{n_{bl}^{\bar{r}_{bl}}}{n_{bl}d_{bl}},\ z_{bl}^{1}=\bar{C}_{bl}^{1},\ z_{bl}^{2}=\bar{C}_{bl}^{2}, \tag{88}$$

where

$$\bar{C}_{bl}^{1}=\frac{1}{n_{(me)}}\sum_{k=1}^{n(me)}C_{bl}^{k},\ \bar{C}_{bl}^{2}=\frac{1}{n_{bl}-n_{(me)}}\sum_{k=n(me)+1}^{nbl}C_{bl}^{k},$$
$$n_{(me)}=\left[\frac{n_{bl}+1}{2}\right], \tag{89}$$

and $[x]$ denotes the entire part of x;

- the exponential distribution with the density function

$$c_{bl}(t)=\begin{cases}0, & t<x_{bl}\\ \alpha_{bl}\exp[-\alpha_{bl}(t-x_{bl})], & t\ge x_{bl},\end{cases} \tag{90}$$

where

$$0<\alpha_{bl}<+\infty;$$

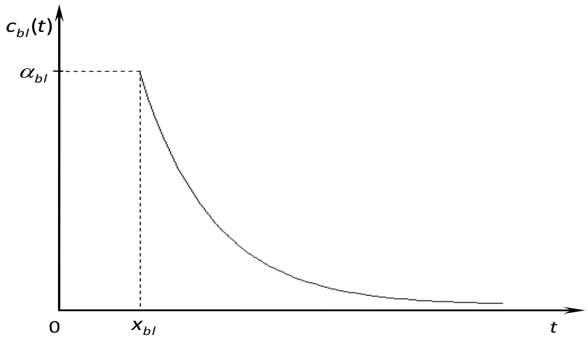


Figure 8. The graph of the exponential distribution's density function

the estimates of the unknown parameters of this distribution are:

$$x_{bl}=a_{bl}^{1},\ \alpha_{bl}=\frac{1}{\bar{C}_{bl}-x_{bl}}; \tag{91}$$

- the Weibull distribution with the density function

$$c_{bl}(t)=$$
$$\begin{cases}0, & t<x_{bl}\\ \alpha_{bl}\beta_{bl}(t-x_{bl})^{\beta bl-1}\exp[-\alpha_{bl}(t-x_{bl})^{\beta bl}], & t\ge x_{bl},\end{cases}$$

where

$$0<\alpha_{bl}<+\infty,\ 0<\beta_{bl}<+\infty;$$

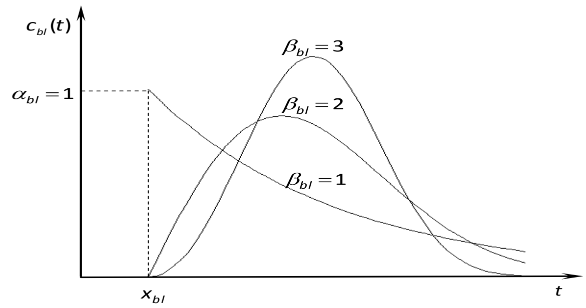


Figure 9. The graphs of the Weibull distribution's density functions

the estimates of the unknown parameters of this distribution are (the expressions for estimates of parameters α_{bl} and β_{bl} are not explicit):

$$x_{bl} = a_{bl}^1,\ \alpha_{bl} = \frac{n_{bl}}{\sum_{k=1}^{n_{bl}} (C_{bl}^k)^{\beta_{bl}}},$$

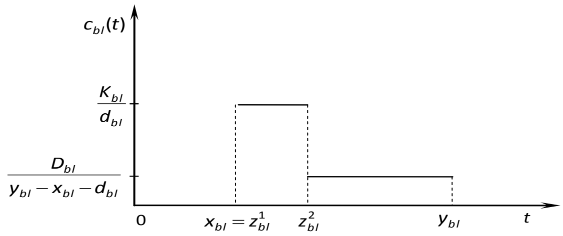
$$\alpha_{bl} = \frac{\frac{n_{bl}}{\beta_{bl}} + \sum_{k=1}^{n_{bl}} \ln(C_{bl}^k - x_{bl})}{\sum_{k=1}^{n_{bl}} (C_{bl}^k)^{\beta_{bl}} \ln(C_{bl}^k - x_{bl})}; \tag{93}$$

