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INTERPRETATION OF FIBONACCI NUMBERS IN BOTANY IN THE CROSS-SECTIONS AND LEAFAGE OF SELECTED VEGETABLES

Abstract

Introduction and aims: The study shows the interpretation of Fibonacci numbers in botany. In particular, it is shown the interpretation of symmetry in the cross-sections of selected vegetables. Also have been presented some definitions of Fibonacci numbers and discuss their interpretation in certain cross-sections of selected vegetables. Therefore, the main aim of this work is to show the interpretation of Fibonacci numbers in the analysis of cross-sections of selected vegetables.
Material and methods: Material consists some pictures of vegetables and their cross-sections which were made by the Authors of this paper. The method of visual and theoretical analysis has been performed in this paper.
Results: In this paper, has been considered a series of interesting images of selected plants vegetables. Presented graphical interpretation of dual, triangular, tetragonal, pentagonal, hexagonal and decagonal symmetry, which shows the occurrence of Fibonacci numbers.
Conclusions: Fibonacci numbers in botany are interpreted in the cross-sections of various vegetables. In some cross-sections of vegetables can be observed some dual, triangular, tetragonal, pentagonal, hexagonal and even decagonal symmetry. The interpretation of Fibonacci numbers may be used to supplement the classification of vegetables plants.

Keywords: Botany, vegetables, cross-section, symmetry, Fibonacci numbers.
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INTERPRETACJA LICZB FIBONACCIEGO W BOTANICE W PRZEKROJACH I ULISTNIENIU WYBRANYCH WARZYW

Streszczenie

Wstęp i cele: *W pracy pokazano interpretację liczb Fibonacciego w botanice. W szczególności pokazano interpretację symetrii występującej przekrojach poprzecznych wybranych warzyw. Podano definicje liczb Fibonacciego oraz omówiono ich interpretację w określonych przekrojach poprzecznych wybranych warzyw. Zatem głównym celem pracy jest pokazanie interpretacji liczb Fibonacciego w analizie przekrojów poprzecznych wybranych warzyw.*
Materiał o metody: *Materiałem są zdjęcia warzyw i ich przekrojów poprzecznych wykonane przez autorów pracy. Zastosowano metodę analizy wizualno-teoretycznej.*
Wyniki: *W pracy otrzymano szereg interesujących zdjęć wybranych warzyw. Przedstawiono interpretację graficzną symetrii dualnej, trójkątnej, czworokątnej, pięciokątnej, sześciokątnej i dziesięciokątnej, w których pokazano występowanie liczb Fibonacciego.*
Wnioski: *Interpretację liczb Fibonacciego można znaleźć w różnych przekrojach wybranych warzyw. W niektórych przekrojach warzyw można zaobserwować symetrię dualną, trójkątną, czworokątną, pięciokątną, sześciokątna a nawet dziesięciokątną. Interpretacja liczb Fibonacciego może być użyteczna w uzupełnieniu klasyfikacji warzyw.*

Słowa kluczowe: *Botanika, warzywa, przekroje poprzeczne, symetria, liczby Fibonacciego.*
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1. Fibonacci numbers

Leonardo Fibonacci
(around 1175 – 1250)

Leonardo of Pisa was born around 1175 years and died in 1250. The Italian mathematician. His father, Guillermo Bonacci family, he was a diplomatic mission in North Africa and the Fibonacci there just was educated. The first lessons in mathematics charge from the Arabic teacher in *Boużia.* He has traveled extensively, first with his father and later alone, visiting and educating themselves in places like Egypt, Syria, Provence, Greece and Sicily. On his travels through Europe and the countries of the East he had a chance to see the achievement of Arab and Hindu mathematicians, especially the decimal system number. Fibonacci ended around 1200 to travel and returned to Pisa [16].

