Acta of Bioengineering and Biomechanics
Vol. 15, No. 3, 2013

Original paper
DOI: 10.5277/abb130312

Applanation pressure function in Goldmann tonometry and its correction

WIESŁAW ŚRÓDKA*

Deformable Body Mechanics Faculty Unit, Wrocław University of Technology, Wrocław, Poland.

So far applanation tonometry has not worked out any theoretical basis for correcting the result of intraocular pressure measurement carried out on a cornea with noncalibration dimensions by means of the Goldmann tonometer. All the tables of instrument reading corrections for cornea thickness or cornea curvature radius are based exclusively on measurements. This paper represents an attempt at creating a mechanical description of corneal apex deformation in Goldmann applanation tonometry. The functional dependence between intraocular pressure and the pressure exerted on the corneal apex by the tonometer was determined from a biomechanical model. Numerical GAT simulations, in which this function was also interrelated with the cornea's curvature radius and thickness were run and a constitutive equation for applanation tonometry, i.e. a full analytical description of intraocular pressure as a function of the above variables, was derived on this basis. The correction factors were defined and an algorithm for correcting the measured pressure was formulated. The presented formalism puts the results of experimental tonometry in new light. Analytical correction factors need not to come exclusively from measurements. A geometric interdependence between them and their dependence on pressure have been revealed. The theoretical description of applanation tonometry contained in the constitutive equation consists of a pressure function developed for a cornea with calibration dimensions and a coefficient correcting this calibration function, dependent exclusively on the cornea's actual thickness and curvature radius. The calibration function is a generalization of the Imbert–Fick law.

Key words: *eyeball, biomechanical model, tonometry, IOP, correction factors*

1. Introduction

A major problem of Goldmann applanation tonometry (GAT) is that intraocular pressure measurement result p_G requires correction for the geometric parameters of the individual cornea. The result is most influenced by central corneal thickness (CCT) and axial radius of anterior corneal curvature *R*. The correction value added to the tonometer reading is usually calculated using correction factors.

The most important for correcting the measurement result in applanation tonometry is a ratio of the pressure increment to the cornea thickness increment. Let us denote this factor as C_t. Its experimentally determined value had ranged widely before Orssengo and Pye [1]. The result reported by Ehlers [2]: 0.71 mmHg per 0.01 mm of corneal thickness change, i.e., 71 mmHg/mm, had then been considered to be the most reliable. The values reported by other researchers were in the range of 0–100 mmHg/mm [3]–[7]. The zero value, indicating no correlation between p_G and CCT, was obtained by Feltgen et al. [8].

Today a value close to C_t = 40 mmHg/mm is favoured. Such a value is reported, besides Orssengo and Pye, by several other authors [9]–[11]. The research done by Kohlhaas et al. [9] deserves special mention. They carried out the investigations on 125 eyes during cataract surgery. Besides CCT and *R* they also controlled intraocular pressure *p*. Prior to GAT measurement, pressure in the anterior chamber had been set to 20, 35 or 50 mmHg. Thus the results are highly important for any attempt to verify the theoretical foundations of applanation tonometry.

* Corresponding author: Wiesław Śródka, Deformable Body Mechanics Faculty Unit, Wrocław University of Technology, ul. Smoluchowskiego 25, 50-372 Wrocław, Poland. E-mail: wieslaw.srodka@pwr.wroc.pl

Received: October 15th, 2012
Accepted for publication: March 11th, 2013

The corneal curvature radius correction factor (C_R) measurement results reported in the literature are inconsistent. Many authors consider the correction table developed by Orssengo and Pye to be correct (e.g., [10], [12]), but not all authors (e.g., [11], [13]). According to the table, at constant intraocular pressure p as R increases, tonometer reading p_G *decreases*.

Whereas Kohlhaas et al. report a significant result: at pressure p = 35 mmHg, indicated pressure p_G *increases* when the curvature radius increases ([9]: fig. 5b). For instance, Elsheikh et al. [10] report a reverse trend, consistent with Orssengo and Pay's predictions, but at a correlation coefficient below 0.02, which practically means the absence of any trend.

Despite the wealth of experimental results, the attempts at developing correction tables for GAT do not provide any theoretical explanation of the results. For example, *why* is the correction factor C_t close to 40 mm Hg/mm or why is the factor C_R over ten times lower than C_t and is it positive or negative? The answer to the question whether p_G increases or decreases as R increases should be theoretically substantiated, regardless of the measurement result and its accuracy.

Another serious problem connected with p_G reading correction appears when $p_G < p$. This inequality usually occurs when the corneal thickness is smaller than the calibration one. For example, for p = 48 mm Hg, CCT = 0.44 mm and R = 8.6 mm, Orssengo and Pye ([1]: table 3) predict reading p_G = 37 mmHg, i.e., by 11 mmHg lower than p. External pressure p_G lower than the intraocular pressure is hard to explain. When attempting to explain why at D_{applan} = 3.06 mm, pressures p_G and p are in equilibrium in the eye with the calibration dimensions, Goldmann thought that the action of the surface tension in the lacrimal film bonding the tonometer tip with the cornea was the cause [14]. He assumed the following equation of equilibrium of the forces acting on the flattened area of the cornea [15]