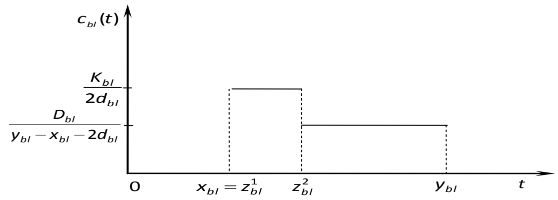
- the chimney distribution with the density function

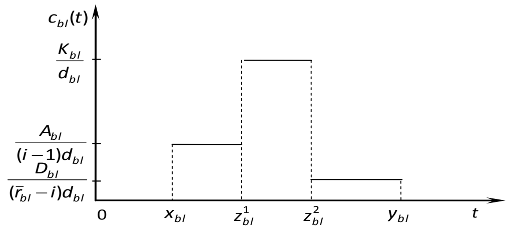
$$c_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \frac{A_{bl}}{z_{bl}^1 - x_{bl}}, & x_{bl} \le t \le z_{bl}^1 \\ \frac{K_{bl}}{z_{bl}^2 - z_{bl}^1}, & z_{bl}^1 \le t \le z_{bl}^2 \\ \frac{D_{bl}}{y_{bl} - z_{bl}^2}, & z_{bl}^2 \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases} \tag{94}$$

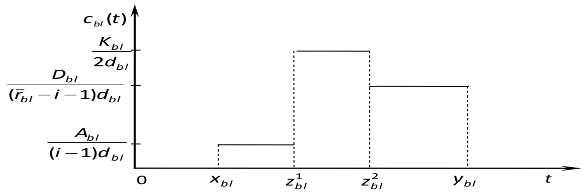
where

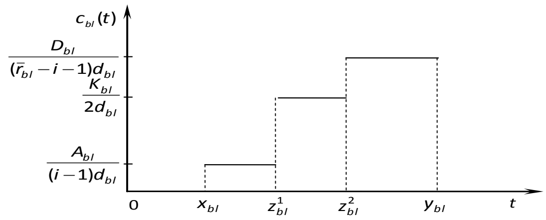
$$0 \le x_{bl} \le z_{bl}^1 \le z_{bl}^2 \le y_{bl} < +\infty, y_{bl} \ne 0,\ A_{bl} \ge 0,$$
$$K_{bl} \ge 0,\ D_{bl} \ge 0,\ A_{bl} + K_{bl} + D_{bl} = 1;$$

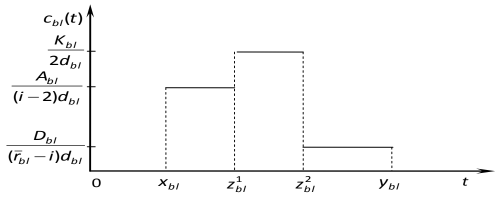


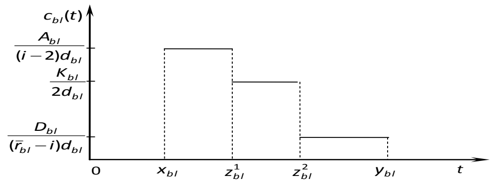


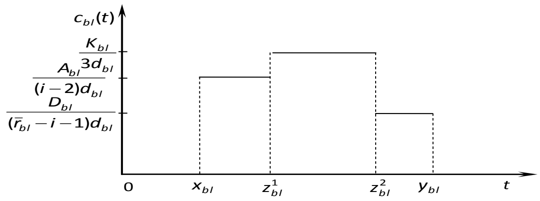












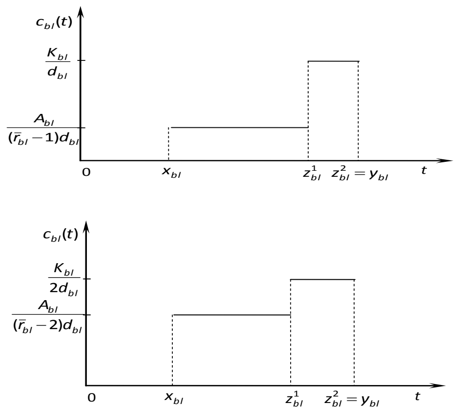


Figure 10. The graphs of the chimney distribution's density functions

the estimates of the unknown parameters of this distribution are:

