SELECTED ENGINEERING PROBLEMS
NUMBER 7
INSTITUTE OF ENGINEERING PROCESSES AUTOMATION
AND INTEGRATED MANUFACTURING SYSTEMS

Marek PŁACZEK

Institute of Engineering Processes Automation and Integrated Manufacturing Systems,
Faculty of Mechanical Engineering, Silesian University of Technology, Gliwice, Poland
marek.placzek@polsl.pl

EXACT AND APPROXIMATE METHODS IN ANALYSIS OF ONE-DIMENSIONAL MECHANICAL SYSTEMS

Abstract: Paper presents comparison of an exact Fourier method and an approximate Galerkin method in analysis of one-dimensional vibrating mechanical systems. Assumptions of the approximate method are presented. The considered systems are beams with different methods of fixing – different boundary conditions. Values of natural frequencies and dynamic flexibilities of considered systems are designated using the approximate method and compared with results obtained using the exact method.

1. Introduction

Considered one-dimensional systems are mechanical subsystems of the mechatronic systems with piezoelectric transducers used as vibration dampers or actuators [2,3]. Mechatronic systems were analyzed in other publications [1,2]. It is impossible to use the exact method to analyze mechatronic systems therefore the approximate method was used. Assumptions and verification of the approximate method was presented in [1]. The analyzed system was a cantilever beam with piezoelectric passive vibration damper. Presented method of system's fixing – one end clamped and one free, was chosen deliberately from all the possible ways of fixing because in this case inaccuracy of the approximate method is the highest among all the possible ways of fixing. The approximate method should be verify and corrected if necessary, so it is important to indicate what determines the uncertainty of the method and how to correct it. In this paper results obtained for two extreme cases are presented. In the first case (a simply supported beam) there is no differences between results obtained using the exact and the approximate method (values of natural frequencies are exactly the same), while in the second case inaccuracies of the approximate method are significant.

2. Analysis of considered mechanical systems

Considered mechanical systems are presented in Fig. 1 and Fig. 2. The first system is a simply supported beam and the second one is a cantilever beam. Both systems are loaded by the external harmonic force that operates perpendicular to the beam's axis. Boundary conditions of considered systems are presented in Tab.1.

Tab.1. boundary conditions of considered mechanical systems

Simply supported beam	Cantilever beam
$y(0,t)=0$, $\frac{\partial^2 y(0,t)}{\partial x^2}=0$, $y(l,t)=0$, $\frac{\partial^2 y(l,t)}{\partial x^2}=0$,	$y(0,t)=0$, $\frac{\partial y(0,t)}{\partial x}=0$, $\frac{\partial^2 y(l,t)}{\partial x^2}=0$, $\frac{\partial^3 y(l,t)}{\partial x^3}=0$,

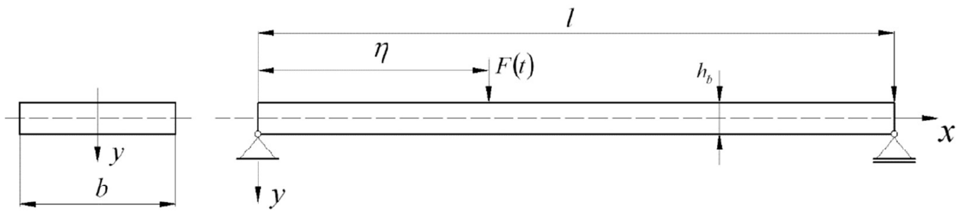


Fig.1. The first considered mechanical system - simply supported beam

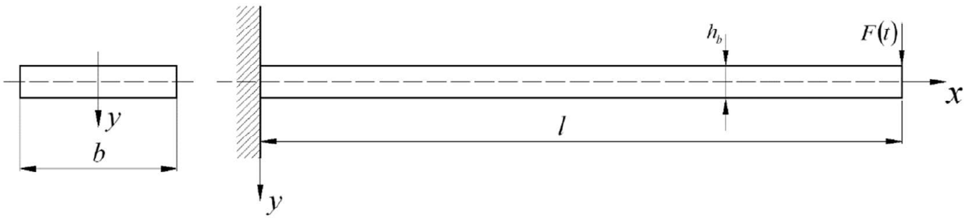


Fig.2. The second considered mechanical system - cantilever beam

In both cases equation of the beam's motion can be described as:

$$\frac{\partial^2 y(x,t)}{\partial t^2} = -\frac{EJ}{\rho b h_b}\frac{\partial^4 y(x,t)}{\partial x^4} + \frac{F(t)\delta(x-\eta)}{\rho b h_b}, \tag{1}$$