$$F_G + F_S - F_{\text{bend}} - F_p = 0. \quad (1)$$

In the membranous state the force F_G exerted by the tonometer counterbalances resultant force F_p originating from intraocular pressure p on the applanation surface, and these are the only forces. After the flexural rigidity of the corneal shell is taken into account, there is still bending force F_{bend} which is counterbalanced by resultant force F_S originating from the surface tension, but only when flattened zone diameter D_{applan} = 3.06 mm and the cornea has the calibration dimensions. When CCT or R differs from the calibration dimensions, force F_{bend} is no longer equal to F_S. By dividing the forces in equation (1) by flattened zone surface area $0.25\,\pi D^2_{\text{applan}}$ one gets a similar equation for the pressures

$$p = p_G + \Delta p, \quad (2)$$

where

$$\Delta p = p_S - p_{\text{bend}}. \quad (3)$$

Can the large differences between the measured pressure and the actual pressure, observed in practice be explained in this way? According to Elsheikh et al. [10], lacrimal film surface tension S = 0.0455 N/m. Since force $F_S = \pi D_{\text{applan}} S$, pressure

$$p_S = 4F_S/(\pi D^2_{\text{applan}}) = 0.45 \text{ mmHg} \quad (4)$$

will appear on the surface of a circle with diameter D_{applan} = 3.06 mm. This result contradicts the possibility, described in the example above, that p_G can be lower than p by as much as Δp = 11 mmHg. As it is apparent, the lowest value which the correction (3) can assume is 0.45 mmHg (when p_{bend} = 0). Thus according to the Goldmann theory it is physically impossible for pressure p_G indicated by the tonometer to be lower than actual pressure p by more than about half a mmHg.

But the actual difference between p and p_G, observed in clinical practice or foreseen in the correction tables, reaches 10 mmHg. This exceeds 20 times the capability of the surface tension force. Then how can this phenomenon be explained?

The answer to this question can be the numerical eyeball model solutions described below. A significant role in the pressure p_G reading problem is played by function $p_{\text{calibr}}(p_G)$ (an equivalent of the Imbert–Fick law) established for the eyeball with the calibration dimensions. The source of knowledge about this function is either the experiment or biomechanical calculations. For the purposes of the present study, $p_{\text{calibr}}(p_G)$ was determined by means of a biomechanical eyeball model.

2. Materials and methods

The numerical model described in detail in papers [16] and [17] was used to determine the functional dependence between the corneal apex flattening pressure and intraocular pressure $p(p_G)$.

2.1. Material

Proper cornea and sclera material parameters were chosen to ensure that the model simulated GAT well. The material characteristic was described by the exponential function

$$\sigma = A(e^{\alpha\varepsilon} - 1), \quad \varepsilon \geq 0\,, \tag{5}$$

for tension, and the linear function

$$\sigma = E_0 \varepsilon, \quad \varepsilon < 0\,, \tag{6}$$

for compression. Young's modulus for the negative part is

$$E_0 = A\alpha\,.$$

The identified cornea and limbus material is (see [17])

$$A = 200 \text{ Pa}, \quad \alpha_{\text{cornea}} = 61.6\,. \tag{7}$$

Sclera constant A is the same and the exponent is (see [16])

$$\alpha_{\text{sclera}} = 5 * 61.6 = 308\,.$$

Poisson ratio $\nu = 0.49$.

2.2. Finite element model

The structure was solved by the finite element method (FEM) in the Cosmos/M system. The adopted solution parameters ensured that all nonlinear physical and geometric effects were covered. The model was built from isoparametric axially symmetric quadrilateral, 8-node SOLID 2D elements. Figure 1 shows the finite element grid of the flattened cornea.

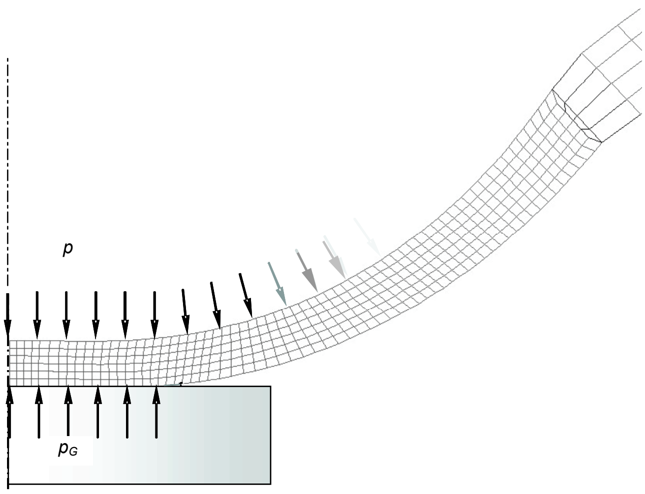

Fig. 1. Finite element grid of loaded structure

2.3. Boundary conditions

Loading with external pressure p_G was kinematically effected by imposing displacements on the external corneal surface nodes so that the latter remained on a single straight line perpendicular to the axis of symmetry. In the direction perpendicular to the symmetry axis the nodes preserved their freedom of displacement. After the FEM solution, pressure p_G was calculated as a ratio between the sum of the nodal forces and the surface area of the flattened region. The sclera was fixed in the polar zone. Apart from that, the corneal and scleral shells remained free.

2.4. Geometry

The basic dimensions of the numerical eyeball model conform to the Gullstrand–Le Grand standard for the human eye [19]: axial radius of anterior corneal curvature R = 7.8 mm, axial radius of interior corneal curvature r = 6.49 mm, central corneal thickness CCT = 0.520 mm, peripheral corneal thickness PCT = 0.720 mm. Nominal intraocular pressure IOP = 16 mmHg (2.14 kPa). It should be noted that as often as 0.520 mm, an average thickness of 0.550 mm can be found in the literature devoted to the eyeball biomechanical model and eyeball diagnostics [9], [20], [22]. However, since standard deviation ±40 µm [22] includes the average assumed in the Gullstrand–Le Grand model, the choice between the two values seems to be a matter of personal conviction.