Fibonacci sequence of the general expression F(n) is determined for each natural number $n \in N$ by the following recursive definition [1], [2]:

$$F(1) = 1, \tag{1}$$

$$F(2) = 1, \tag{2}$$

$$F(n+2) = F(n+1) + F(n). \tag{3}$$

From above definition we obtain the well known sequence of Fibonacci numbers:

$$1, \ 1, \ 2, \ 3, \ 5, \ 8, \ 13, \ 21, \ 34, ... \tag{4}$$

By definition, the first two numbers in the Fibonacci sequence are either 1 and 1, or 0 and 1, depending on the chosen starting point of the sequence, and each subsequent number is the sum of the previous two.

2. Symmetry in vegetables

Thus symmetry relates to the geometric properties that occur in the world around us. There is beauty in symmetry. According to botanists (specialists who study plants) a fruit is the part of the plant that develops from a flower. It is also the section of the plant that contains the seeds. The other parts of plants are considered vegetables. These include the stems, leaves and roots, and even the flower bud.

The interpretation of Fibonacci numbers can be found in various issues of botany. Fibonacci sequences appear in biological settings, in two consecutive Fibonacci numbers, such as branching in trees, arrangement of leaves on a stem, the fruit-lets of a pineapple, the flowering of artichoke, an uncurling fern and the arrangement of a pine cone, and the family tree of honeybees [3]-[6], [8].

In particular case, Fibonacci numbers can be read not only the cross-section of vegetables, but also in their foliage [7], [9]-[15]. In this paper, we show the interpretation of Fibonacci numbers in the symmetry of selected vegetables. We will present the dual *(cocktail tomato),* triangular *(tomato and ice lettuce),* tetragonal *(cucumber and yellow pepper),* pentagonal *(eggplant),* hexagonal *(red tomato and yellow tomato)* and decagonal symmetry *(garlic).*

Moreover it will be indicated the angle of rotation about the central axis of the vegetable cross-section. We will also show examples of vegetables, in which there is some symmetry of leaves pattern. There is considered a triangular symmetry *(Chinese cabbage)* and pentagonal symmetry *(ordinary cabbage, tomato and long cucumber).*

3. Symmetry in cross-sections of vegetables

3.1. Dual symmetry

This variant of radial symmetry, also called dual, with rotation angle of 180° about the central axis there is in cocktail tomato (Fig. 1a). In cross-section of the cocktail tomato is observed Fibonacci number 2 (Fig. 1b).

Fig. 1a. Cocktail tomato, general view
Photo: G.P. Skorny, J. Śledziowski

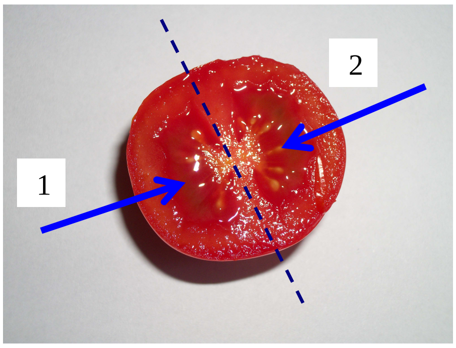

Fig. 1b. Cocktail tomato, cross-section, Fibonacci number 2
Photo: G.P. Skorny, J. Śledziowski

3.2. Triangular symmetry

A triangular symmetry with rotation angle 120° is presented in another tomatoes (Fig. 2a). We have interpretation of Fibonacci number 3 in the cross-section (Fig. 2b).

Fig. 2a. Tomatoes, general view
Photo: G.P. Skorny, J. Śledziowski

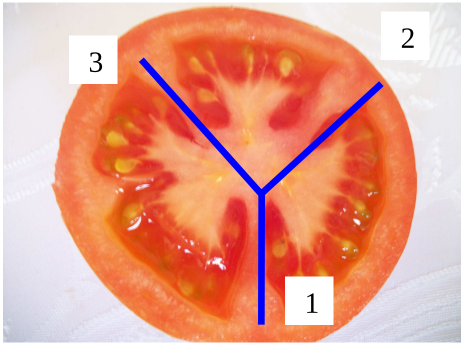

Fig. 2b. Tomato, cross-section, Fibonacci number 3
Photo: G.P. Skorny, J. Śledziowski

Also a triangular symmetry with rotation angle 120° is presented in the ice lettuce (Fig. 3a). We have interpretation of Fibonacci number 3 in the cross-section (Fig. 3b).

