

Arch. Min. Sci., Vol. 58 (2013), No 3, p. 805–822

Electronic version (in color) of this paper is available: http://mining.archives.pl

DOI 10.2478/amsc-2013-0056

PIOTR SOBOTA*

DETERMINATION OF THE FRICTION WORK OF A LINK CHAIN INTERWORKING WITH A SPROCKET DRUM

WYZNACZENIE PRACY TARCIA ŁAŃCUCHA OGNIWOWEGO WE WSPÓŁDZIAŁANIU Z BĘBNEM ŁAŃCUCHOWYM

The significant abrasive wear of sprocket drum teeth and seats bottoms is observed during the exploitation of longwall scraper conveyors. For this reason, it is important to determine friction work in sliding conditions of the horizontal link on the tooth seat bottom and on the tooth flank and friction work in the joint of links in the context of such nodes' abrasive wear. The different construction variants of sprocket drums can be compared by determining friction work in the sliding positions of the horizontal link on the drum. The determination of the losses of the power transmitted is a requisite condition in such situation for determining the efficiency values of chain meshing.

The friction work of the friction couple of a sprocket drum – link chain consists of friction work of the horizontal link in the places where it contacts with the seat bottom A_g and the tooth flank A_f and friction work in the joints of a horizontal link in the contact place with vertical links: in the front joint A_p and the rear joint A_t . The article presents dependencies enabling to determine the value of such work for specific geometric relations between the chain and the drum and different friction conditions. The curves of relative friction work and the values of total friction work on the seat bottom, on the tooth flank and in a front and rear joint of links are presented for examples of friction conditions.

Keywords: longwall conveyor, sprocket drum, scraper chain, friction work

W czasie eksploatacji ścianowych przenośników zgrzebłowych obserwuje się znaczne zużycia ścierne powierzchni zębów i den gniazd bębnów łańcuchowych. Z tych powodów ważne jest określenie pracy tarcia w warunkach poślizgu ogniwa poziomego na dnie gniazda i na flance zęba oraz pracy tarcia w przegubach ogniw w aspekcie zużycia ściernego tych węzłów. Wyznaczenie pracy tarcia w miejscach poślizgu ogniwa poziomego na bębnie daje możliwość porównania różnych wariantów konstrukcyjnych bębnów łańcuchowych. Określenie strat przenoszonej mocy jest przy tym istotnym warunkiem określenia wartości sprawności zazębienia łańcuchowego.

Na pracę tarcia pary ciernej bęben łańcuchowy – łańcuch ogniwowy składa się praca tarcia ogniwa poziomego w miejscach jego kontaktu z dnem gniazda A_g i flanką zęba A_f oraz praca tarcia w przegubach

^{*} INSTITUTE OF MINING MECHANISATION, FACULTY OF MINING AND GEOLOGY, SILESIAN UNIVERSITY OF TECH-NOLOGY, AKADEMICKA 2, 44-100 GLIWICE, POLAND

ogniwa poziomego w miejscach kontaktu z ogniwami pionowymi: w przegubie przednim A_p i w przegubie tylnym A_r . W artykule przedstawiono zależności umożliwiające wyznaczenie wartości tych prac dla określonych relacji geometrycznych pomiędzy łańcuchem a bębnem i różnych warunków tarcia. Dla przykładowych warunków tarcia, zaprezentowano przebiegi względnej pracy tarcia oraz wartości sumarycznej pracy tarcia na dnie gniazda, na flance zęba oraz w przegubie przednim i tylnym ogniw.

W czasie eksploatacji ścianowego przenośnika zgrzebłowego następuje – głównie na skutek zużycia – zwiększenie podziałki łańcucha ogniwowego. Zwiększenie długości podziałki łańcucha wynoszące Δp najczęściej opisuje się względnym zwiększeniem podziałki odniesionym do podziałki technologicznej $\Delta p/p$ i wyrażonym w procentach. Podczas współdziałania bębna łańcuchowego o wymiarach normowych z łańcuchem o zwiększonej podziałce nabiegające ogniwo poziome nie styka się z dnem gniazda na całej swej długości. To zazębienie charakteryzuje się tym, że ogniwa poziome łańcucha znajdujące się na bębnie łańcuchowym o liczbie zębów z są nachylone względem den gniazd pod kątem ε tak, że ich torusy przednie stykają się dnami gniazd a torusy tylne stykają się z bokami roboczymi segmentów zębów bębna o kącie pochylenia względem dna gniazda β . W celu jednoznacznego opisu położenia ogniw łańcucha w gniazda bębna (rys. 1) wyznaczyć należy kąt nachylenia ogniw względem den gniazd koła ε , odległość środka przegubu przy torusie przednim ogniwa poziomego od początku boku wieloboku foremnego u oraz kąt obrotu ogniwa poziomego względem poprzedzającego ogniwa poziomego w środku przegubu przy torusie przednim ogniwa względene podziałki tym większe wartości osiągają parametry opisujące położenie ogniw w gniazdach koła łańcuchowego (ε , u oraz α^u).