$$x_{bl} = a_{bl}^1,\ y_{bl} = x_{bl} + \bar{r}_{bl} d_{bl}, \tag{95}$$

and moreover, if

$$\hat{n}_{bl} = \max_{1 \le j \le \bar{r}_{bl}} \{n_{bl}^j\} \tag{96}$$

and i, $i \in \{1, 2, \ldots, \bar{r}_{bl}\}$, is the number of the interval including the largest number of realizations i.e. such as that

$$n_{bl}^i = \hat{n}_{bl}, \tag{97}$$

then:

- for $i = 1$

either

$$z_{bl}^1 = x_{bl} + (i-1)d_{bl},\ z_{bl}^2 = x_{bl} + id_{bl},$$
$$A_{bl} = 0,\ K_{bl} = \frac{n_{bl}^i}{n_{bl}},\ D_{bl} = \frac{n_{bl}^{i+1} + \ldots + n_{bl}^{\bar{r}_{bl}}}{n_{bl}}, \tag{98}$$

while

$$n_{bl}^{i+1} = 0 \text{ or } n_{bl}^{i+1} \ne 0 \text{ and } \frac{n_{bl}^i}{n_{bl}^{i+1}} \ge 3, \tag{99}$$

or

$$z_{bl}^1 = x_{bl} + (i-1)d_{bl},\ z_{bl}^2 = x_{bl} + (i+1)d_{bl},$$
$$A_{bl} = 0, \tag{100}$$

$$K_{bl} = \frac{n_{bl}^i + n_{bl}^{i+1}}{n_{bl}},\ D_{bl} = \frac{n_{bl}^{i+2} + \ldots + n_{bl}^{\bar{r}_{bl}}}{n_{bl}}, \tag{101}$$

while

$$n_{bl}^{i+1} \ne 0 \text{ and } \frac{n_{bl}^i}{n_{bl}^{i+1}} < 3; \tag{102}$$

- for $i = 2, 3, \ldots, \bar{r}_{bl} - 1$

either

$$z_{bl}^1 = x_{bl} + (i-1)d_{bl},\ z_{bl}^2 = x_{bl} + id_{bl},$$
$$A_{bl} = \frac{n_{bl}^1 + \ldots + n_{bl}^{i-1}}{n_{bl}},\ K_{bl} = \frac{n_{bl}^i}{n_{bl}}, \tag{103}$$

$$D_{bl} = \frac{n_{bl}^{i+1} + \ldots + n_{bl}^{\bar{r}_{bl}}}{n_{bl}},$$

while

$$n_{bl}^{i-1} = 0 \text{ or } n_{bl}^{i-1} \ne 0 \text{ and } \frac{n_{bl}^i}{n_{bl}^{i-1}} \ge 3 \tag{105}$$

and while

$$n_{bl}^{i+1} = 0 \text{ or } n_{bl}^{i+1} \ne 0 \text{ and } \frac{n_{bl}^i}{n_{bl}^{i+1}} \ge 3, \tag{106}$$

or

$$z_{bl}^1 = x_{bl} + (i-1)d_{bl},\ z_{bl}^2 = x_{bl} + (i+1)d,$$
$$A_{bl} = \frac{n_{bl}^1 + \ldots + n_{bl}^{i-1}}{n_{bl}}, \tag{107}$$

$$K_{bl} = \frac{n_{bl}^i + n_{bl}^{i+1}}{n_{bl}},\ D_{bl} = \frac{n_{bl}^{i+2} + \ldots + n_{bl}^{\bar{r}_{bl}}}{n_{bl}}, \tag{108}$$