Fig. 3a. Ice lettuce, general view
Photo: G.P. Skorny, J. Śledziowski

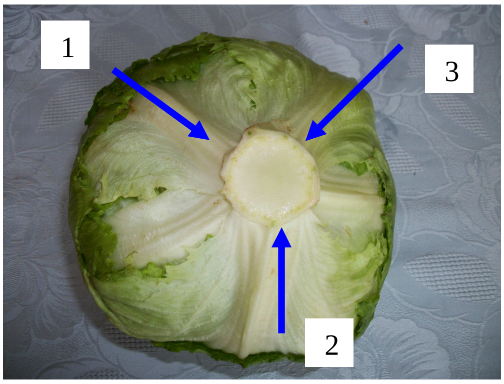

Fig. 3b. Ice lettuce, cross-section, Fibonacci number 3
Photo: G.P. Skorny, J. Śledziowski

3.3. Tetragonal symmetry

A tetragonal *(i.e. tetraradial)* symmetry with rotation angle 90° is shown in the cucumber (Fig. 4a). Here we see an illustration of double Fibonacci number 2 i.e. 4 = 2·**2** where for ease of interpretation was indicated by the dashed line outline of baffles (Fig. 4b).

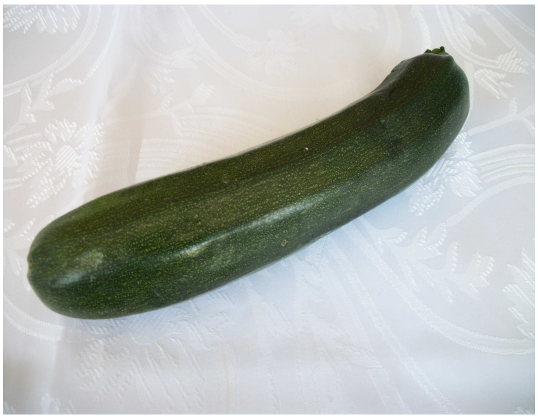

Fig. 4a. Cucumber, general view
Photo: G.P. Skorny, J. Śledziowski

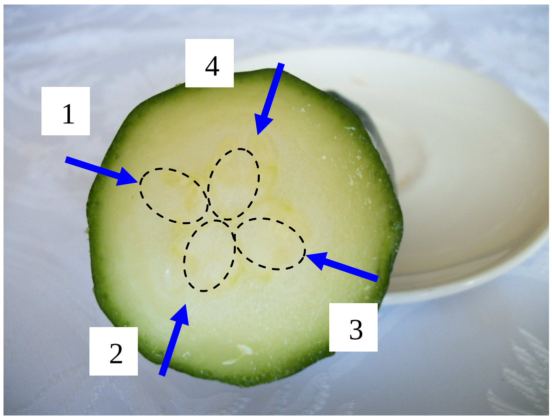

Fig. 4b. Cucumber, cross-section, Fibonacci number 4 = 2·**2**
Photo: G.P. Skorny, J. Śledziowski

Also a tetragonal symmetry with rotation angle 90° is shown in the yellow pepper (Fig. 5a). Here we have also a double Fibonacci number 2, i.e. 4 = 2·**2** and a cross-section with the number of baffles and chambers after 4 (Fig. 5b). Here we see also double number 2 both the number of partitions and the number of chambers.

