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Aerial Target Flight Path Modeling using B-Spline Curves

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Abstract. A method is presented for generating the flight path of an aerial target using B-spline curves developed for studying the combat effectiveness of anti-aircraft missiles and air defense systems. A procedure is described for determining the kinematic quantities (velocity, accelerations, angles of spatial orientation) that characterize the motion of an object along a set trajectory. Exemplary simulation results are included.
Keywords: kinematics, trajectory, path modeling, spline theory, aerial target

1. INTRODUCTION

Modern technological solutions provide an attack aircraft with the possibility of reacting effectively to fire, including by the performance of anti-missile maneuvers or the use of various types of interference. It is thus especially important to test the effectiveness of air defence systems in a variety of potential conditions and scenarios.

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Testing under field conditions, although desirable as regards the reliability of the results, is not only extremely expensive but also entails many organizational and logistical problems. An important role is therefore assigned to simulation tests.

Simulation testing of the combat effectiveness of anti-aircraft missiles and air defense systems is carried out within the framework of an extensive computer modeling process. One of the elements of the simulation is the model of the aerial target, whose motion in space (depending on the degree of simplification adopted) is described by means of kinematic or dynamic equations for a material point, or dynamic equations for a rigid body. In the kinematic model, the movement of the target is controlled by means of explicit changes in velocity and the pitch and yaw angles. In the dynamic model, the changes in velocity and in the angles of spatial orientation are a consequence of the determination of applied g-loading or inclinations of the control surfaces and ailerons. These equations provide a simple way to describe single maneuvers; however, employing them to construct complex trajectories or whole sets of trajectories can be laborious and inefficient in terms of time. A natural approach to planning the motion of an aerial target is to plot its flight path over the analyzed area. In doing this, use may be made of spline functions, whose advantages have long been exploited in computer graphics and simulations [1, 2, 5-11].

This paper describes a way of generating the flight path of an aerial target using B-spline curves, as well as a procedure for determining the kinematic quantities (velocity, accelerations, angles of spatial orientation) that characterize an object's motion along a set trajectory.

The content is organized as follows. Section 2 contains the mathematical foundations for the description of parametric curves. Section 3 contains a commentary on the procedures for numerical differentiation and coordinate system transformation used in the computation of the kinematic quantities, as well as the constraints imposed on the model of the aerial target. Section 4 presents selected simulation results. The final section sets out the conclusions and indicates some possible uses of the developed solution.

2. AERIAL TARGET TRAJECTORY REPRESENTATION

A generalization of de Casteljau's algorithm for Bézier curves known as "de Boor's algorithm" is used in the proposed solution [2, 10]. The algorithm is a fast and numerically stable means of finding a point on the B-spline curve $\mathbf{s}(u)$ at the position u. The curve $\mathbf{s}(u)$ is defined as a linear combination of the basis functions $f_{i,p}$ of the order p in the variable u and $(n+1)$ control points $\mathbf{c}_i$ as follows:

$$\mathbf{s}(u) = \sum_{i=0}^{n} \mathbf{c}_i f_{i,p}(u) \qquad u \in U \tag{1}$$

where U is the given non-decreasing real sequence of locations (knots)

$$U \in \mathbf{R}: \quad U = \{u_0, u_1, ..., u_m\} \tag{2}$$

and the control point $\mathbf{c}_i$ is a vector-valued constant defined as

$$\mathbf{c}_i = \begin{bmatrix} c_{xi} \\ c_{yi} \\ c_{zi} \end{bmatrix} \tag{3}$$

Any spline function $\mathbf{s}(u)$ of the order p in the variable u on the given set U can be expressed as a sum of the p-order basis splines. Expressions for the polynomial pieces $f_{i,p}$ are described by means of the Cox-de Boor recursion formula [2]:

$$\begin{gathered} f_{i,0}(u) = \begin{cases} 1 & \Leftrightarrow \quad u_i \le u \le u_{i+1} \\ 0 & \quad\text{otherwise} \end{cases} \\ f_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} f_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} f_{i+1,p-1}(u) \end{gathered} \tag{4}$$