while

$$n_{bl}^{i-1} = 0 \text{ or } n_{bl}^{i-1} \ne 0 \text{ and } \frac{n_{bl}^i}{n_{bl}^{i-1}} \ge 3 \tag{109}$$

and while

$$n_{bl}^{i+1} \ne 0 \text{ and } \frac{n_{bl}^i}{n_{bl}^{i+1}} < 3, \tag{110}$$

or

$$z_{bl}^1 = x_{bl} + (i-2)d_{bl},\ z_{bl}^2 = x_{bl} + id_{bl},$$
$$A_{bl} = \frac{n_{bl}^1 + ... + n_{bl}^{i-2}}{n_{bl}},\ K_{bl} = \frac{n_{bl}^{i-1} + n_{bl}^i}{n_{bl}}, \quad (111)$$
$$D_{bl} = \frac{n_{bl}^{i+1} + ... + n_{bl}^{\bar{r}_{bl}}}{n_{bl}}, \quad (112)$$

while

$$n_{bl}^{i-1} \neq 0 \text{ and } \frac{n_{bl}^i}{n_{bl}^{i-1}} < 3, \quad (113)$$

and while

$$n_{bl}^{i+1} = 0 \text{ or } n_{bl}^{i+1} \neq 0 \text{ and } \frac{n_{bl}^i}{n_{bl}^{i+1}} \geq 3, \quad (114)$$

or

$$z_{bl}^1 = x_{bl} + (i-2)d_{bl},\ z_{bl}^2 = x_{bl} + (i+1)d_{bl},$$
$$A_{bl} = \frac{n_{bl}^1 + ... + n_{bl}^{i-2}}{n_{bl}}, \quad (115)$$

$$K_{bl} = \frac{n_{bl}^{i-1} + n_{bl}^i + n_{bl}^{i+1}}{n_{bl}},$$
$$D_{bl} = \frac{n_{bl}^{i+2} + ... + n_{bl}^{\bar{r}_{bl}}}{n_{bl}}, \quad (116)$$

while

$$n_{bl}^{i-1} \neq 0 \text{ and } \frac{n_{bl}^i}{n_{bl}^{i-1}} < 3 \quad (117)$$

and while

$$n_{bl}^{i+1} \neq 0 \text{ and } \frac{n_{bl}^i}{n_{bl}^{i+1}} < 3; \quad (118)$$

- for $i = \bar{r}_{bl}$

either

$$z_{bl}^1 = x_{bl} + (i-1)d_{bl},\ z_{bl}^2 = x_{bl} + id_{bl},$$
$$A_{bl} = \frac{n_{bl}^1 + ... + n_{bl}^{i-1}}{n_{bl}}, \quad (119)$$

$$K_{bl} = \frac{n_{bl}^i}{n_{bl}},\ D_{bl} = 0, \quad (120)$$

while

$$n_{bl}^{i-1} = 0 \text{ or } n_{bl}^{i-1} \neq 0 \text{ and } \frac{n_{bl}^i}{n_{bl}^{i-1}} \geq 3, \quad (121)$$

or

$$z_{bl}^1 = x_{bl} + (i-2)d_{bl},\ z_{bl}^2 = x_{bl} + id_{bl},$$
$$A_{bl} = \frac{n_{bl}^1 + ... + n_{bl}^{i-2}}{n_{bl}}, \quad (122)$$

$$K_{bl} = \frac{n_{bl}^{i-1} + n_{bl}^i}{n_{bl}},\ D_{bl} = 0, \quad (123)$$

while

$$n_{bl}^{i-1} \neq 0 \text{ and } \frac{n_{bl}^i}{n_{bl}^{i-1}} < 3. \quad (124)$$

