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Operating characteristic curves in monitoring of strength in ready-mixed concrete

Keywords

quality, safety, operating-characteristic curve, concrete, compliance criteria

Abstract

Statistical conformity criteria for compressive strength of concrete are the matter debate. Statistical criteria can have prejudicial effects to quality and safety of construction. The paper is concerned with the application of operating-characteristic curves to the monitoring of compressive strength in ready-mixed concrete. In this paper the crude Monte Carlo simulation method has been applied to obtain OC-curves for different compound conformity criteria and normal distribution type of concrete strength.

1. Introduction

The PN-EN 206-1:2003, Concrete – Part 1: Specification, performance, production and conformity [1] gives specification and requirements for quality control of ready mixed concrete. These requirements determine the safety and durability of concrete structures. In the European codes EN 206-1 this is taken into account by choosing of optimal parameters for the conformity criteria and the value of the partial safety factor for concrete. In the European Standard EN 206-1 the statistical compliance criteria for compressive strength of concrete for continuous production are given as follows:

- for n=3 of results in the group

$$f_{cm} \geq f_{ck} + 4 \, , \; f_{ci} \geq f_{ck} - 4 \qquad (1)$$

- for n≥15 of results in the group

$$f_{cm} \geq f_{ck} + 1{,}48\sigma \; , \; f_{ci} \geq f_{ck} - 4 \qquad (2)$$

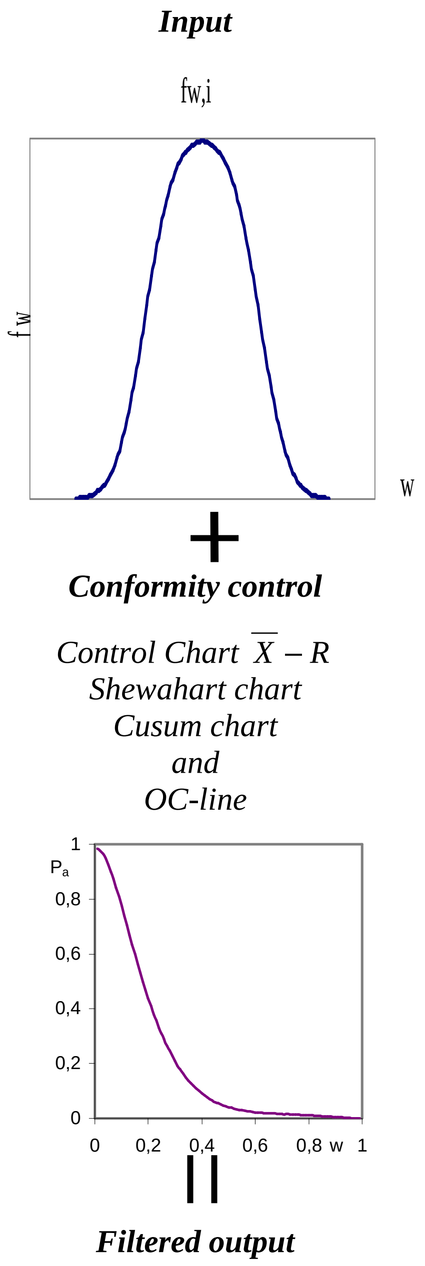
These criteria are far from perfection and there is always the risk that the suitable lot of concrete can be rejected (producer's risk and loss) or that the defective lot can be accepted (contractor's or/and investor's risk). Each supplied lot includes a random fraction defective (w) of sample units, which have strength characteristics worse than the required. The probability density function of the fraction defective corresponding to the supplied lots of concrete is $f_{w,i}$ and the density function of the fraction defective of the accepted lots is $f_{w,o}$. The function $f_{w,i}$ characterizes the production process, it is called the process curve and the function $f_{w,o}$ is called the filtered process curve. The conformity control acts as a filter and transforms the distribution $f_{w,i}$ into the distribution $f_{w,o}$ (*Figure 1*) .

The filtered process curve can be calculated using (3):

$$P_a(w) = P_a \frac{f_{w,o}}{f_{w,i}} \qquad (3)$$

where $P_a(w)$ is the probability of acceptance in the offered lots of concrete and P_a is a constant probability of acceptance.

The function $P_a(w)$ expresses the probability that a lot of concrete with known fraction defective *w* can be accepted using any statistical conformity criterion. The diagram of this function is called the operating characteristic curve or OC-curve.