Assuming that the final simulation time t_f is estimated *via* the total distance d travelled by the target, its average velocity $\bar{v}$, and rounded up to the nearest integer:

$$t_f = \left\lceil \frac{d}{\bar{v}} \right\rceil \tag{5}$$

the variable m used in Eq. (2) is given as follows:

$$m = \frac{t_f}{t_s} \tag{6}$$

where t_s is the sampling time. It is also assumed that

$$t_s = 10^{-k} \qquad k \in \mathbf{N} \tag{7}$$

therefore, the values of m are limited to the set of natural numbers. For such assumptions the set U is defined as

$$U = \left\{ u : u = \frac{j}{m},\ j \in \mathbf{N} \wedge j \le m \right\} \tag{8}$$

and Eq. (1) can be rewritten in form of

$$\mathbf{s}(u)=\begin{bmatrix} x(u) \\ y(u) \\ z(u) \end{bmatrix}=\begin{bmatrix} \sum_{i=0}^{n} c_{xi} f_{i,p}(u) \\ \sum_{i=0}^{n} c_{yi} f_{i,p}(u) \\ \sum_{i=0}^{n} c_{zi} f_{i,p}(u) \end{bmatrix} \tag{9}$$

in which $x(u)$, $y(u)$ and $z(u)$ represent the flat components of the spatial curve $\mathbf{s}(u)$. By entering a set of time instants

$$T=\left\{ t : t = jt_s,\ j \in \mathbf{N} \wedge j \le m \right\} \tag{10}$$

and mapping

$$U \mapsto T \tag{11}$$

the component functions are redefined by the change of their domain from

$$x : U \to X \qquad y : U \to Y \qquad z : U \to Z \tag{12}$$

to

$$x : T \to X \qquad y : T \to Y \qquad z : T \to Z \tag{13}$$

where the codomains X, Y, Z are given as

$$X,Y,Z \in \mathbf{R}: \quad X=\left\{x_0, x_1, ..., x_m\right\} \quad Y=\left\{y_0, y_1, ..., y_m\right\} \quad Z=\left\{z_0, z_1, ..., z_m\right\} \tag{14}$$

The resultant discrete functions $x(t)$, $y(t)$, $z(t)$ are both time dependent and differentiable with respect to time.

3. AERIAL TARGET KINEMATICS

The target is taken as a material point moving along a trajectory defined by the functions $x(t)$, $y(t)$ and $z(t)$. These functions are twice differentiated numerically by a finite difference method. This gives the vectors

$$\mathbf{v}=\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}=\begin{bmatrix} \frac{dx(t)}{dt} \\ \frac{dy(t)}{dt} \\ \frac{dz(t)}{dt} \end{bmatrix} \qquad \mathbf{a}=\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}=\begin{bmatrix} \frac{d^2x(t)}{dt^2} \\ \frac{d^2y(t)}{dt^2} \\ \frac{d^2z(t)}{dt^2} \end{bmatrix} \tag{15}$$

describing the velocity and the acceleration of the aerial target in an inertial Cartesian coordinate system *Oxyz*, respectively. Knowing the vector **v**, it is possible to calculate the course angle β and the pitch angle ε. The course angle is obtained from the formula

$$\beta = \arctan\left(\frac{v_y}{v_x}\right) \tag{16}$$

and the pitch angle from the equation

$$\varepsilon = \arctan\left(\frac{v_z}{\sqrt{v_x^2 + v_y^2}}\right) \tag{17}$$

where it is assumed that the target's pitch and yaw angles are equal to the pitch and yaw angles of the velocity vector **v**.