Ze względu na powtarzalność położenia ogniw w gniazdach bębna łańcuchowego o liczbie zębów z podczas nabiegania łańcucha następuje cykliczne obciążanie kolejnych den gniazd, flanek zębów i ogniw w czasie obrotu bębna łańcuchowego o kąt podziałowy $\varphi = 2\pi/z$. W zakresie obrotu bębna łańcuchowego o kąt podziałowy wyróżniono trzy przedziały charakteryzujące się odmiennym sposobem obciążenia elementów bębna łańcuchowego (rys. 2÷4).

Poślizg torusa przedniego ogniwa poziomego na dnie gniazda ma miejsce w pierwszym przedziale obciążenia bębna o kąt podziałowy. W przedziale tym wyróżnić można w przegubie przednim dwie fazy: toczenia i poślizgu ogniw. Wyznaczono drogę tarcia oraz pracę tarcia na dnie gniazda podczas toczenia się ogniw oraz podczas poślizgu ogniw w przegubie przednim (zależności 5÷17). Wykorzystując te zależ-ności wyznaczono drogę tarcia i pracę tarcia na dnie gniazda bębna łańcuchowego o liczbie zębów z = 7, współdziałającego z łańcuchem wielkości 34×126 mm o podziałkach ogniw wydłużonych o $\Delta p/p = 0,5\%$ i $\Delta p/p = 3,0\%$. Pracę tarcia na dnie gniazda A_g wyznaczono całkując numerycznie iloczym drogi tarcia i odpowiedniej wartości siły *R* w punkcie styku ogniwa z dnem gniazda. Z powodu względnego określenie siły w punkcie styku do wartości siły nabiegającej R/S_{H} również pracę tarcia dnie gniazda wyznaczono jako względną w stosunku do siły nabiegającej jako A_g/S_H . Względną pracę tarcia na dnie gniazda (rys. 8÷9).

Od chwili, w której wartość siły *R* spada do zera przy kącie obrotu bębna φ_{R0} , rozpoczyna się przedział trzeci kąta podziałowego, w którym na flance zęba pojawia się siła *T* prostopadła do reakcji *F* i skierowana w stronę głowy zęba, niezbędna do utrzymania ogniwa poziomego w równowadze (rys. 4). Zapobiega ona poślizgowi torusa tylnego ogniwa poziomego po flance zęba w stronę dna gniazda. Sformułowano warunek wystąpienia poślizgu torusa tylnego ogniwa poziomego po flance zęba i wyznaczono drogę poślizgu i względną pracę tarcia A_f/S_H dla tego przypadku (zależności 20÷21).

W czasie obrotu bębna łańcuchowego o kąt podziałowy następuje wzajemny obrót ogniwa poziomego względem poprzedzającego go ogniwa pionowego w przegubie przednim oraz wzajemny obrót ogniwa pionowego następującego po ogniwie poziomym wokół torusa tylnego ogniwa poziomego w przegubie tylnym. W obydwóch przegubach wyróżnić można dwie fazy obrotu ogniw: toczenia się i poślizgu ogniw w przegubie. Wyznaczono sumaryczną pracę tarcia w przegubie przednim A_p , będącą sumą pracy tarcia przy toczeniu i poślizgu ogniw (zależności 22÷26) oraz pracę tarcia w przegubie tylnym A_t (zależności 30÷39). Dla określonych warunków tarcia oraz obciążenia ogniw wyznaczono względną pracę tarcia w przegubie przednim A_p/S_H i tylnym A_t/S_H przy obrocie bębna łańcuchowego o kąt podziałowy (rys. 12÷13).

Słowa kluczowe: przenośnik ścianowy, bęben łańcuchowy, łańcuch zgrzebłowy, praca tarcia

1. Introduction

Sprocket drums are one of the most important parts of a longwall conveyor drive system as the hauling force of a scraper chain transporting the mined rock in a pan line is transmitted onto such drums. At present, sprocket drums are one of the least durable parts of scraper conveyors. Despite various technical solutions applied, numerous adverse effects are observed during the interwork of sprocket drums with chain links, such as: chain links are skipping on the drum, links are seizing in the seats hindering their demeshing, links are positioned in seats inappropriately, the chain and the drums exhibit excessive wear. This last phenomenon is especially an important functional issue in the operation of scraper conveyors. The significant abrasive wear has been observed of the surface of teeth and chain drums' seats interworking with chain links at high friction coefficients (Schaeffer, 1976). For this reason, it is important to determine friction work in sliding conditions of the horizontal link on the tooth seat bottom and on the tooth flank and friction work in the joint of links in the context of analysing such nodes' abrasive wear for the following configuration: sprocket drum - link chain. Different construction variants of sprocket drums can be compared by determining friction work in the sliding locations of the horizontal link on the drum. The determination of the losses of the power transmitted is a requisite condition in such situation for determining the efficiency values of chain meshing.