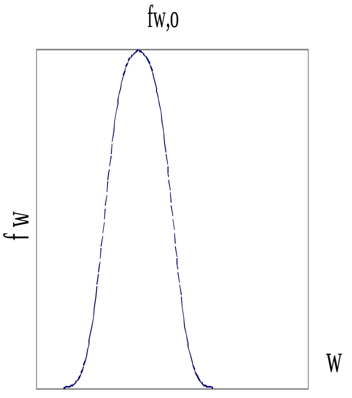


Figure 1. Filtering effect of the conformity control

Investigations of the efficiency of the statistical conformity criteria can be performed using statistical reasoning, Bayesian method, OC-curves , Shewahart chart or Cusum chart analysis. However, a general analytical expression for the $P_a(w)$ - value of compound criteria are not available and the only possibility is numerical simulation. In this paper the crude Monte Carlo simulation method has been applied to obtain OC-curves for different compound conformity criteria and normal distribution type of concrete strength.

2. The operating characteristic curves

Performance and efficiency of the statistical conformity criteria are investigated using the operation characteristic curves (OC curves). The probability of acceptance P_a is calculated by means of numerical simulation using the Monte Carlo technique and simulation programs.

The simulation programs for this purpose were elaborated for normal distribution with several known standard deviation, obtaining the operating characteristic curves shown in *Figures 2* and *Figure 3* for compressive strength criteria specified in PN-EN 206-1. Similar procedure has been adopted by Taerwe [3].

OC curves calculated of single criterion and compound criterion. Denote by:

A - the event that $\bar{x}_n \geq f_{ck}+4$,

B - the event that $x_{\min} \geq f_{ck}-4$

and $\bar{A},\bar{B}$ - opposite event for A, B. Each group of three simulated strength values can be classified. In one of the mutually exclusive subsets of the sample: AB, $\bar{A}B$, $A\bar{B}$, $\bar{A}\bar{B}$.

The probability of acceptance of compound criteria can be evaluated in this relationship:

$$P_a = P(AB) \Rightarrow P_a = 1 - P(\bar{A}B) - P(A\bar{B}) - P(\bar{A}\bar{B}) \quad (4)$$

For standard deviation 5MPa and n=3, the contributions of the different subset to the probability of rejection are show in *Figure 2*.

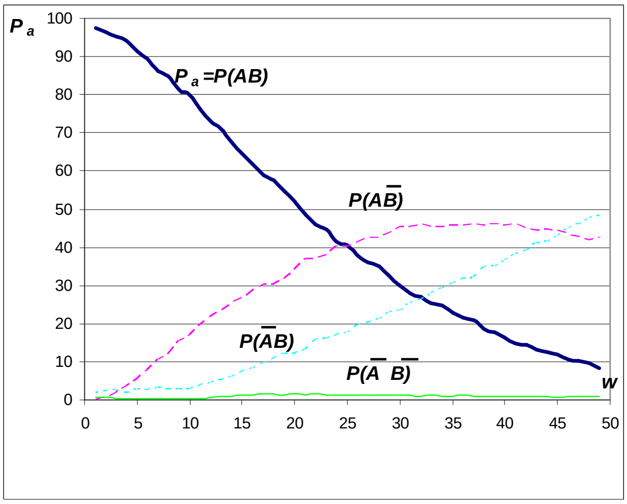


Figure 2. Contributions to the probability of rejection of criterion $\bar{x}_n \geq f_{ck} + 4$, $x_{min} \geq f_{ck} - 4$ for *σ=5MPa, n=3.*

2.1. Analysis of the operating characteristic curves

Using Monte Carlo [2] simulation technique OC-curves for the compound criterion of conformity were investigated for three different types of distribution of the compressive concrete strength: N, different standard deviation σ_{fc} of concrete strength and small sample size: *n*= *3*. The total number of simulation *N* = 100000 were used to obtain OC curves in all case.

The following conformity criteria were used:

- for the sample size *n* = *3*;

$$f_{cm} \geq f_{ck} + 4 \text{ and } f_{ci} \geq f_{ck} - 4 .$$

Comparisons of the rules for judging the quality of concrete (OC curves) with boundaries for unsafe and uneconomic regions have been presented by *Figure 3* and *Figure 4*. These regions were given as mathematical basis and justification by Taerwe [3].

Taerwe's definition of the boundary of the unsafe region is:

$$w \cdot P_a = 500 \quad (5)$$

and of the uneconomic region is:

$$\frac{w}{100 - P_a} = 0{,}05 \quad (6)$$

These boundaries are shown on *Figure 3* as dashed lines.

If conformity rule gives an operating characteristic that passes through the unsafe region then the protection it gives the specifies would be too weak. If a rule gives an operating characteristic that passes through the uneconomic region it causes producers to use excessively large margins, even then, accept high risks of non-conformity [5].