3. Results

The FEM solutions for intraocular pressure p versus "measured" pressure p_G for models with dimensions different than the calibration ones are shown in Figs. 2 and 3.

Each of the figures includes a graph (the same) for the model with the calibration dimensions. As one can see, the other curves reproduce the shape of the function obtained for the calibration model. For the next, increasingly larger, corneal thicknesses given in Fig. 2 the pressure equilibrium point (p = p_G) shifts upwards, successively amounting to 6, 13, 16, 27 and 37 mmHg.

The evolution of the diagram of increasing corneal curvature radius R (Fig. 3) is somewhat different than in the case of CCT. As R increases, function $p(p_G)$ straightens and better fits the Imbert–Fick straight line

$p = p_G$. Close to the nominal pressure, the influence of R on measured pressure p_G is weak. It becomes stronger above 16 mmHg, but even at p = 45 mmHg the simulated deviation of tonometer p_G reading from the calibration model reading does not exceed 3 mmHg (but then the difference between pressures p and p_G is large).

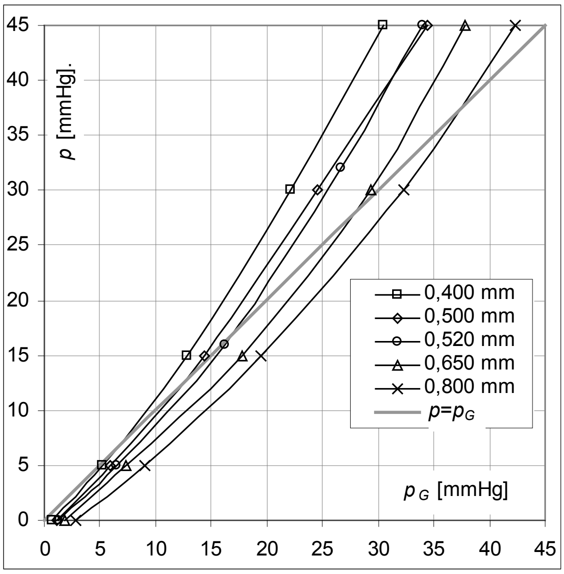

Fig. 2. Intraocular pressure functions for models with CCT in the range of 0.4–0.8 mm. R = 7.8 mm. The Imbert–Fick law is represented by grey line

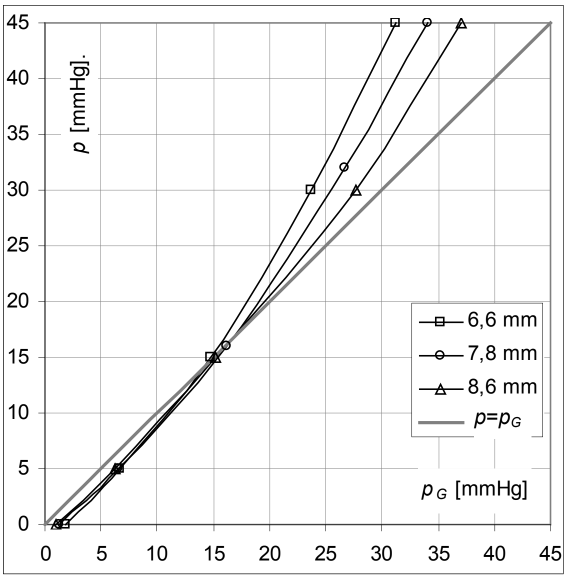

Fig. 3. Intraocular pressure functions for models with curvature radii 6.6, 7.8 and 8.6 mm. CCT = 0.520 mm. Middle graph comes from previous figure

Such a corneal shell material (7) was chosen that the structure with the calibration dimensions satisfied the condition

$$p_G = p = 16 \text{ mmHg (2.14 kPa)}. \tag{8}$$

This is actually the case: the calibration model graph in Figs. 2 and 3 passes through this point. But except for this point, the internal pressure is nowhere equal to the external pressure. Initially, as intuition suggests (in agreement with Goldmann's postulates), $p_G > p$, but as the internal pressure increases, difference $p_G - p$ becomes increasingly smaller and above the equilibrium point (8) the direction of the inequality reverses: $p_G < p$. The trend here is obvious – difference $p_G - p$ decreases as p increases. Thus the two pressures cannot be equal to each other in the calibration cornea, except for the equilibrium point. Whereas the other graphs in Fig. 2 also have their pressure equilibrium points for thicknesses other than the calibration one – the higher, the larger the CCT.

The final stage in the calculations concerns the role which material parameters play in GAT. Figure 4 shows applanation pressure functions for the eyeball model in which the cornea has calibration dimensions and four sets of material parameters A and α are substituted one after another. Exponent α assumes values from an interval of 34–82 experimentally determined for the human cornea [21], material parameter A is fitted to satisfy condition (8). Results of the calculations in Fig. 4 come from [17].