The friction work of the friction couple of a sprocket drum – link chain consists of friction work of the horizontal link in the places where it contacts with the seat bottom A_g and the tooth flank A_f and friction work in the joints of a horizontal link in the contact place with vertical links: in the front joint A_p and the rear joint A_t .

2. Load on sprocket drum and link chain parts

The link chain pitch is increasing and the sprocket drum pitch is decreasing during the operation of a longwall scraper conveyor – mainly due to wear. The actual pitch of chain links due to friction on the link joints while the chain is running on and unwinding from the sprocket drum is increasing over the technological pitch p of the link chain as the hauling system of the chain is operated. The increased length of the chain pitch resulting from chain production tolerances and from the abrasive wear of links in the joints of Δp is most often described by a relative increase in the pitch in relation to the technological pitch $\Delta p/p$ and expressed in per cents.

When a sprocket drum with standard dimensions is interworking with a chain with the elongated pitch, the running-on horizontal link does not contact the seat bottom along its entire length. This meshing variant is characterised by a fact that the horizontal chain links positioned on the sprocket drum with the number of teeth z are inclined relative to the seat bottoms under the angle ε so that their front toruses are contacting the seat bottoms and the rear toruses are contacting the working sides of the drum teeth segments with the inclination angle relative to the seat bottom β . The following parameters are determined in order to clearly describe the position of the chain links in the drum seats (Fig. 1) (Dolipski et al., 2010, 2011):

- the links' inclination angle relative to the bottoms of the drum seats ε ;
- the distance between the centre of the joint with the front torus of the horizontal link from the beginning of the side of a regular polygon u;
- the rotation angle of the vertical link relative to the preceding horizontal link in the middle of the joint with the horizontal link rear torus α^{u} .

The longer relative elongation of the pitch the greater values achieved by the parameters describing the position of links in the sprocket wheel seats (ε , u and α^{u}).



Fig. 1. Position of chain links in the sprocket drum seats

The mobility effect of links in joints, when the links are tilting mutually, has been considered when analysing the interworking of the sprocket drum with the link chain, the result of which is the displacement of the contact point of the links (Dolipski, 1997). The tilting of the horizontal link relative to the vertical link is accompanied by link rolling or slide in the joint. When the links are rolling in the joint, the contact point of the links is displacing in the joint, and the position of their contact point in the joint remains unchanged during the slide of the links.

Due to the repeatability of the links' position in the sprocket drum seats with the number of teeth *z*, when the teeth are running on, the relevant seats bottoms, teeth flanks and chain links are loaded cyclically when the sprocket drum is rotated by the pitch angle of $\varphi = 2\pi/z$. It was assumed when analysing the loading of the sprocket drum elements that the drum rotation angle is changing from the moment the front torus of the running-on horizontal link is contacting the seat bottom ($\varphi = 0$) until the front torus of another horizontal link is contacting the bottom of the next seat ($\varphi = 2\pi/z$). Three ranges, characterised by the varied loading of the sprocket drum parts, are differentiated for sprocket drum rotation by the pitch angle (Dolipski et al., 2012).

The first pitch angle range lasts from the moment the front torus of the horizontal front link contacts the seat bottom until the moment the rear torus of the horizontal link contacts the tooth flank. Within this range, the link that is meshed is loaded with the run-on force S_H , with the force S_V conveyed onto the preceding vertical link and with the reactive force R between the front torus of the horizontal link and the seat bottom (Fig. 2). The slide of the front torus on the bottom seat is causing the friction force perpendicular to the reaction R. The tilting of the horizontal link in the joint is accompanied by link rolling or slide in the joint resulting in the displacement of the contact point of the links by the angle γ^P . The mobility of links in the front joint and the friction force on the seat bottom are forcing the tilting of the running-on vertical link in the rear joint by the inclination angle λ . This is accompanied by link rolling or slide in the rear joint resulting in the rear joint resulting in

the displacement of the contact point of the links by the angle γ_t . The first range comprises drum rotation by the angle φ changing within the range of:

$$0 \le \varphi \le \frac{2 \cdot \pi}{z} - \alpha^u + \lambda_k \tag{1}$$

where:

 λ_k — value of inclination angle λ at the end of the first range.