The vectors **v** and **a** are then projected onto the axes of the non-inertial coordinate system $Ox'y'z'$ associated with the target. The origin of this system is the same as the origin of the velocity vector **v**, the x' axis coincidence with the direction and sense of that vector, and the orientation of the y' and z' axes, which are normal to the x' axis, dependent on the instantaneous values of the angles β and ε. The projection of the vectors **v** and **a** onto the axes of the local coordinate system $Ox'y'z'$ is performed using the operations

$$\begin{bmatrix} v'_x \\ v'_y \\ v'_z \end{bmatrix} = \mathbf{M} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \mathbf{Mv} \qquad \begin{bmatrix} a'_x \\ a'_y \\ a'_z \end{bmatrix} = \mathbf{M} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \mathbf{Ma} \tag{18}$$

where v'_x, v'_y, v'_z and a'_x, a'_y, a'_z are the components of the velocity vector **v** and the acceleration vector **a** in the $Ox'y'z'$ coordinate system, respectively, and **M** is the rotation matrix given by

$$\mathbf{M} = \begin{bmatrix} \cos\beta\cos\varepsilon & -\sin\beta\cos\varepsilon & \sin\varepsilon \\ \sin\beta & \cos\beta & 0 \\ -\cos\beta\sin\varepsilon & \sin\beta\sin\varepsilon & \cos\varepsilon \end{bmatrix} \tag{19}$$

Since the x' axis coincides with the direction and sense of the velocity vector **v**, we have

$$\mathbf{v} = \begin{bmatrix} v'_x \\ 0 \\ 0 \end{bmatrix} \qquad v = |\mathbf{v}| = v'_x \tag{20}$$

The modulus of the vector **a** is obtained using the Euclidean norm

$$a = |\mathbf{a}| = \sqrt{(a'_x)^2 + (a'_y)^2 + (a'_z)^2} \tag{21}$$

The roll angle φ is approximated using the formula

$$\varphi = \arctan\left(\frac{a'_y}{g}\right) \tag{22}$$

where g is the acceleration due to gravity.

The computed kinematic values are subjected to the constraints

$$\begin{gathered} 0 < z < h_{\max}(v) \\ v_{\min}(z) \le v \le v_{\max}(z) \\ a_{\min}(v,z) \le a \le a_{\max}(v,z) \end{gathered} \tag{23}$$

where $h_{\max}$ is the function describing the permissible flying altitude of the target, $v_{\min}$ and $v_{\max}$ are the functions describing the target's minimum and maximum velocity, and $a_{\min}$ and $a_{\max}$ are the functions describing its minimum and maximum acceleration. The values of the constraining functions result from the typical maneuvering capabilities of a modern multirole fighter (Fig. 1). These values were tabulated for the purposes of computer simulation.

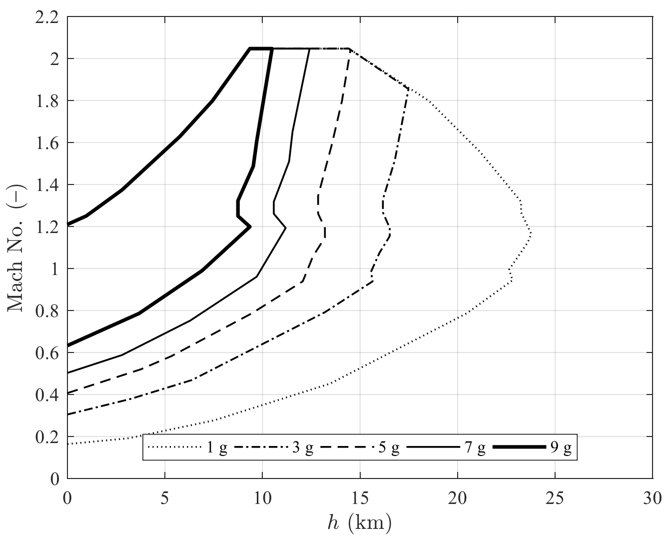

Fig. 1. Exemplary maneuvering capabilities of a multirole fighter [3]

Based on the calculated velocity and acceleration values related to kinematic constraints (23), the software determines the set of trajectories that meet the imposed conditions.