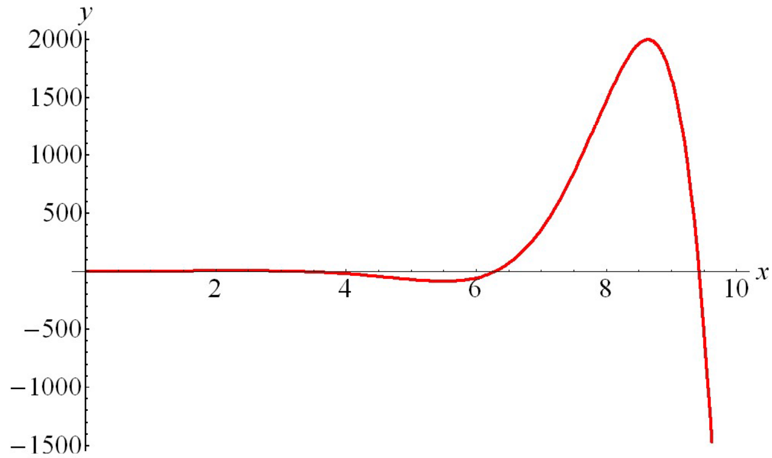
where: E, J, ρ are Young's modulus, moment of inertia and density of the beam. Dirac delta function $\delta(x-\eta)$ was introduced to describe the distribution of the externally applied force. Taking into account boundary conditions and equation of the beam's motion solutions of characteristic equations of both system were designated in agreement with the exact Fourier's method of separation of variables:

$\sin k_n l \cdot \sinh k_n l = 0,$ - for the simply supported beam and (2)

$$\cos k_n l = -\frac{1}{\cosh k_n l}, \qquad \text{- for the cantilever beam.} \qquad (3)$$

Graphical solutions of these equations are presented in Fig. 3.

a)



b)

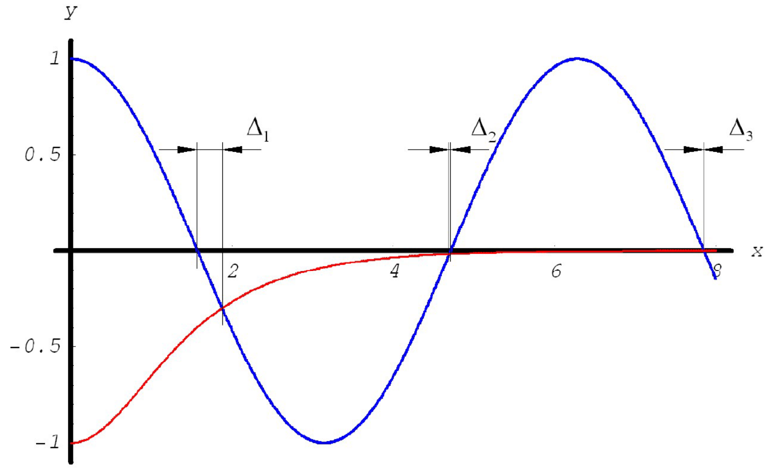


Fig.3. Graphic solutions of the characteristic equation, a) simply supported beam, b) cantilever beam

Taking into account graphical solutions of equations of motion of considered mechanical systems the equation of beams deflection in the approximate method for both systems was assumed as:

$$y(x,t) = \sum_{n=1}^{\infty} A \cdot \sin k_n x \cdot \cos \omega t, \qquad (4)$$

where:

$$k_n = \frac{n\pi}{l}, \qquad n = 1,2,3... \qquad \text{- for the simply supported beam and} \tag{5}$$

$$k_n = (2n-1)\frac{\pi}{2l}, \qquad n = 1,2,3... \qquad \text{- for the cantilever beam} \tag{6}$$

A is an amplitude of the beam's vibration.

Equations of natural frequencies of considered systems obtained using the exact and approximate methods are presented in table 2. In this table inaccuracies of the approximate method are presented in percentage.