Fig. 2. Load on horizontal link in the first range

The second range (Fig. 3) lasts from the moment the rear torus of the horizontal link is contacting the tooth flank until the reaction R reaches a zero value and includes drum rotation by the angle φ changing within the following range:

$$\frac{2 \cdot \pi}{z} - \alpha^u + \lambda_k < \varphi \le \varphi_{R0} \tag{2}$$

where:

 φ_{R0} — drum rotation angle at which the reaction R attains a zero value.

The horizontal link is loaded with the run-on force S_H , reactive force R at the contact point with the seat bottom, with the reactive force F in the contact point of the rear torus with the tooth flank and with the force S_V transmitted onto the preceding vertical link (Fig. 3).

The first range commences from the moment where the value of the reaction R falls to zero and lasts until the front torus contacts the next horizontal link with the bottom of the next seat (Fig. 4):

$$\varphi_{R0} < \varphi \le \frac{2 \cdot \pi}{z} \tag{3}$$



Fig. 3. Load on horizontal link in the second range



Fig. 4. Load on the horizontal link in the third range

In the third range, the force T occurs on the tooth flank necessary for maintaining the balance of the horizontal link, preventing the slide of the rear torus of the horizontal link at the tooth flank towards the seat bottom. If the reaction R in the second range does not reach the zero value until the moment the front torus of the next horizontal link contacts the bottom of the next seat, i.e. for:

$$\varphi_{R0} \ge \frac{2 \cdot \pi}{z} \tag{4}$$

then the third pitch range of the sprocket drum does not occur.

The examples of curves of forces acting on the chain horizontal link and a drum during drum rotation by a pitch angle have been determined for a sprocket drum with the parameters conforming to the standard PN-G-46703, with the number of teeth of z = 7 and the tooth flank inclination angle of $\beta = 60^{\circ}$, interworking with a chain sized 34×126 mm with the joint module of $m_p = 0.85$ for the friction coefficient in the joint of $\mu_p = 0.1$ and the friction coefficient in seat bottom of $\mu_g = 0.15$. The individual ranges of load for sprocket drums interworking with links with their pitch enlarged by $\Delta p/p = 0.5\%$ (Fig. 5) occur for the drum rotation angle of φ changing within the range of:

- first range $0 \le \varphi \le 21.26^\circ$;
- second range $21.26^\circ < \varphi \le 50.13^\circ$;
- third range $50.13^\circ < \varphi \le 51.43^\circ$.



Fig. 5. Load curves of horizontal link and sprocket drum for $\Delta p/p = 0.5\%$ ($\mu_p = 0.1$; $\mu_g = 0.15$)

The individual ranges of load for a drum interworking with links with their pitch enlarged by $\Delta p/p = 3.0\%$ (Fig. 6) occur within the range of:

- first range $0 \le \varphi \le 10.85^\circ$;
- second range $10.85^\circ < \varphi \le 42.24^\circ$;
- third range $42.24^\circ < \varphi \le 51.43^\circ$.

The division limits of load may change over a wide range depending on the extension of the pitches of links of the chain interworking with a sprocket drum and on friction conditions.



Fig. 6. Load curve of horizontal link and sprocket drum for $\Delta p/p = 3.0\%$ ($\mu_p = 0.1$; $\mu_g = 0.15$)

3. Friction work for horizontal link slide on the seat bottom

The slide of the horizontal link front torus on the seat bottom takes place in the first range of loading the drum by the pitch angle. Two phases can be distinguished in the first range in a front joint: a links rolling and slide phase. The rolling of links in the front joint takes place for the drum rotation angle within the range of:

$$0 \le (\varphi - \lambda) \le \varphi_{gr} \tag{5}$$

whereas:

$$\varphi_{gr} = \frac{(1 - m_p)}{m_p} \cdot \arctan(\mu_p) \tag{6}$$

$$\gamma_p = \frac{m_p}{(1 - m_p)} \cdot (\varphi - \lambda) \tag{7}$$

$$m_p = \frac{d}{c} \tag{8}$$

where:

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$$n_p$$
 — joint module,

- d link rod diameter,
- c internal link dimension (Fig. 2).

When links are rolling in the front joint, the contact point of the links is displacing, and the vertical link is lowered in the drum tooth groove and the centre of the horizontal link front torus

is displaced relative to the seat bottom. The horizontal link front torus is sliding on the seat bottom as a result of such phenomena. The front torus sliding path on the bottom seat bottom L_{g1} during the rolling of links in the front joint is:

$$L_{g1} = \frac{d}{2} \cdot \left(\varphi - \lambda\right) + \frac{d}{2} \left(1 - \cos \gamma_p\right) \cdot \cos \nu \tag{9}$$

whereas:

$$v = \frac{2\pi}{z} + \varepsilon - \alpha^u \tag{10}$$

Friction work on the seat bottom during the rolling of links in the front joint A_{g1} is expressed with the following formula:

$$A_{g1} = \int_{0}^{\varphi_{g1} + \lambda} L_{g1} \cdot R \cdot \mu_g \cdot d\varphi$$
(11)