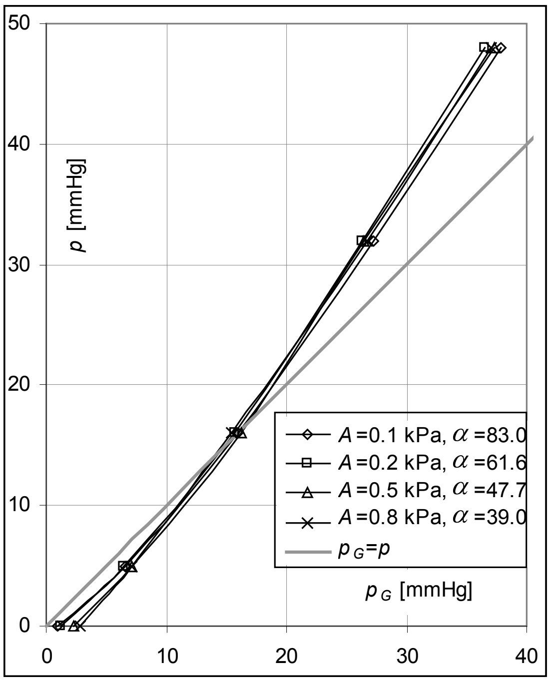

Fig. 4. Applanation pressure for four cornea materials. Material characteristic is defined by equations (5) and (6). Cornea with calibration dimensions

3.1. Pressure function $p(p_G)$

In its original form, given by Goldmann, function $p(p_G)$ was linear with a slope coefficient equal to 1 and passed through the origin of the coordinates (the Imbert–Fick law)

$$p = p_G. \tag{9}$$

The authors of the modified Imbert–Fick law [1]

$$p = K_g^{-1} \cdot p_G \tag{10}$$

questioned slope coefficient K_g^{-1} equal to 1 and interrelated it with CCT and R. But it follows from the solutions presented in Figs. 2 and 3 that also the other constraints are no longer valid (in the light of the numerical solutions). The graph of function $p(p_G)$ is neither linear nor passes through the origin of the coordinates.

It is convenient to set the calculation results in the existing tonometry convention, adding the quadratic component

$$p = a + bp_G + cp_G^2 \tag{11}$$

to the linear dependence between pressure p and p_G. For each curve shown in Fig. 2 it is possible to find a quadratic regression function, i.e., a set of coefficients a, b and c. The coefficients change for the successive curves due to changes in CCT. The dependence between the coefficients in equation (11) and CCT, approximated by the linear function, leads to the following expressions

$$a = -2.053 \text{ CCT} - 0.482, \tag{12a}$$

$$b = -1.092 \text{ CCT} + 1.541, \tag{12b}$$

$$c = -0.0053 \text{ CCT} + 0.014. \tag{12c}$$

The series of functions (11) for a CCT of 0.4–0.8 every 0.1 mm are plotted (fine lines) in Fig. 5 (the graphs from Fig. 2 are included for comparison).

Equations (11) and (12) describe the general model which takes into account the influence of corneal thickness on the form of function $p(p_G)$. It turns out, however, that condition (8) is best satisfied by the 0.550 mm thick cornea. Since in the literature this value is assumed as often as 0.520 mm, it will still be referred to as the calibration thickness.

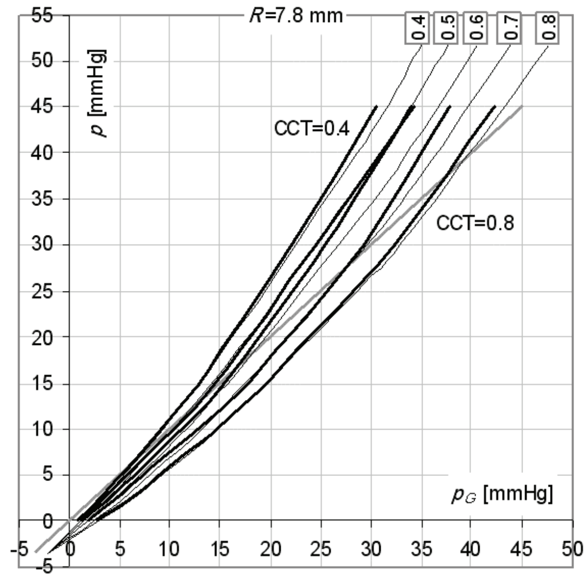

Fig. 5. Lines of constant CCT according to (11) and (12) – fine lines

3.2. Analytical form of function $p(p_G)$

The family of functions (11), parameterized with corneal thickness (12), shows several peculiar properties. All the functions intersect at one point

$$(p_G^{(0)}, p^{(0)}) = (-2.05, -3.44)\,. \tag{13}$$

It also turns out that in coordinate system p'_G, p' such that

$$p'_G = p_G - p_G^{(0)} \tag{14a}$$

$$p' = p - p^{(0)}\,, \tag{14b}$$

polynomial (11) satisfies the condition

$$\frac{p'|_{\text{CCT}_2} - p'|_{\text{CCT}_1}}{p'|_{\text{CCT}_{\text{calibr}}}} \approx \text{const} \tag{15}$$

irrespective of pressure p'_G. This means that the difference between the neighbouring functions in Fig. 5 $(p'|_{\text{CCT}_2} - p'|_{\text{CCT}_1})$ is proportional to function $p'|_{\text{CCT}_{\text{calibr}}}$ of the model with the calibration corneal thickness. For the calibration cornea this peculiar pressure function will be denoted by p'_{calibr}. Therefore one can write

$$p' = Wp'_{\text{calibr}}\,. \tag{16}$$

W is defined as a coefficient linearly dependent on CCT and equal to 1 when CCT assumes the calibration value

$$W = t\,(\mathrm{CCT_{calibr}} - \mathrm{CCT}) + 1, \tag{17}$$

where t is a constant.