- the Gamma distribution with the density function

$$c_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \dfrac{(t - x_{bl})^{\alpha_{bl}-1} \exp[-(t - x_{bl})/\beta_{bl}]}{\beta_{bl}^{\alpha_{bl}} \cdot \Gamma(\alpha_{bl})}, & t \geq x_{bl}, \end{cases}$$

where

$$0 < \alpha_{bl} < +\infty,\ 0 < \beta_{bl} < +\infty.$$

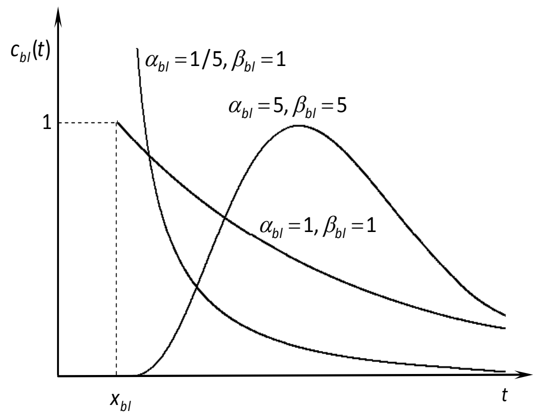


Figure 11. The graphs of the Gamma distribution's density functions

the estimates of the unknown parameters of this distribution are given as follows

$$x_{bl} = a_{bl}^1, \quad \alpha_{bl} = \frac{(\bar{C}_{bl} - x_{bl})^2}{(S_{bl}^*)^2},$$

$$\beta_{bl} = \frac{(S_{bl}^*)^2}{\bar{C}_{bl} - x_{bl}}, \tag{126}$$

where $(S_{bl}^*)^2$ is the variance of the conditional sojourn times C_{bl}, given by

$$(S_{bl}^*)^2 = \frac{1}{n_{bl} - 1}\sum_{k=1}^{n_{bl}}(C_{bl}^k - \bar{C}_{bl})^2.$$

3.5. Procedure of identifying the distribution functions of climate-weather change process conditional sojourn times at climate-weather states for the critical infrastructure

To formulate and next to verify the non-parametric hypothesis concerning the form of the distribution of the climate-weather change process conditional sojourn time C_{bl} at the climate-weather state c_b when the next transition is to the climate-weather state c_l, on the basis of at least 30 its realizations C_{bl}^k, $k = 1,2,\ldots,n_{bl}$, it is due to proceed according to the following scheme:

- to construct and to plot the realization of the histogram of the climate-weather change process conditional sojourn time C_{bl} at the climate-weather state c_b, defined by the following formula

$$\bar{h}_{n_{bl}}(t) = \frac{n_{bl}^j}{n_{bl}} \text{ for } t \in I_j, \tag{127}$$

- to analyze the realization of the histogram $\bar{h}_{n_{bl}}(t)$, comparing it with the graphs of the density functions $c_{bl}(t)$ of the previously distinguished in Section 4.2.4 distributions, to select one of them and to formulate the null hypothesis H_0, concerning the unknown form of the distribution of the conditional sojourn time C_{bl} in the following form:

H_0: The climate-weather change process conditional sojourn time C_{bl} at the climate-weather state c_b when the next transition is to the climate-weather state c_l, has the distribution with the density function $c_{bl}(t)$

- to join each of the intervals I_j that has the number n_{bl}^j of realizations less than 4 either with the neighbour interval I_{j+1} or with the neighbour interval I_{j-1} this way that the numbers of realizations in all intervals are not less than 4;
- to fix a new number of intervals $\bar{\bar{r}}_{bl}$;
- to determine new intervals

$$\bar{I}_j = < \bar{a}_{bl}^j, \bar{b}_{bl}^j), \quad j = 1,2,..,\bar{\bar{r}}_{bl};$$

- to fix the numbers $\bar{n}_{bl}^j$ of realizations in new intervals $\bar{I}_j$, $j = 1,2,..,\bar{\bar{r}}_{bl}$;
- to calculate the hypothetical probabilities that the variable C_{bl} takes values from the interval $\bar{I}_j$, under the assumption that the hypothesis H_0 is true, i.e. the probabilities

$$p_j = P(C_{bl} \in \bar{I}_j) = P(\bar{a}_{bl}^j \le C_{bl} < \bar{b}_{bl}^j)$$
$$= C_{bl}(\bar{b}_{bl}^j) - C_{bl}(\bar{a}_{bl}^j), \; j = 1,2,..,\bar{\bar{r}}_{bl}, \tag{128}$$

where $C_{bl}(\bar{b}_{bl}^j)$ and $C_{bl}(\bar{a}_{bl}^j)$ are the values of the distribution function $C_{bl}(t)$ of the random variable C_{bl} corresponding to the density function $c_{bl}(t)$ assumed in the null hypothesis H_0;