The second phase starts from the moment the horizontal link starts sliding in the front joint and ends when the horizontal link rear torus contacts the tooth flank. This occurs for the drum rotation angle of:

$$\varphi_{gr} + \lambda \le \varphi \le \frac{2 \cdot \pi}{z} - \alpha^u + \lambda_k \tag{12}$$

whereas:

$$\gamma_p = \arctan(\mu_p) \tag{13}$$

When links are sliding in the front joint, the contact point on the vertical link remains unchanged. The horizontal link front torus is sliding in the front joint and the torus is sliding at the same time on the seat bottom. The torus sliding path on the seat bottom L_{g2} and friction work A_{g2} :

$$L_{g2} = \frac{d}{2} \cdot \left(\varphi - \varphi_{gr} - \lambda\right) \tag{14}$$

$$A_{g2} = \int_{\varphi_{gr}}^{\frac{2\pi}{z} - \alpha^{u} + \lambda_{k}} L_{g2} \cdot R \cdot \mu_{g} \cdot d\varphi$$
(15)

The total horizontal link front torus sliding path on the seat bottom is:

$$L_g = L_{g1} + L_{g2} \tag{16}$$

and the total friction work for horizontal link slide on the seat bottom is:

$$A_g = A_{g1} + A_{g2} \tag{17}$$

A friction path and friction work have been determined using the dependency $(5\div17)$ on the sprocket drum socket bottom with the number of teeth of z = 7 interworking with a chain sized 34×126 mm with the pitches of links elongated by $\Delta p/p = 0.5\%$ and $\Delta p/p = 3.0\%$ for $\mu_p = 0.1$. The front torus sliding path on the seat bottom in the first phase of links' rolling in the front joint is $L_{e1} = 0.34$ mm, and in the second phase of sliding in the joint it reaches the value of $L_{g2} = 5.81$ mm for $\Delta p/p = 0.5\%$ and $L_{g1} = 0.34$ mm and $L_{g2} = 2.77$ mm for $\Delta p/p = 3.0\%$. The total sliding path of the horizontal link front torus on the seat bottom is $L_{g} = 6.15$ mm for $\Delta p/p = 0.5\%$ and $L_g = 3.13$ mm for $\Delta p/p = 3.0\%$. (Fig.7). The reason for shortening the sliding path in the second phase of sliding for a chain with more strongly elongated links is that the first range of drum load is shortened by pitch angle.



Fig. 7. Horizontal link front torus sliding path on the seat bottom

Friction work during the slide of the horizontal link front torus on the seat bottom was determined by integrating numerically the product of the friction path and the respective reactive force R value in the contact point between the link and the seat bottom. Friction work was also determined as relative to the run-on force as A_g/S_H [J/kN] because the reactive force in the contact point was determined relatively in relation to the run-on force R/S_H (Dolipski et al., 2012).

Shown in Fig. 8 is relative friction work on the seat bottom determined in the function of the drum rotation angle for different coefficient values of friction on the seat bottom ($\mu_g = 0.15$; $\mu_g =$ 0.30; $\mu_g = 0.45$) and for $\Delta p/p = 0.5\%$. Friction work on the seat bottom is rising in a nonlinear manner along with drum rotation thus reaching the higher values the higher is the friction coefficient value on the seat bottom. The value of relative total friction work with front torus slide on the seat bottom in such case is:

•	$A_g/S_H =$	0.196 J/kN	(for $\mu_g =$	0	.15	red	line)	;

- $A_g/S_H = 0.425 \text{ J/kN}$ $A_g/S_H = 0.696 \text{ J/kN}$ (for $\mu_g = 0.30$ green line);
- (for $\mu_g = 0.45$ blue line).



Fig. 8. Friction work on drum seat bottom for $\Delta p/p = 0.5\%$, $\mu_p = 0.1$

As the first phase of drum rotation is shortened by pitch angle, when the elongation of links pitch is growing, the sliding paths of the horizontal link on the seat bottom are reduced and the values of forces are smaller in the contact point of the joint with the seat bottom in the initial drum rotation period. Relative total friction work in the function of drum rotation angle for a chain with the pitches of links elongated by $\Delta p/p = 3.0\%$ is shown in Fig. 9. The value of relative total friction work on the seat bottom in such case is:

•	$A_g/S_H = 0.049 \text{ J/kN}$	(for $\mu_g = 0.15$);
•	$A_g/S_H = 0.103 \text{ J/kN}$	(for $\mu_g = 0.30$);
•	$A_g/S_H = 0.163 \text{ J/kN}$	(for $\mu_g = 0.45$).

