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STUDENTS' MATHEMATICAL COMPETENCE AFTER COMPLETING THE FIRST STAGE OF EDUCATION

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Abstract. In the school year 2009/10 a mathematics achievement test was conducted on a sample of 576 students in the 3th grade of school. A mathematical skills test examined the ability to complete both simple and complex tasks. It also tested their aptitude for the application of knowledge to practical and problematic situations. The skills checked were mastered at a level of about 65%. Skills in geometrics were at a lower level - around 37%. Problem solving ability was mastered at a very low level. One of the reasons is that school pupils are usually under teacher guidance while solving simple typical tasks. Rarely do they solve them independently or collectively. The teacher does not allow the student to err in search of a solution, and problematic tasks rarely be found in textbooks.

Poor results in mathematics in the matriculation exam, the final gymnasium exam and the final primary school test continuously concern everyone, especially the people responsible for Mathematical education, including maths teachers. Constant changes to the *Curriculum*, which sometimes give the impression of manipulation of it, are not beneficial to solid mathematical education. The four stages of it, where teachers of one stage often do not know the mathematical content learnt by the students in earlier or later stages, do not support consistent mathematical education, but they often create disparate fragments of knowledge. School textbook publishers tend to "relieve" teachers by the introduction of exercise books in which a child is limited to filling in the "blank spaces". This fact does not encourage student's self-reliance during the solution of tasks. The limitation of the new *Curriculum* for the early school education stage, according to which not more than 1/3 of students' notes can be made in exercise books, has not brought satisfactory changes. The reason is that a teacher has already got used to "easy" work based on "prepared materials".

What are the effects of this situation?

After the first school term 2009/10, the research on students' mathematical achievement was conducted on a sample of 576 students attending the III grade of primary school. A school mathematical achievement test, checking the range of material learnt by a student at school, was conducted [1]. It was a one stage test (without distinguishing tasks for particular requirement levels), however it contained tasks both easy and complex as well as of varied difficulty, which demanded the skill to apply knowledge in practical and atypical problematic situations. The test examined the following skills:

- arithmetic,
- geometric,
- practical,
- solving text-based tasks.

Two tasks, one text-based and the other related to geometric problems were atypical and required the students to notice inter-relationships, skills which are not always developed during classes in mathematical education. They will be analyzed separately. A precise, yet straightforward and clear, answer coding key showing correct and incorrect responses has been developed, which helped to detect the mistakes most commonly made by students. Basic indicators depicting the results of the research are shown in Table 1 as well as in the graph (Figure 1).

Table 1. Collation of basic indicators describing the test results

Number of subjects	n	576
Maximum possible result	X	33
Maximum result achieved	$X_{\max}$	32
Mean result	$\bar{X}$	20.6
Median	X_{me}	22
Mode	X_{mod}	23
Standard deviation	S_x	6.8

Source: Self study

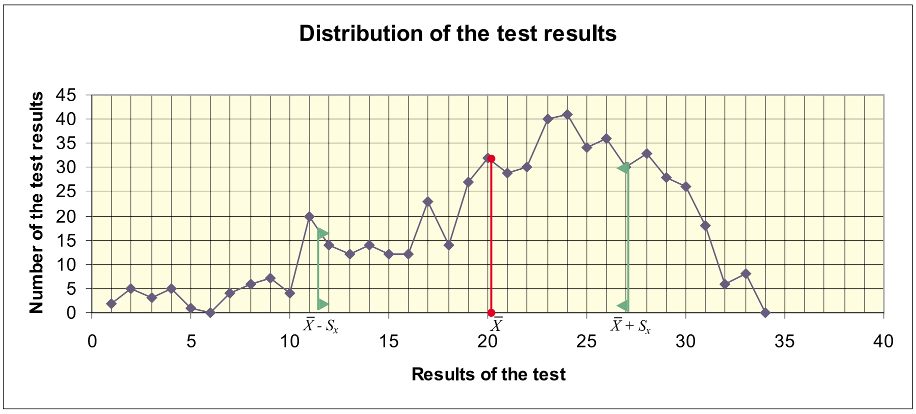


Figure 1. Graph showing the distribution of the test results.

The median value, 22 points – with the mean result being 20.6 – indicates that more than half of the subjects (61.5%) achieved a result higher than the mean. There were 71 poor results – below 12 points ($\bar{X} - S_x$) – constituting 12.3% of all the subjects and there were 86 results higher than 27 points ($\bar{X} + S_x$) constituting 15% of all the subjects. The above distribution of the test results could be considered to be satisfactory, although it is worthwhile analyzing the results achieved in the particular skills categories.