Using these, it then selects the route described by the lowest-order polynomials, thus preventing potential large oscillations in the trajectory that can result from high-order polynomials [1].

4. PERFORMANCE ANALYSIS

The first scenario chosen to illustrate the performance of the presented solution is the 90-degrees banking turn maneuver of an aerial target flying at the constant altitude of 3,000 m. The final simulation time is t_f = 20 s. Five control points (marked as '×') and two additional boundary points ('o') are declared for this maneuver. The target trajectories in the horizontal plane are given in Fig. 2. The software tool described in [4] was used to visualize the shape of the aerial target on the chart. The velocity and acceleration histories corresponding to this simulation are illustrated in Figs. 3-4.

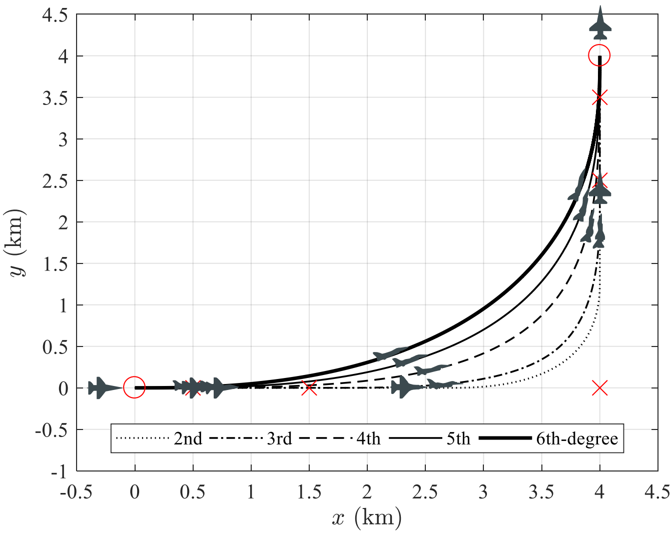

Fig. 2. Spline trajectories of various degrees

Note that the received velocity and acceleration profiles are strongly connected with the order of the basis splines. As it can be seen in Figs. 3-4, the 2nd-, 3rd- and 6th-degree B-spline approximations give partly non-physical solutions. In the case of the 3rd-degree curve, the high value of the a'_y component of the acceleration vector **a** in the midcourse results in the maximum acceleration constraint a_{max} being exceeded. On one hand, further reducing the degree of approximating polynomials produces significant instances of maximum constraint values being exceeded; on the other, it causes a disturbance in the continuity of the velocity and acceleration profiles.

The option to use the 6th-degree curve was thus discarded after analyzing the velocity profile.