4. Conditions for the occurrence of horizontal link slide on the tooth flank

The reactive force F (Fig. 3) is working in the contact point in the second range of pitch angle between the horizontal link and tooth flank. The system of forces acting on the horizontal link is balanced as long as the value of the reaction R in the contact point of the horizontal link with the seat bottom does not fall to zero. Starting with the moment when the value of the reaction Rfalls to zero for the drum rotation angle of φ_{R0} , the third range of pitch angle begins (Fig. 4). The force T occurs on the tooth flank then perpendicular to the reaction F and positioned towards the tooth head, necessary for maintaining the balance of the horizontal link. The force prevents the slide of the rear torus of the horizontal link at the tooth flank towards the seat bottom. If the value of the friction force induced by the pressing force of the reaction F on the tooth flank equals at least the value of the force T, then the system of forces is in balance and the horizontal link does not change its position in relation to the chain drum. If, however, the friction force coming from the pressing force F on the tooth flank is smaller than the value of the force T, then the slide of the horizontal link rear torus on the tooth flank towards the seat bottom occurs.



Fig. 9. Friction work on drum seat bottom for $\Delta p/p = 3.0\%$, $\mu_p = 0.1$

The slide condition of the horizontal link rear torus on the tooth flank depends on the friction coefficient value between the link and the tooth flank μ_f and can be expressed as:

$$T > F \cdot \mu_f \tag{18}$$

The slide occurrence condition can also be recorded as below considering the relative determination of reactive forces on the tooth flank in relation to the value of the run-on force F/S_H and T/S_H .

$$\frac{\frac{T}{S_H}}{\frac{F}{S_H}} > \mu_f \tag{19}$$

The slide occurrence condition may arise only in the third range of drum rotation by pitch angle for the drum rotation angle of $\varphi > \varphi_{R0}$. The force *F*, preventing the slide of the horizontal link rear torus on the tooth flank, occurs only in this range of drum rotation.

The maximum friction coefficient value μ_{f_2} at which a link is sliding along the flank, is determined by the maximum value of the ratio of forces $(T/F)_{max}$. The value of this ratio of forces is higher the higher is the friction coefficient value in the joints of links μ_p and the larger is the elongation of the links' pitches $\Delta p/p$ (Fig. 10). Hence, the occurrence probability of sliding of the horizontal link front torus on the tooth flank is rising as the elongation of the pitch is growing and as the friction coefficient value is growing in the joints of the links.

Link sliding on the tooth flank may not occur, with an appropriately high friction coefficient value, between the tooth flank and the horizontal link. It should be noted, however, that as the tooth flank is wearing, so is the flank inclination angle growing, which is dramatically increasing the *T* force value (Sobota, 2012). Conditions are then emerging for the sliding of the horizontal link front torus on the tooth flank, even for high values of the friction coefficient μ_f .



Fig. 10. Influence of elongation of chain links' pitches and friction coefficient in the joints of links on the value of the ratio of forces $(T/F)_{max}$

5. Friction work for horizontal link slide on the tooth flank

As the value of the ratio of forces T/F is changing during sprocket drum rotation by the pitch angle, horizontal link sliding on the tooth flank may take place for different values of the drum rotation angle φ_p within the third range. Sliding occurs at the moment when the value of the ratio of forces T/F exceeds the friction coefficient value between the tooth flank and the link.

If horizontal link sliding takes place on the tooth flank, the friction work A_f is described with the following dependency:

$$A_f = L_f \cdot F(\varphi_p) \cdot \mu_f \tag{20}$$

where:

 L_f — the sliding path of the horizontal link rear torus on the tooth flank,

 $F(\varphi_p)$ — the F force value at the moment of sliding,

 μ_f — the friction coefficient between the tooth flank and the link.

The sliding path of the rear torus on the tooth flank is:

$$L_f = \frac{(p+d)\cdot\sin\varepsilon}{\sin\beta} + \frac{d}{2}\cdot\varepsilon$$
(21)

Curves for the values of the ratio of force T/F in the function of the chain drum rotation angle for friction conditions $\mu_p = 0.2$, $\mu_g = 0.15$ depend on the growth of pitch for the links (Fig. 11). If the friction coefficient value between the tooth flank and the horizontal link will be $\mu_f = 0.05$, then link sliding on the tooth flank will take place for the value of the ratio of forces of T/F > 0.05. This will take place for the drum rotation angle φ_p depending on the growth of pitches for the links. Therefore:

•	for $\Delta p/p = 1.0\%$	$\varphi_p = 50.6^{\circ}$	$F(\varphi_p)/S_H = 0.970$
		$L_f = 4.86 \text{ mm}$	$A_f / \dot{S}_H = 0.236 \text{ J/kN}.$
•	for $\Delta p/p = 2.0\%$	$\varphi_p = 47.3^{\circ}$	$\dot{F}(\varphi_p) / S_H = 0.960$
		$L_f = 9.12 \text{ mm}$	$A_f / \dot{S}_H = 0.438 \text{ J/kN}.$
•	for $\Delta p/p = 3.0\%$	$\phi_{p} = 44.1^{\circ}$	$\vec{F}(\varphi_p) / S_H = 0.945$
		$L_f = 12.83 \text{ mm}$	$A_f / \dot{S}_H = 0.606 \text{ J/kN}.$