Table 2. Distribution of the results achieved in particular skills categories

Category of the skills being tested	Arithmetic	Geometric	Practical	Text based tasks	Problem based tasks
Maximum possible result	9	4	8	11	5
Mean result	5.72	1.49	5.68	7.60	1.60
Percentage of tasks solved in a category	63.60	37.15	71.03	69.10	31.94

Source: Self study

Skills in the category *the application of mathematics in practical situations* have been the best acquired. The tasks tested the skill to read a thermometer, to read a watch and to convert units of weights and measures. The level of arithmetic skills can be considered as low (63.6% of tasks solved) – the skills to arrange numbers in order and to calculate the results of arithmetical operations up to the number 100 were being tested.

Text based tasks, especially those regarding practical situations, were not too difficult for the students either. An atypical task, requiring students to notice a certain inter-relationship, was an exception. The task originated from research led by Miroslaw Dabrowski in 2005 in the project: **Third grade primary school students' basic skills test** [2]. Let us focus on the task.

Task 14
For 4 chocolate bars and 4 chocolates you have to pay 28 PLN. 3 of the same chocolate bars and 4 of the same chocolates cost in total 23 PLN. How much is a chocolate bar and how much is a chocolate?
Solution ....
Answer ....

Students of elementary education, who were also solving the task, most often started by making this set of equations:

$$\begin{cases} 4c + 4b = 28 \\ 3c + 4b = 23 \end{cases}$$

A third grade pupil is fortunately not familiar with this method and solves the task by applying the available information. In order to solve the task the pupil had to notice the difference between the two purchases. In the second purchase one less chocolate bar was bought and the difference in the cost of the two purchases represents the cost of one chocolate bar.

28 PLN − 23 PLN = 5 PLN	cost of one chocolate bar
5 PLN · 4 chocolate bars = 20 PLN	total cost of chocolate bars from the first purchase
28 PLN − 20 PLN = 8 PLN	total cost of 4 chocolates
8 PLN : 4 = 2 PLN	cost of one chocolate

The cost of one chocolate can also be worked out by proper calculation of the second purchase. Some pupils depicted the described situation which helped them to find the hidden values. A correct solution was the one in which the calculation or the result had been worked out on the basis of the depiction and there was a written answer. In this way 22.9% of the students tested solved the task. Twenty students (3.5%) limited themselves to giving the solution without writing the answer. These solutions can also be considered to be partly correct. A group of students (8.5%) made an attempt to solve the task, however they were unable to complete it.

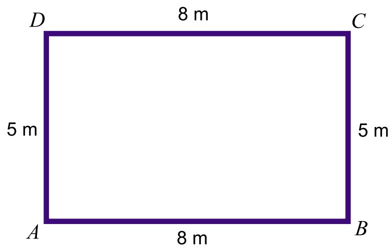
Unfortunately, as many as 65% of the students tested did not solve the task correctly, including 39.4% of the students who did not make an attempt to solve it. For such a large group the task proved to be too difficult. As mentioned above, the difficulty did not lie in complicated calculation, but in noticing a relatively simple inter-relationship. The conclusion may be that the students are not familiar with solving tasks which require independent analysis as well as noticing the inter-relationships between the described variables.

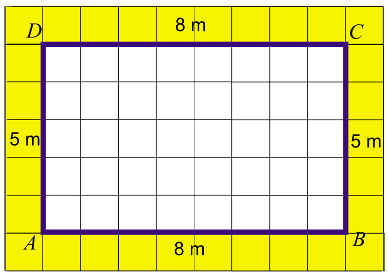
The geometric skills were tested by the use of two tasks. The first one concerned the identification of parallel and perpendicular segments – it was solved by 35% of the students. The second one required the students to notice an inter-relationship in a practical but atypical situation.

Task 5.

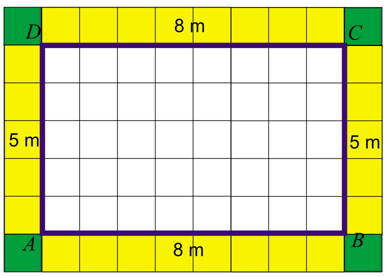
Around the swimming pool of measurements given in the picture, a pavement of square paving stones with the measure of 1 m per side was made. How many paving stones were used?