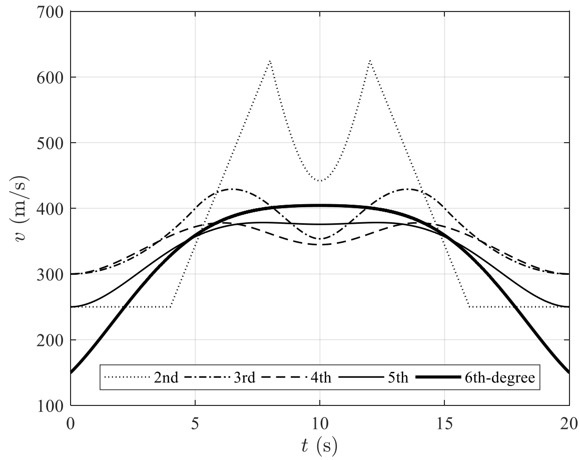

Fig. 3. Velocity profiles for spline trajectories of various degrees

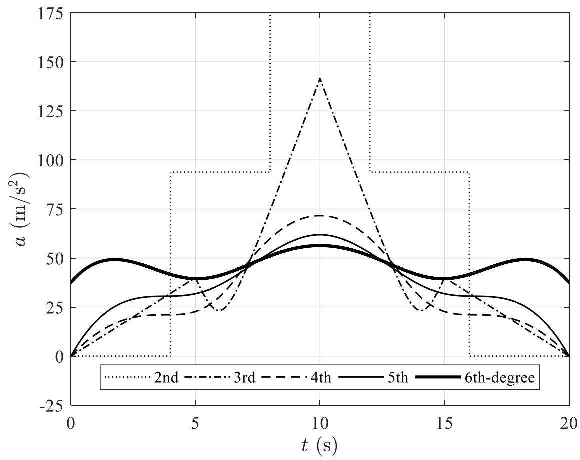

Fig. 4. Acceleration profiles for spline trajectories of various degrees

During the first two seconds of flight at the considered altitude with the acceleration a > 40 m/s^2, the target velocity is slightly lower than the minimum permissible limit $v_{\min}$ = 180 m/s (cf. Fig. 1). In this example, the 4th-degree spline curve was proposed by the program as adequate for the approximation of the discussed maneuver. The next scenario illustrates another trajectory of degree 4 obtained for a complex aerial target maneuver in space.

The final simulation time is t_f = 25 s. The route is defined by three control points ('×') and two boundary points ('o'). The target trajectory is presented in Fig. 5.

The velocity and acceleration histories corresponding to this simulation are illustrated in Figs. 6-7. The target yaw, pitch and roll angle histories are given in Fig. 8. With this trajectory, all values are within the limits imposed, i.e. all conditions for achieving desirable solution are satisfied.