If the friction coefficient value between the tooth flank and the horizontal link will be $\mu_f = 0.10$, then link sliding on the tooth flank will take place for the value of the ratio of forces of T/F > 0.10, i.e.:

•	for $\Delta p/p = 1.0\%$	$\varphi_p > 2\pi/z$	(for conditions for slide)
•	for $\Delta p/p = 2.0\%$	$\varphi_p = 50.9^{\circ}$	$F(\varphi_p)/S_H = 0.971$
		$L_f = 9.12 \text{ mm}$	$A_f / \dot{S}_H = 0.886 \text{ J/kN}.$
•	for $\Delta p/p = 3.0\%$	$\phi_p = 47.9^{\circ}$	$\dot{F}(\varphi_p) / S_H = 0.961$
		$L_f = 12.83 \text{ mm}$	$A_f / S_H = 1.233 \text{ J/kN}.$



Fig. 11. Curve of the ratio of forces T/F in drum interworking with elongated links ($\mu_p = 0.2, \mu_g = 0.15$)

6. Friction work in the joints of links

Friction work in the horizontal link joints at the interface with vertical links results in the abrasive wear of the links causing an increased pitch of the chain links. A link chain joint is a kinematic couple of two links: a horizontal and vertical link. When a chain drum is rotated by the pitch angle, a horizontal link is rotated relatively in relation to the preceding vertical link in the front joint and the relative rotation takes place of a vertical link following the horizontal link around the horizontal link rear torus in the rear joint.

The rotation of links in the front joint takes place in the first range of sprocket drum rotation by the pitch angle (Fig. 2). Two rotation phases of links in the front joint can be distinguished in this range: links rolling and link sliding in the joint.

The rolling of links in the front joint takes place for the drum rotation angle and conditions described with the dependencies (5)÷(8): The contact point of links in the front joint is displaced during the rolling of links. The rolling path L_{p1} is:

$$L_{p1} = \frac{d}{2} \cdot \gamma_p = \frac{d}{2} \cdot \frac{m}{1-m} (\varphi - \lambda)$$
(22)

Rolling work in the front joint being the product of the rolling way and rolling friction force is:

$$A_{p1} = \int_{0}^{\varphi_{gr} + \lambda} L_{p1} \cdot S_V \cdot \cos \gamma_p \cdot \frac{2f_p}{d} \cdot d\varphi$$
(23)

where:

 f_p — rolling friction force

The second phase, in which a horizontal link is sliding in the front joint, takes place for the sprocket drum rotation angle described with the dependency (12). The horizontal link front torus sliding path in the joint L_{p2} is:

$$L_{p2} = \frac{d}{2} \cdot \left(\varphi - \varphi_{gr} - \lambda\right) \tag{24}$$

while friction work during the sliding of links in the front joint A_{p2} :

$$A_{p2} = \int_{\varphi_{gr}+\lambda}^{2\pi} L_{p2} \cdot S_{V} \cdot \cos \gamma_{p} \cdot \mu_{p} \cdot d\varphi$$
(25)

Total friction work in the front joint A_p is a sum of friction work at the rolling and sliding of joints:

$$A_p = A_{p1} + A_{p2} \tag{26}$$

The phase of relative rotation of links in the rear joint starts already in the first range of sprocket drum rotation, and in this rotation range, the axis of the vertical link deflects from the axis of the horizontal link by the inclination angle λ (Fig. 2). The rolling of links takes place in the first phase of relative rotation of links in the rear joint, and the slide of links in the joint takes place in the second phase. The rolling of links in the rear joint occurs within the entire first range of drum pitch angle due to relatively small values of the inclination angle λ . This takes place when:

$$\lambda_k \le \lambda_{gr} \tag{27}$$

whereas:

$$\lambda_{gr} = \left(\frac{1-m}{m}\right) \cdot \arctan\left(\mu_p\right) \tag{28}$$

Vertical link rolling angle is:

$$\gamma_t = \frac{m}{1-m} \cdot \lambda \tag{29}$$

The contact point of links in the rear joint is displaced during the rolling of links. The rolling path L_{t1} is:

$$L_{t1} = \frac{d}{2} \cdot \gamma_t = \frac{d}{2} \cdot \frac{m}{1 - m} \cdot \lambda \tag{30}$$