After transformation (14b) equation (16) assumes the form

$$p(p_G) = W \cdot (p_{\mathrm{calibr}}(p_G) - p^{(0)} + p^{(0)}. \tag{18}$$

By comparing function (18) with the solutions illustrated in Fig. 2 one can determine the value of constant t, which happens to be equal to unity ($[t]$ = 1/mm). Another comparison with the results presented in Fig. 3 gives a justification for taking the linear influence of curvature radius R into account in expression (17)

$$W = t\left(\mathrm{CCT_{calibr}} - \frac{R}{R_{\mathrm{calibr}}}\mathrm{CCT}\right) + 1. \tag{19}$$

Thus, ultimately

$$p(p_G) = \left[t\left(\mathrm{CCT_{calibr}} - \frac{R}{R_{\mathrm{calibr}}}\mathrm{CCT}\right) + 1\right] \cdot (p_{\mathrm{calibr}}(p_G) - p^{(0)}) + p^{(0)}. \tag{20}$$

Function $p_{\mathrm{calibr}}(p_G)$ is a 2nd degree polynomial with coefficients calculated from (12). Substituting there CCT = 0.55 mm one gets

$$p_{\mathrm{calibr}}(p_G) = -1.61 + 0.94\,p_G + 0.0111\ p_G^2. \tag{21}$$

In Fig. 6, after substituting R = 6.6 or R = 8.6 mm, function (20) can be compared with the previous solutions shown in Fig. 3.

3.3. Linear version of $p(p_G)$

It is also possible to present the calibration function in the linear form. In such a case, it could assume the form similar to the one shown in Fig. 7.

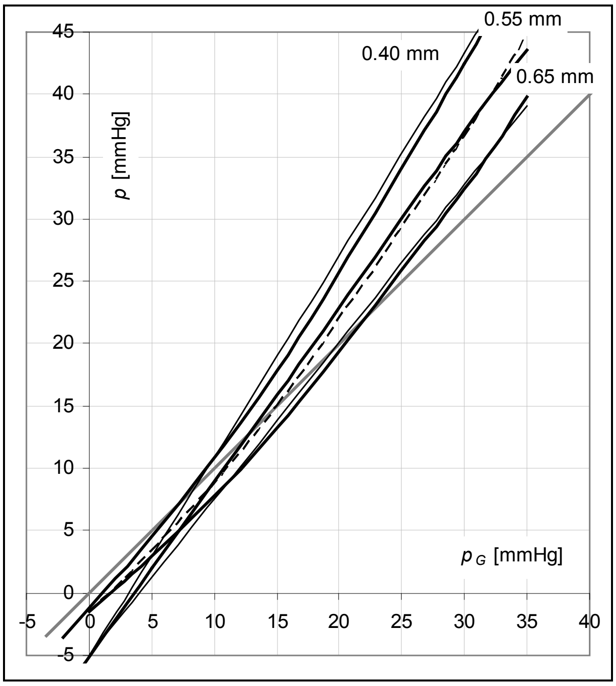

Fig. 7. Calibration function and its linear approximation – denoted as 0.55 mm. Functions for extreme corneal thicknesses and their linear approximations are denoted as 0.4 mm and 0.65 mm. Grey line represents Imbert–Fick law (9)

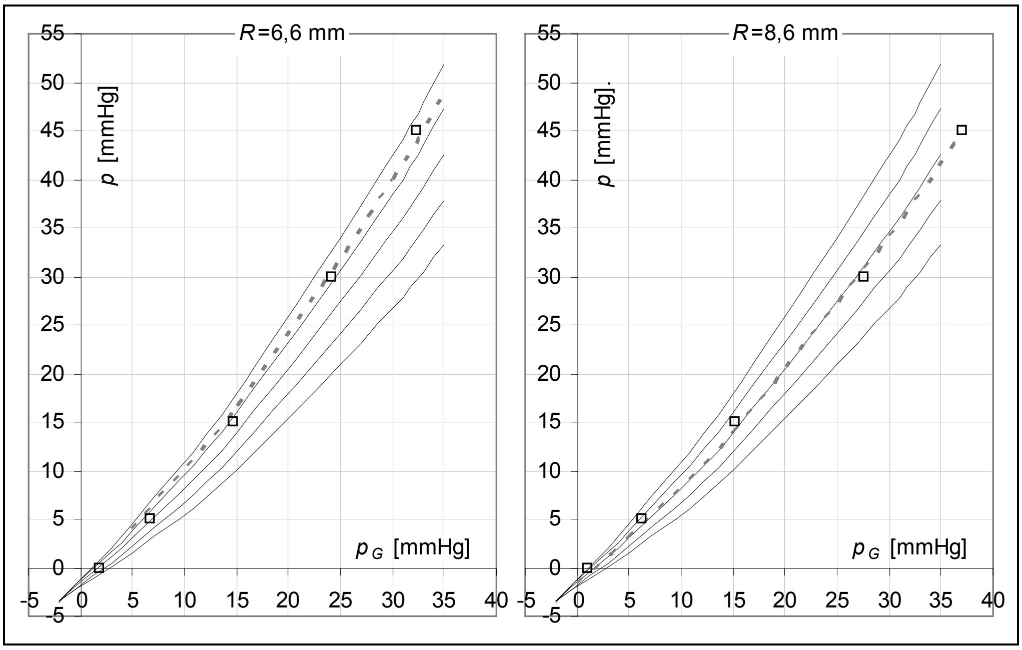

Fig. 6. Squares mark solutions for model with corneal curvature radius 6.6 mm (on left) and 8.6 mm (on right) previously presented in Fig. 3. Grey broken line represents function (20), calculated for each model. In the background one can see CCT = const lines from Fig. 5