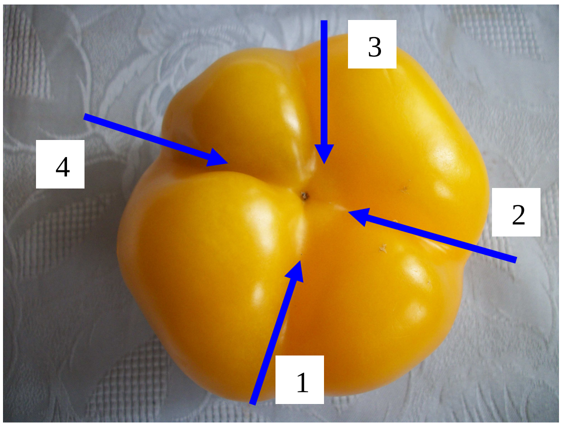

Fig. 5a. Yellow pepper, general view
Photo: G.P. Skorny, J. Śledziowski

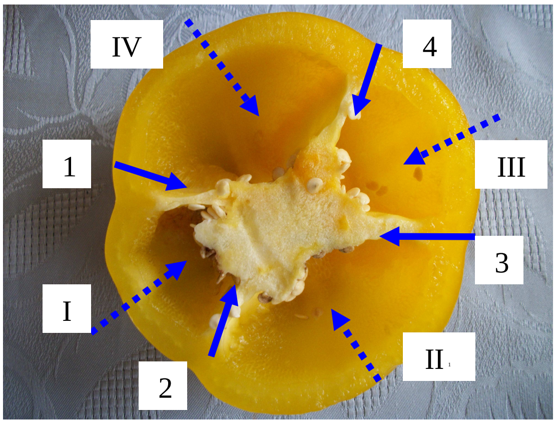

Fig. 5b. Yellow pepper, cross-section, number of baffles 4 = 2·**2**
Photo: G.P. Skorny, J. Śledziowski

Most peppers are green, yellow, orange, and red (between stages of ripening). More rarely, color can be brown, white, lavender and dark purple, depending on the variety of pepper.

3.4. Pentagonal symmetry

This variant of pentagonal symmetry, also called *pentaradial symmetry*, with rotation angle of 72° about the central axis show some leaves pattern of eggplant (Fig. 6a). Here there is shown Fibonacci number 5. However, in the inner chambers of eggplant occurs an untypical symmetry i.e. heptagonal symmetry 7 = 8 – 1 = 4·**2** – 1. We have here the Fibonacci number 8 which is reduced by 1 (Fig. 6b).

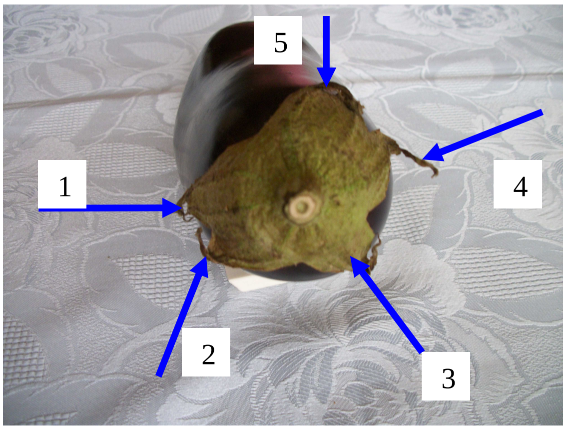

Fig. 6a. Eggplant, number of leaves 5
Photo: G.P. Skorny, J. Śledziowski

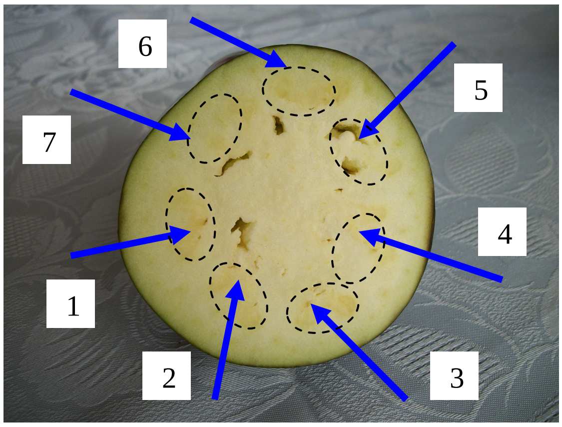

Fig. 6b. Eggplant, cross-section, number of leaves 7 = **8**–1
Photo: G.P. Skorny, J. Śledziowski

Eggplant has different varieties of size, shape, and color, though typically purple.