Friction work during rolling A_{t1} is:

$$A_{t1} = \int_{0}^{\frac{2\pi}{z} - \alpha^{u} + \lambda_{k}} L_{t1} \cdot S_{H} \cdot \cos \gamma_{t} \cdot \frac{2f_{p}}{d} \cdot d\varphi$$
(31)

The rolling of links in the rear joint is continued in the second range of drum load in the case where:

$$\lambda_k < \lambda_{gr} \tag{32}$$

This occurs for the drum rotation angle of:

$$\frac{2\pi}{z} - \alpha^{u} + \lambda_{k} \le \varphi \le \frac{2\pi}{z} - \alpha^{u} + \lambda_{gr}$$
(33)

The rolling path of links in the rear joint in this drum rotation angle L_{t2} in such case is:

$$L_{t2} = \frac{d}{2} \cdot \frac{m}{1-m} \cdot \left(\varphi - \frac{2\pi}{z} + \alpha^{u} - \lambda_{k}\right)$$
(34)

while friction work during the rolling of links in the rear joint A_{t2} is described with the relationship:

$$A_{t2} = \int_{\frac{2\pi}{z} - \alpha^u + \lambda_k}^{\frac{2\pi}{z} - \alpha^u + \lambda_{gr}} L_{t2} \cdot S_H \cdot \cos \gamma_t \cdot \frac{2f_p}{d} \cdot d\varphi$$
(35)

The sliding of links in the rear joint takes place during further sprocket drum rotation for the drum rotation angle:

$$\frac{2\pi}{z} - \alpha^u + \lambda_{gr} \le \varphi \le \frac{2\pi}{z} \tag{36}$$

The sliding path of links in the rear joint L_{t3} in such case is:

$$L_{t3} = \frac{d}{2} \cdot \left(\varphi - \frac{2\pi}{z} + \alpha^u - \lambda_{gr} \right)$$
(37)

$$A_{t3} = \int_{\frac{2\pi}{z} - \alpha^{u} + \lambda_{gr}}^{\frac{2\pi}{z}} L_{t3} \cdot S_{H} \cdot \cos \gamma_{t} \cdot \mu_{p} \cdot d\varphi$$
(38)

Total friction work in the rear joint A_t in this case is:

$$A_t = A_{t1} + A_{t2} + A_{t3} \tag{39}$$

Friction work in the front and rear joint for sprocket drum rotation by the pitch angle can be determined from the dependency (27)÷(39) for specific friction conditions and the known load on links. Friction work was also presented as relative to the run-on force as A_p/S_H [J/kN] because the reactive force in the front joint was determined relatively in relation to the run-on force value S_V/S_H . Relative friction work in the joints of links was determined for different sliding friction coefficient values in the joints of links ($\mu_p = 0.1$; $\mu_p = 0.2$; $\mu_p = 0.3$) for a sprocket drum with the number of teeth z = 7 interworking with a chain sized 34×126 mm for $\Delta p/p = 1.0\%$, $\mu_{\sigma} = 0.15$ and $f_p = 0.05$ mm.

The curve of relative friction work in the front joint of links in the function of drum rotation angle is presented in Fig.12. The phase of links rolling in the front joint is characterised by a low friction work value. This phase is extending as the sliding friction coefficient values in the joint μ_p are growing. The higher value μ_p , the later links slide phase begins and the faster growth of relative friction work A_p/S_H in the function of the drum rotation angle (Fig. 12). Total relative friction work in the front and rear joint for different friction coefficient values in the joints is in such case:

• for $\mu_p = 0.1$ $A_p/S_H = 0.548$ [J/kN] $A_t/S_H = 0.961$ [J/kN]. • for $\mu_p = 0.2$ $A_p/S_H = 1.050$ [J/kN] $A_t/S_H = 1.853$ [J/kN].

for
$$\mu_p = 0.3$$
 $A_p/S_H = 1.526 [J/kN]$ $A_t/S_H = 2.670 [J/kN].$



Fig. 12. Progress of relative friction work in joint for $\Delta p/p = 1.0\%$ and $\mu_g = 0.15$

Total relative friction work in the front joint A_p/S_H reaches values lower than friction work in the rear joint A_t/S_H irrespective of the friction coefficient value in the joint (Fig. 13). This results first of all from a large portion of the angle α^u in the drum pitch angle causing a high link slide path value in the rear joint.



Fig. 13. Total relative work in front and rear joint for $\Delta p/p = 1.0\%$ and $\mu_g = 0.15$

7. Conclusion

It is necessary to determine friction work in the slide conditions of the horizontal link on the seat bottom and on the tooth flank and friction work in the joints of links in the context of such nodes' abrasive wear. This allows to compare different construction variants of chain drums.

The assignment carried out under the development project No. N R09 0026 06/2009 financed by the Ministry of Science and Higher Education under decision No. 0481/R/T02/2009/06

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Received: 25 January 2013