Tab.2. Obtained equations of natural frequencies of considered systems

Simply supported beam		Cantilever beam		
The exact and the approximate method	Δ[%]	The exact method	The approximate method	Δ[%]
$\omega_1 = \sqrt{\frac{EJ}{\rho A}}\left(\frac{\pi}{l}\right)^2$	0	$\omega_1 = \sqrt{\frac{EJ}{\rho A}}\left(\frac{1{,}8751}{l}\right)^2$	$\omega_1 = \sqrt{\frac{EJ}{\rho A}}\left(\frac{\pi}{2l}\right)^2$	29,8
$\omega_2 = \sqrt{\frac{EJ}{\rho A}}\left(\frac{2\pi}{l}\right)^2$	0	$\omega_2 = \sqrt{\frac{EJ}{\rho A}}\left(\frac{4{,}6941}{l}\right)^2$	$\omega_2 = \sqrt{\frac{EJ}{\rho A}}\left(\frac{3\pi}{2l}\right)^2$	-0,782
$\omega_3 = \sqrt{\frac{EJ}{\rho A}}\left(\frac{3\pi}{l}\right)^2$	0	$\omega_3 = \sqrt{\frac{EJ}{\rho A}}\left(\frac{7{,}85477}{l}\right)^2$	$\omega_3 = \sqrt{\frac{EJ}{\rho A}}\left(\frac{5\pi}{2l}\right)^2$	0,023
$\omega_{n>3} = \sqrt{\frac{EJ}{\rho A}}\left(\frac{n}{l}\pi\right)^2$	0	$\omega_{n>3} = \sqrt{\frac{EJ}{\rho A}}\left(\frac{2n-1}{2l}\pi\right)^2$	$\omega_{n>3} = \sqrt{\frac{EJ}{\rho A}}\left(\frac{2n-1}{2l}\pi\right)^2$	0

In the exact method equation of the beam's natural frequency was designated as the solution of the characteristic equation of the system. In the approximate method these equations were determined taking into account assumed equations (2) and (3). For the simply supported beam obtained equations are exactly the same, while for the cantilever beam there are inaccuracies for the first three natural frequencies.

Dynamic flexibility of considered mechanical system was designated using the exact and corrected approximate method. The dynamic flexibility α_Y is defined by equation [1,2]:

$$y(x,t)=\alpha_Y \cdot F(t). \tag{7}$$

Geometrical and material parameters of considered systems are presented in Tab. 3. Absolute value of obtained dynamic flexibility Y for the first three natural frequencies are presented in Fig. 4 for the simply supported beam and in Fig. 5 for the cantilever beam.

Tab. 3. Parameters of considered mechanical systems

Geometrical parameters	Material parameters
$l=0{,}24[m]$, $b=0{,}04[m]$, $h_b=0{,}002[m]$, $\eta=0{,}01[m]$,	$E=210000[MPa]$, $\rho=7850\left[\frac{kg}{m^3}\right]$.