In a p range of 5–44 mmHg, the difference between quadratic function (21) and its linear approximation

$$p_{\text{calibr}} = 1.4\,p_G - 5 \quad [\text{mmHg}] \qquad (22)$$

does not exceed 1.3 mm Hg. Shifting the point with coordinates (13) to position (0, –5) one can write expression (20) as follows

$$p(p_G) = \left[t\left(\text{CCT}_{\text{calibr}} - \frac{R}{R_{\text{calibr}}}\text{CCT} \right) + 1 \right] 1.4 p_G - 5. \quad (23)$$

It is apparent that even for the calibration cornea the slope coefficient of the straight line, here equal to 1.4, is considerably different from unity.

3.4. Pressure correction factors

In GAT each of the correction factors, for CCT and for central corneal radius R, is defined as a ratio of the difference between pressures p to the increase of the parameter which causes the difference. They should obviously be treated as derivatives of the pressure function

$$C_t = \frac{\partial p}{\partial \text{CCT}}, \qquad (24)$$

$$C_R = \frac{\partial p}{\partial R}. \qquad (25)$$

Differentiating function (20), for the calibration cornea one gets

$$C_t = -t(p(p_G)_{\text{calibr}} - p^{(0)}), \qquad (26)$$

$$C_R = -\frac{\text{CCT}_{\text{calibr}}}{R_{\text{calibr}}} t(p_{\text{calibr}}(p_G) - p^{(0)}). \qquad (27)$$

The two expressions depend on pressure. Thus it becomes apparent that the factors are actually functions. One can also easily notice the dependence between them

$$C_R = \frac{\text{CCT}_{\text{calibr}}}{R_{\text{calibr}}} C_t. \qquad (28)$$

Corrective function C_t is numerically equal to (accurately to the sign) p', i.e., to the intraocular pressure in coordinate system (14).

For linear law (22) one gets

$$C_t = -1.4 t p_G \ [\text{mmHg/mm}]. \qquad (29)$$

3.5. Example

Pressure p_G = 35 mmHg was measured on an eye with the following geometric parameters: R = 8.60 mm, CCT = 0.44 mm. What is the intraocular pressure?

3.5.1. Corrective formula (20)

Calibration function (21) assumes the value

$$p_{\text{calibr}} = -1.61 + 0.94*35 + 0.0111*35^2$$
$$= 44.9 \text{ mmHg}.$$

If the eye had the calibration dimensions, then according to (20), the intraocular pressure would be

$$p = 1*(44.9 - (-3.44)) - 3.44 = 44.9 \text{ mmHg}. \quad (30)$$

Since, however, the eye does not have the calibration dimensions, the cornea geometry is introduced through coefficient (19)

$$W = 1\,(0.55 - 8.6/7.8 * 0.44) + 1 = 1.06.$$

Now, according to (20),

$$p = 1.06\,[44.9 - (-3.44)] - 3.44 = 48 \text{ mmHg}. \quad (31)$$

Intraocular pressure p is by 13 mmHg higher than the one indicated by the applanation tonometer, 9.9 mm Hg of which is due to pressure – see formula (30), and the rest is due to the combined action of CCT and R.

3.5.2. Corrective functions (linear approximation)

1. The influence of corneal thickness is:

$$C_t * \Delta\,\text{CCT} = (-1.4\,t\,p_G)*(\text{CCT} - \text{CCT}_{\text{calibr}})$$
$$= -49\,(0.44 - 0.55) = 5.4 \text{ mmHg},$$

where C_t = –1.4*1*35 = – 49 mmHg, according to (29).

2. The influence of the radius of curvature:

$$C_R * \Delta R = (\text{CCT}_{\text{calibr}}/R_{\text{calibr}} * C_t)*(R - R_{\text{calibr}})$$
$$= 0.55/7.8*(-49)*(8.6 - 7.8) = -2.8 \text{ mmHg}.$$

3. The influence of pressure – the calibration model, according to (23):

$$p - p_G = (1.4\,p_G - 5) - p_G = (1.4*35 - 5) - 35$$
$$= 9.0 \text{ mmHg}.$$

Even though the total correction amounts to 11.6 mmHg, linear approximation seems to be sufficiently accurate (in comparison with the previous result).

3.5.3. Orssengo and Pye correction factor

For the above data the correction factor is K_g = 0.77 ([1]: table 3]. It follows from (10) that p = 35/0.77 = 45.5 mmHg, i.e., p differs from the tonometer reading by 10.5 mmHg. If only the deviation of CCT from the calibration value were taken into account, then K_g = 0.78 and p would amount to 35/0.78 = 45 mm Hg. As it is apparent, the influence of the deviation of R from the calibration value is here reverse – a curvature radius larger than the calibration value introduces a positive correction of 0.5 mmHg. Thus,

1. the component due to CCT +10 mmHg.
2. the component due to R +0.5 mmHg.