- to calculate the realization of the χ^2 (chi-square)-Pearson's statistics $U_{n_{bl}}$, according to the formula

$$u_{n_{bl}} = \sum_{j=1}^{\bar{\bar{r}}_{bl}} \frac{(\bar{n}_{bl}^j - n_{bl}p_j)^2}{n_{bl}p_j}; \tag{129}$$

- to assume the significance level α ($\alpha = 0.01$, $\alpha = 0.02$, $\alpha = 0.05$ or $\alpha = 0.10$) of the test;
- to fix the number $\bar{\bar{r}}_{bl} - l - 1$ of degrees of freedom, substituting for I for the distinguished in Section 4.2.4 distributions respectively the following values: $l = 0$, for the uniform, triangular, double trapezium, quasi-trapezium and chimney distributions, $l = 1$, for the exponential distribution and $l = 2$, for the Weibull distribution and Gamma distribution;
- to read from the Tables of the χ^2 Pearson's distribution the value u_α for the fixed values of the significance level α and the number of degrees of freedom $\bar{\bar{r}}_{bl} - l - 1$ such that the following equality holds

$$P(U_{n_{bl}} > u_\alpha) = \alpha, \tag{130}$$

and next to determine the critical domain in the form of the interval $(u_\alpha, +\infty)$ and the acceptance domain in the form of the interval $< 0, u_\alpha >$,

- to compare the obtained value $u_{n_{bl}}$ of the realization of the statistics $U_{n_{bl}}$ with the read from the Tables critical value u_α of the chi-square random variable and to decide on the previously formulated null hypothesis H_0 in the following way: if the value $u_{n_{bl}}$ does not belong these to the critical domain, i.e. when $u_{n_{bl}} \le u_\alpha$, then we do not reject the hypothesis H_0, otherwise if the value $u_{n_{bl}}$ belongs to the critical domain, i.e. when $u_{n_{bl}} > u_\alpha$, then we reject the hypothesis H_0.

3.6. Procedure of identifying the mean values of the climate-weather change proces conditional sojourn times at the climate-weather states for the critical infrastructure

After identifying the matrix of the conditional density functions of the climate-weather change process conditional sojourn times C_{bl}, b, $l = 1,2,\ldots,$ w, $b \neq l$, at the climate-weather states

$$[c_{bl}(t)]_{w\mathrm{x}w} = \begin{bmatrix} c_{11}(t)\, c_{12}(t) \ldots c_{1w}(t) \\ c_{21}(t)\, c_{22}(t) \ldots c_{2w}(t) \\ \ldots \\ c_{w1}(t)\, c_{w2}(t) \ldots c_{ww}(t) \end{bmatrix} \quad (131)$$

it is possible to determine the mean values of the climate-weather change process conditional sojourn times at the particular climate-weather change states using the following formula

$$M_{bl} = E[C_{bl}] = \int_0^\infty t c_{12}(t) dt, \quad (132)$$
$$b, l = 1,2,\ldots,w, \ b \neq l.$$

If there is lack of sufficient numbers of realizations of the climate-weather change process conditional sojourn times at the climate-weather states, it will not be possible to identify statistically their distributions. In those cases of not identified distributions, using formula (103), it is possible to find the approximate empirical values of the mean values of the conditional sojourn times at the particular climate-weather states or use their approximate values coming from experts.
If there are no realizations of the climate-weather change process conditional sojourn times at the climate-weather states of the critical infrastructure, then it is impossible to estimate their empirical conditional mean values.

4. Conclusions

The proposed statistical methods of identification of the unknown parameters of the climate-weather change processes allow us for the identification of the models discussed in [EU-CIRCLE Report D3.3-GMU12, 2016] and next their practical applications in evaluation, prediction and optimization safety of real complex critical infrastructures.

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