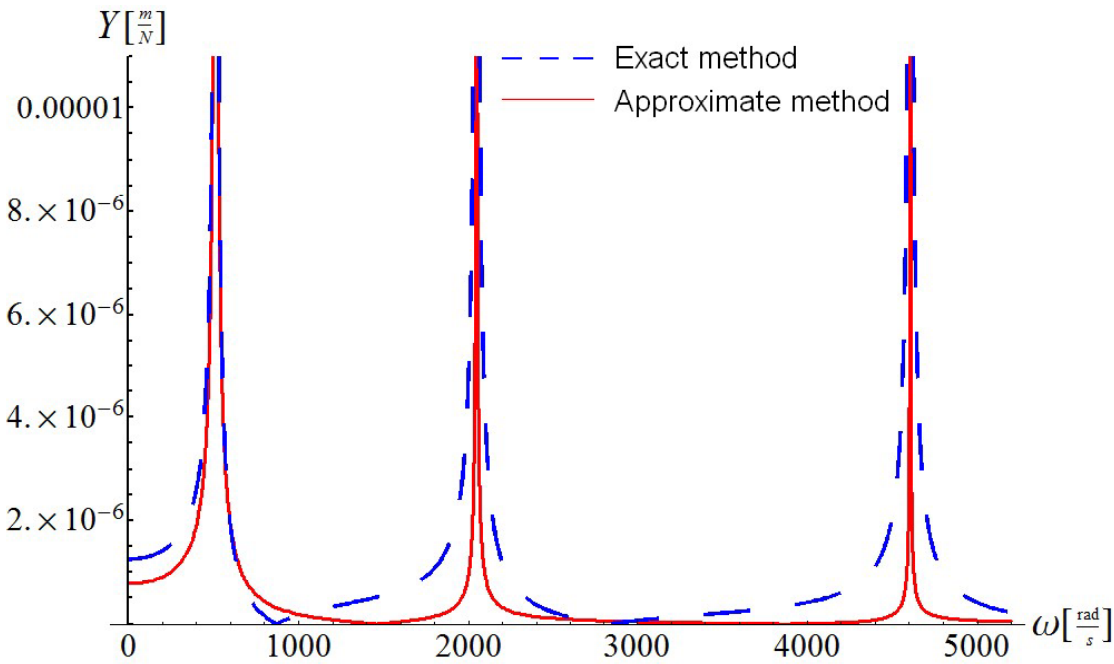


Fig. 4. The dynamic flexibility of the simply supported beam, for n=1,2,3

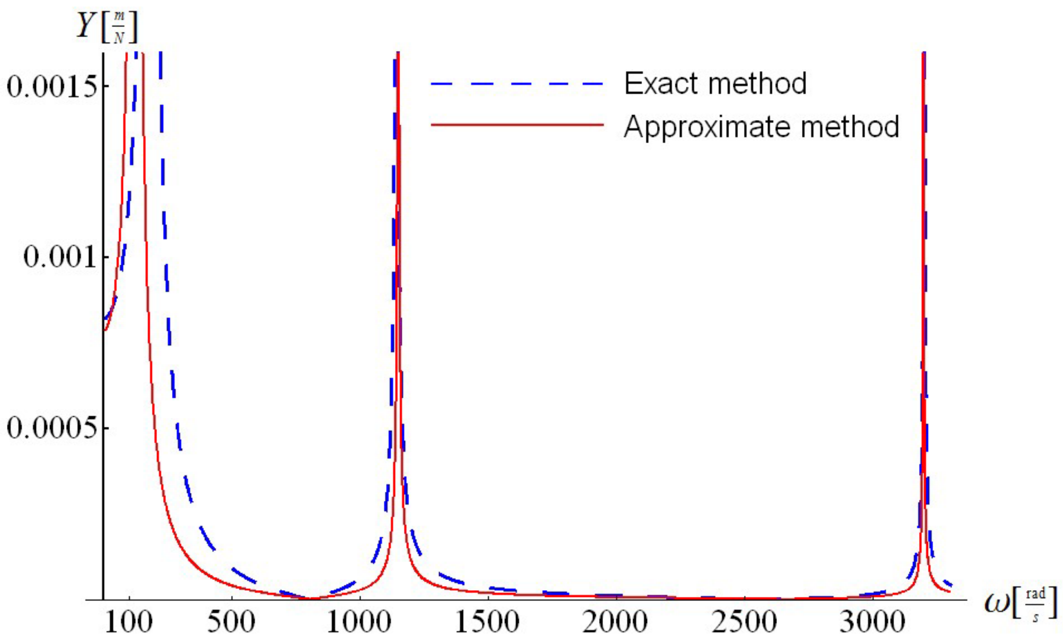


Fig.5. The dynamic flexibility of the cantilever beam, for n=1,2,3

3. Conclusions

It was proved that inexactness of the Galerkin method depends on the boundary conditions of the analysed system and assumed equation of the beam's deflection (equation 4). Considered systems were chosen purposely to show that in the first system the approximate method does not require correction while in the second one inaccuracy has to be corrected.

References

1. Buchacz A., Płaczek M.: The analysis of vibrating systems based on the exact end approximate method, International Journal of Modern Manufacturing Technologies, Vol. II, No 1, (2010), pp. 19-24
2. Buchacz A., Płaczek M.: Development of Mathematical Model of a Mechatronic System, Solid State Phenomena Vol. 164 (2010), Trans Tech Publications, Switzerland, pp. 319-322.
3. Maxwell N. D., Asokanthan S. F.: Modal characteristics of a flexible beam with multiple distributed actuators. Journal of Sound and Vibration 269 (2004), pp. 19-31.