3.5.4. Commentary

Seemingly the Orssengo and Pye correction little differs from the previous results. Their correction table gives a correction of 10.5 mmHg, which, in comparison with 11.6 mmHg is a difference allowable in tonometry. But let us examine what makes up this result. GAT ascribes this correction to mainly the influence of CCT – as much as 10 mmHg, whereas according to numerical calculations this component amounts to merely half this value (5.4 mmHg). The real cause of such a large deviation of p_G from the actual pressure – the pressure alone (formula (30) or point 3 in *Corrective functions*) – is completely neglected in GAT. The appearance of correctness is additionally strengthened in GAT by the correction (with the opposite sign) dependent on R. Readers who would like to find out what this leads to should do the above calculations for: CCT = 0.44 mm, R = 7.0 mm and p_G = 35 mmHg. Then, K_g = 0.82.

4. Discussion

Kohlhaas et al. [9] experimentally obtained C_t = –40 mmHg/mm at p = 35 mm Hg and C_t = –50 mmHg at p = 50 mm Hg. The theoretical value of C_t, calculated from (26), is numerically equal to pressure p increased by 3.44 mmHg. Thus for the two pressures, C_t assumes the values of –38 mmHg/mm and –53 mmHg/mm. The results are surprisingly close. The other correction factor, calculated from (27), at an intraocular pressure of 35 mmHg should amount to C_R = –38*0.55/7.8 = –2.7 mmHg/mm. The results reported by Kohlhaas et al. (–2.5 mmHg/mm in their fig. 5B) confirm not only the sign of this coefficient (as R increases, tonometer reading p_G increases – unlike in Orssengo and Pye) but also its value.

But an apparent inconsistency emerges at p = 20 mmHg. Then, according to (26), function C_t reaches –23 mm Hg/mm. Kohlhaas et al. Obtained –40 mmHg/mm (at a correlation coefficient considered by them to be too low), but this particular result is incorrect as can be demonstrated within GAT. According to the definition of C_t (24) function p (10) should be differentiated over CCT. Then one gets

$$C_t = \frac{\partial p}{\partial \text{CCT}} = \frac{\partial (K_g^{-1} p_G)}{\partial \text{CCT}}$$

$$= \frac{\partial (K_g^{-1})}{\partial \text{CCT}} p_G = -\frac{1}{K_g^2} \frac{\partial K_g}{\partial \text{CCT}} p_G, \qquad (32)$$

since the dependence between p and CCT is covered by coefficient K_g. From the Orssengo and Pye correction table ([1]: tab. 3) the following derivative can be calculated for the calibration model (K_g = 1)

$$\frac{\partial K_g}{\partial \text{CCT}} \approx 3 \ [\text{mm}^{-1}] \qquad (33)$$

hence

$$C_t \approx -3 p_G \quad [\text{mmHg/mm}]. \qquad (34)$$

Function C_t turns out to be *proportional* to p_G. Therefore, if at pressure p = 35 mmHg the Kohlhaas measurement result C_t = –40 mmHg/mm is correct, then C_t at p = 20 mmHg should be equal to –40*20/35 = 23, and not to –40 mmHg/mm. Besides, we find another error in the correction table: measurement result C_t = –40 mmHg/mm at p_G = 35 mmHg ($p = p_G$ since K_g = 1) means that in function (34) the factor is equal to 40/35 = 1.14, and not to 3 mm^{-1}.

The theoretical basis of GAT proposed here makes it possible to predict new dependencies and to avoid simple errors. It turns out that correction factors C_t and C_R need not to be measured. They are the consequence of geometric proportion (19) and are described analytically. Moreover, they remain in mutual relationship (28). Formula (20) allows one to skip this stage of simplified calculations, directly yielding the sought value of the actual pressure. For this one needs to know function $p(p_G)$ for the cornea with the calibration dimensions. Here the function was determined numerically. If it turned out that calibration function (21) does not describe correctly the calibration eye, it could be replaced by, for example, an experimental function. But let us note that at a low pressure (below the nominal one) calibration function (21) ensures that the numerical model functions in agreement with Goldmann's postulates (the slope coefficient equal to 1, p_G slightly higher than p, the trend of the dependence of p_G on CCT and R, in agreement with the Orssengo and Pye table). Whereas, when the pressure is high (above the nominal one), function $p_{\text{calibr}}(p_G)$ is clearly larger than p_G. This turns out to be the nature of the nonlinear corneal shell flattened at a relatively high internal pressure. It is this effect – buckling – which is the cause of inequality $p_G < p$ which the presence of surface tension in the lacrimal film does not explain. This phenomenon has been investigated in [18].

Finally, let us have a closer look at the highly valuable property of applanation tonometry, which clearly distinguishes it from spherical tonometry (DCT, ART), i.e., the calibration function is almost insensitive to the variation of the cornea material parameters, which occurs in clinical practice. Graphs $p_G(p)$ in Fig. 4 differ only slightly, even though the materials used in the model solutions cover nearly the entire spectrum found in people.