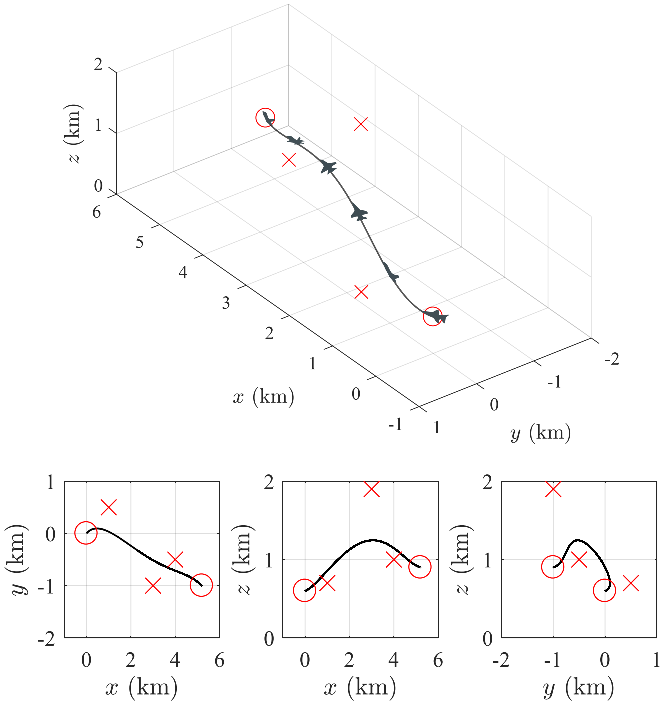

Fig. 5. Aerial target trajectory

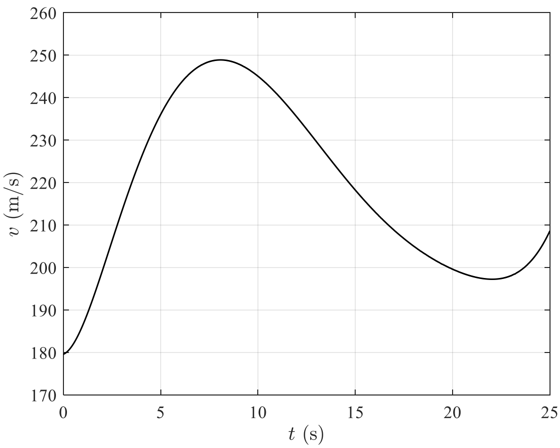

Fig. 6. Velocity profile

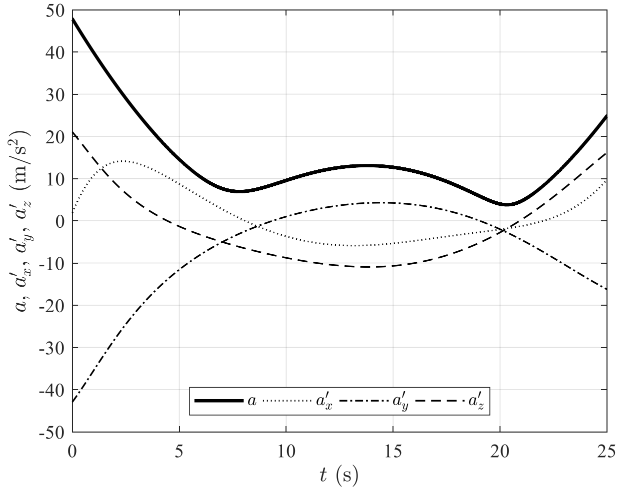

Fig. 7. Acceleration profiles

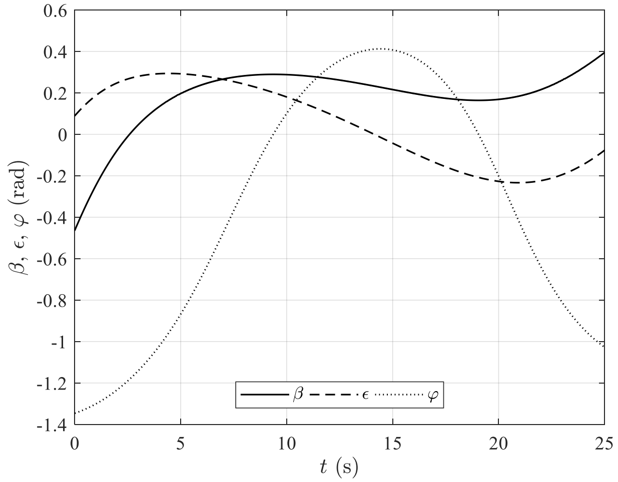

Fig. 8. Yaw, pitch and roll angle histories

5. CONCLUDING REMARKS

We have described a way of generating aerial target flight paths using B-spline curves. A procedure for determining the kinematic quantities that characterize the motion of an object along a set trajectory was similarly expanded.

In simulation tests, two sample scenarios were presented in which the determined trajectories, along with corresponding kinematic quantities, were compared with the constraints resulting from the typical maneuvering capabilities of modern multirole aircraft. This made it possible to identify, from among the set of solutions, the flight path that best reflects the real behaviour of an aerial target.

The numerical results obtained confirm the high effectiveness and usefulness of the proposed method of modelling the flight paths of an attack aircraft. The method may find applications in software tools for path editing for the purposes of numerical analysis of the combat effectiveness of anti-aircraft missiles and air defense systems.

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Modelowanie trajektorii lotu celu powietrznego z wykorzystaniem krzywych B-sklejanych

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Streszczenie. W artykule przedstawiono sposób generowania trajektorii lotu celu powietrznego z wykorzystaniem krzywych B-sklejanych, opracowany na potrzeby badań skuteczności bojowej rakiet przeciwlotniczych i zestawów rakietowych obrony powietrznej. Opisano procedurę wyznaczania wielkości kinematycznych (prędkości, przyspieszeń, kątów orientacji przestrzennej), charakteryzujących ruch obiektu po zadanej trajektorii. Zamieszczono przykładowe wyniki badań symulacyjnych.

Słowa kluczowe: kinematyka, modelowanie trajektorii, krzywe B-sklejane, cel powietrzny