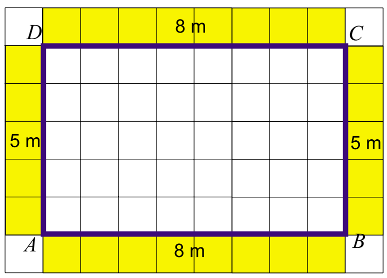
The task seems to be extremely easy as it is enough simply to draw a picture of the pavement surrounding the swimming pool, which helps to count the number of paving stones required. Several students dealt with the task in this manner. They marked the square paving stones in the picture and gave the result: **30 paving stones are needed**.





The second approach is to calculate by any method the perimeter of the swimming pool, which is a rectangle, and then add the number of corner paving stones to the result.





However, it turned out that the number of paving stones required was properly calculated by merely 12 of the students tested, which represents 2.1% of all the participants. Perhaps, if the initial picture had been drawn on graph paper, more students would have noticed the missing corner paving stones.

The vast majority of the students calculated the perimeter of the rectangle by the use of one of the following equations:

$$\text{Per.} = 2 \times a + 2 \times b \qquad \text{or} \qquad \text{Per.} = 2 \times (a + b)$$

or simply by performing the following calculations:

$$2 \times 8 + 2 \times 5 = 26 \quad \text{or} \quad 2 \times (8 + 5) = 26 \quad \text{or even} \quad 5 + 5 + 8 + 8 = 26.$$

71.4% of the tested students completed the calculation of the task with the above result. As many as 14.4% of the students did not make an attempt to solve the task, and 12% tried to solve it, however, they made errors which proved that they had not understood the instructions, or made errors in arithmetic calculation. Both of the presented tasks required the skill to seek the inter-relationships, namely research to discover mutual inter-relationships between the items of the given information. At school children solve simple typical tasks usually with the guidance of a teacher. Rarely do they seek the solution independently or collectively. The problem often first needs to be noticed and then individually formulated into a task. A teacher does not allow a student to err in search of a solution, and tasks with too little, too much or mutually exclusive data, which compelled a student to think, have been removed from school textbooks. Dorota Klus-Stańska has repeatedly been highlighting the necessity to seek and research within the framework of mathematical education in her studies [3].

Can a lack of a particular skill, namely the disability to solve problem tasks – the lack of the ability to notice the inter-relationships between the items of the provided information be pronounced on the basis of poor results in the two chosen tasks? After all, as it has been shown above, the overall results are not poor, they can even be considered to be satisfactory. The only problem is that the majority of the tasks regarded the so-called simple typical skills connected with arithmetic calculations.

However, the acquisition of mathematics is not only learning arithmetic and the skill to apply it to simple situations, but primarily the process of thinking, sometimes also referred to as mathematical thinking. Thinking or reasoning is a complex mental process which consists of seeking the inter-relationships between concepts and deductions. **Mathematical thinking** is unique in its logical thinking based on the defined assumptions, logical rules – definitions, theorems, but at the same time in its need to pose questions – hypotheses, although it is not always possible to answer them. It requires analysis and synthesis. Logical thinking, often considered as mathematical thinking, is necessary in every field of science which requires the skill to associate facts with their mutual inter-relationships [4].

Therefore, a question should be posed: *Are we concerned with students who acquire knowledge at the level of simple schemes or with creative students who have been prepared to solve even uncomplicated problem tasks?* The size of the group allows us to make a certain generalization: **students are unable to solve atypical tasks**. Where does the reason for this situation lie – in the low effectiveness of mathematical education in relation to the skills from higher categories of teaching aims?

In my opinion, the answer is relatively simple. The reason lies in schools. A teacher is limited by the *Curriculum* which "must be completed" by means of textbooks and workbooks provided, which allow a student only to "fill in blank spaces" instead of solving tasks independently.

Additionally, the introduced *Curriculum* reform and the retention of only simple text tasks in elementary mathematical education, as well as the change of system where, at least for the present, there are six and seven year old children in one class of over twenty pupils, do not improve the effectiveness of school education in the first stage of primary school.

The consequences of this situation at the end of the VI grade are reflected in the results of the test which takes place at the end of the second stage of education. They are available on the website of the Central Examination Board.

There is still the issue of the teacher – is he/she definitely prepared for creative work with a child within the framework of mathematical education?

However, this suspicion requires independent research.

References

[1] In the research a test conducted by a group of students in the V year of elementary education at the University of Opole was used. It was carried out under the guidance of prof. dr. hab. Gabriela Kapica, with the consultation of Jerzy Nowik.

[2] M. Dąbrowski. *Pozwólmy dzieciom myśleć. O umiejętnościach matematycznych polskich trzecioklasistów.* CKE, Warszawa 2008.

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[4] J. Mason, L. Burton, K. Stacey. *Matematyczne myślenie*. WSiP, Warszawa 2005.