5. Conclusions

1. The proposed technique of correcting tonometer readings requires going through two calculation stages: Calibration Function and Correction. The former applies to a model of the cornea with calibration dimensions and the form of the pressure function should be taken from the real cornea, whereas the latter stage concerns only the deviation of the cornea dimensions from the calibration values and when a reading correction is calculated there is no need to refer to experiment. In GAT it is the other way round: the calibration function is given a priori and the effect of R and CCT on the correction value is determined experimentally. For this reason arriving at correction factors is such an arduous process.
2. Contrary to what is assumed in GAT, the applanation pressure function is not a linear function of IOP.
3. The so-called "correction factors", i.e., C_t – a ratio of an increment in IOP to an increment in CCT, and C_R – a ratio of an increment in IOP to an increment in corneal radius R, depend on IOP and so they are not "coefficients" (constants) as is commonly believed.
4. The calibration formulas derived here are based on mechanical foundations and expressed in a functional form. Thanks to this, for the first time one can see how the particular cornea model parameters, the cornea's geometry and material and also IOP affect the correction value. (The numerical values of the correction factors currently used in GAT come solely from experiment and so they have no theoretical basis and, of course, are not described by functions.)
5. Quantities C_t and C_R are not independent. In order to determine C_R one should multiply C_t by the ratio of calibration values of CCT and R, i.e., by 0.07.
6. GAT does not require reading correction due to the cornea material parameters, to which the tonometer's flat tip contributes.

References

[1] ORSSENGO G.J., PYE D.C., *Determination of the true intraocular pressure and modulus of elasticity of the human cornea*, B. Math. Biol., 1999, Vol. 61(3), 551–572.

[2] EHLERS N., BRAMSEN T., SPERLING S., *Applanation tonometry and central corneal thickness*, Acta Ophthalmol. Scand., 1975, Vol. 53, 34–43.

[3] DAMJI K.F., MUNGER R., *Influence of central corneal thickness on applanation intraocular pressure*, J. Glaucoma., 2000, Vol. 9, 205–207.

[4] DOUGHTY M.J., ZAMAN M.L., *Human corneal thickness and its impact on intraocular pressure measures: a review and meta-analysis approach*, Surv. Ophthalmol., 2000, Vol. 44, 367–408.

[5] SHAH S., CHATTERJEE A., MATHAI M., *Relationship between corneal thickness and measured intraocular pressure in a general ophthalmology clinic*, Ophthalmology, 1999, Vol. 106, 2154–2160.

[6] SHAH S., *Accurate intraocular pressure measurement – the myth of modern ophthalmology?* Ophthalmology, 2000, Vol. 107, 1805–1807.

[7] STODTMEISTER R., *Applanation tonometry and correction according to corneal thickness*, Acta Ophthalmol. Scand., 1998, Vol. 76, 319–324.

[8] FELTGEN N., LEIFERT D., FUNK J., *Correlation between central corneal thickness, applanation tonometry, and direct intracameral IOP readings*, Brit. J. Ophthalmol., 2001, Vol. 85, 85–87.

[9] KOHLHAAS M., BOEHM A.G., SPOERL E., PÜRSTEN A., GREIN H.J., PILLUNAT L., *Effect of Central Corneal Thickness, Corneal Curvature, and Axial Length on Applanation Tonometry*, Archives of Ophthalmology, 2006, Vol. 124(4), 471–476.

[10] ELSHEIKH A., WANG D., KOTECHA A., BROWN M., GARWAY-HEATH G., *Evaluation of Goldmann applanation tonometry using a nonlinear finite element ocular model*, Ann. Biomed. Eng., 2006, Vol. 34(10), 1628–1640.

[11] GUNVANT P., O'LEARY D.J., BASKARAN M., BROADWAY D.C., WATKINS R.J., VIJAYA L., *Evaluation of tonometric correction factors*, J. Glaucoma, 2005, Vol. 14(5), 337–343.

[12] LIU J., ROBERTS C.J., *Influence of corneal biomechanical properties on intraocular pressure measurement*, J. Cataract. Refr. Surg., 2005, Vol. 31, 146–155.

[13] KIRSTEIN E.M., HUSLER A., *Evaluation of the Orssengo–Pye IOP corrective algorithm in LASIK patients with thick corneas*, Optometry, 2005, Vol. 76(9), 536–543.

[14] GOLDMANN H., SCHMIDT T., *Weiterer Beitrag zur Applanationstonometrie*, Ophthalmologica, 1961, Vol. 141, 441–456.

[15] GOLDMANN H., SCHMIDT T., *Über Applanations-tonometrie*, Ophthalmologica, 1957, Vol. 134, 221–242.

[16] ŚRÓDKA W., PIERSCIONEK B.K., *Effect of material properties of the eyeball coat on optical image stability*, J. Biomed. Opt., 2008, Vol. 13(5), 054013.

[17] ŚRÓDKA W., *Evaluating the material parameters of the human cornea in a numerical model*, Acta Bioeng. Biomech., 2011, Vol. 13(3), 77–85.

[18] ŚRÓDKA W., *Goldmann applanation tonometry – not as good as gold*, Acta Bioeng. Biomech., 2010, Vol. 12(2), 39–47.

[19] Le GRAND Y., el HAGE S.G., *Physiological Optics*, Springer Series in Optical Sciences, Vol. 13, Springer-Verlag, Berlin, Heidelberg, New York, 1980.

[20] HALLBERG P., EKLUND A., SANTALA K., KOSKELA T., LINDAHL O., LINDEN C., *Underestimation of intraocular pressure after photorefractive keratectomy: a biomechanical analysis*, Med. Biol. Eng. Comput., 2006, Vol. 44, 609–618.

[21] NASH L.S., GREENE P.R., FOSTER C.S., *Comparison of mechanical properties of keratoconus and normal corneas*, Experimental Eye Research, 1982, Vol. 35, 413–423.

[22] WOLLENSAK G., *Stress-strain measurements of human and porcine corneas after riboflavin-ultraviolet-A-induced cross-linking*, J. Cataract. Refr. Surg., 2003, Vol. 29, 1780–1785.