2.2. Normal distribution of concrete strength

OC curve for criteria (2) shown in *Figure3* to *Figure 4* for samples n=3 for normal (N) distributions with known standard deviation.

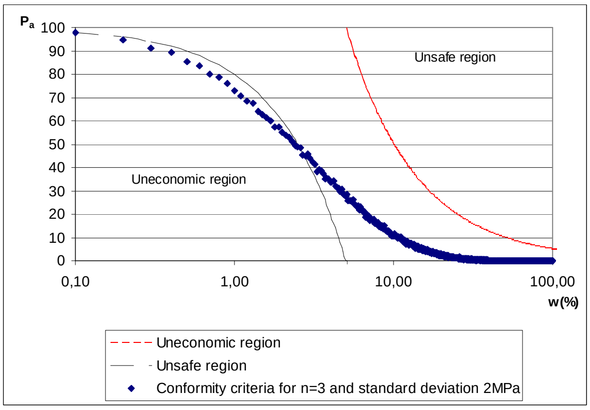


Figure 3. Compliance criteria for group n=3 samples with known standard deviation 2MPa and for compound criterion $f_{cm} \geq f_{ck} + 4$ and $f_{ci} \geq f_{ck} - 4$

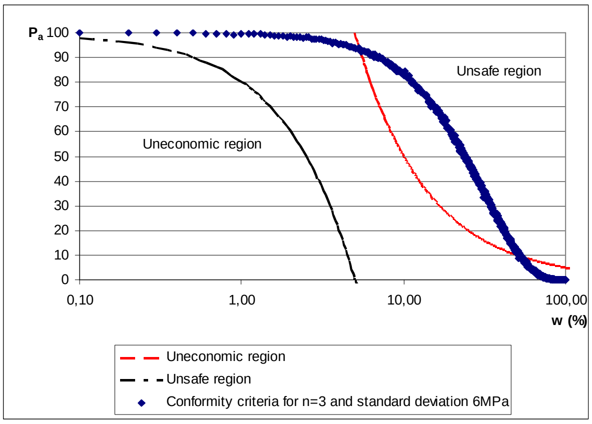


Figure 4. Compliance criteria for group n=3 samples with known standard deviation 6MPa and for compound criterion $f_{cm} \geq f_{ck} + 4$ and

$$f_{ci} \geq f_{ck} - 4 .$$

Pprediction accuracy of statistical acceptance criteria has been determinate with use of normal distribution. Taking the compliance compound criterion into consideration, higher values of acceptance probability P_a correspond to lots of bigger strength variability. The lower probability of acceptance has been for N distributions of higher strength variability.

High values of probability of acceptance have been of productions with more than 5% of defects, which can be around 80% in some cases.

Inference about the quality of produced concrete can be supplemented through the use of fuzzy sets. With the membership functions to different classes of concrete C_{i-1}, C_i, C_{i+1} and the mean strength value estimated on the basis of samples produced batch of concrete, you can calculate the degree of membership of the party to different classes of concrete (*Figure 5*).

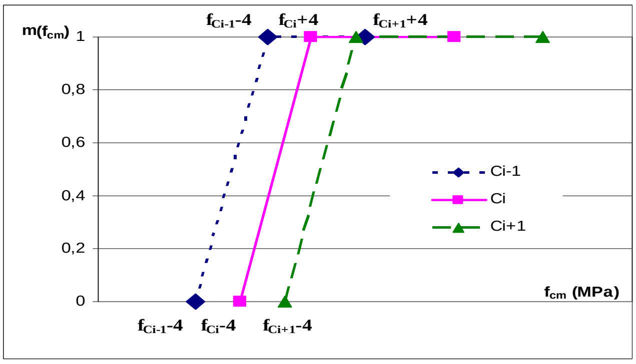


Figure 5. Membership functions for concrete class C_{i-1}, C_i, C_{i+1}

Depending on the value of $\mu(f_c)$, may be decided a pass the considered batch of concrete to the appropriate class. This decision may be more or less cautious, for example depending on the qualitative assessment and its impact on durability and safety of the structure.

3. Conclusions

The compound conformity criteria for concrete strength for *n* =3; $f_{cm} \geq f_{ck} + 4$ and $f_{ci} \geq f_{ck} - 4$, which were recommended in PN-EN 206 are not perfect.

The analyses presented in this paper showed that the compound criteria of small sample sizes *n*=3 can have prejudicial effects to both - producer and contractor, for example groundless increase of production costs and high level of contractor's risk.

Application of fuzzy methods gives possibility of taking reasonable decision on the concrete classification.

References

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