3.5. Hexagonal symmetry

Variant of hexagonal *(i.e. hexaradial)* symmetry with rotation angle of 60° about the central axis is shown both in the leaves pattern of yellow tomato (Fig. 7a) and also in cross-section (Fig. 7b). It is an illustration of Fibonacci number 3 (a cross-section with the number of baffles), and its doubling i.e. 6 = 2·**3** (a cross section with the number of chambers).

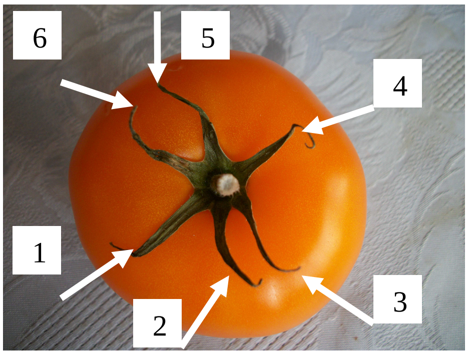

Fig. 7a. Yellow tomato, general view, number of leaves 6 = 2·**3**
Photo: G.P. Skorny, J. Śledziowski

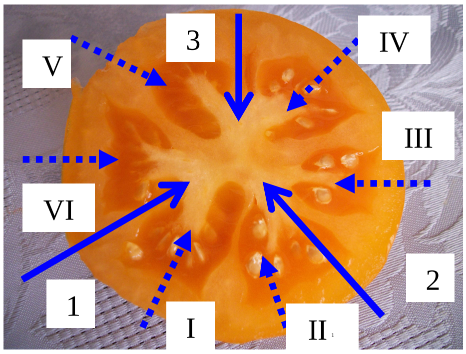

Fig. 7b. Yellow tomato, cross-section, number of baffles 3, number of chambers 6 = 2·**3**
Photo: G.P. Skorny, J. Śledziowski

This variant of symmetry is in the red tomato, both for the number of leaves 6 (Fig. 8a) and the number of internal walls 6 (Fig. 8b). There is doubled Fibonacci number 3, that is, 6 = 2·**3**.

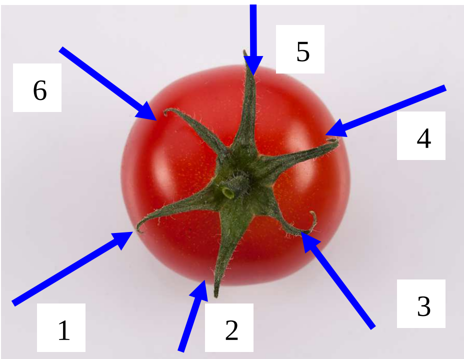

Fig. 8a. Red tomato, leafage, number 6 = 2·**3**
Photo: G.P. Skorny, J. Śledziowski

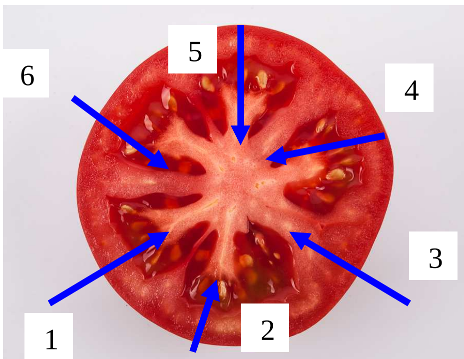

Fig. 8b. Red tomato, cross-section, number 6=2·**3**
Photo: G.P. Skorny, J. Śledziowski

3.6. Decagonal symmetry

A decagonal symmetry *(i.e. decaradial)*, with rotation angle 36° about central axis occurs in the right selected cross-section in a garlic (Fig. 9a). There is double of Fibonacci number 5 in the certain cross-sections of the garlic, i.e. 10 = 2·**5**.

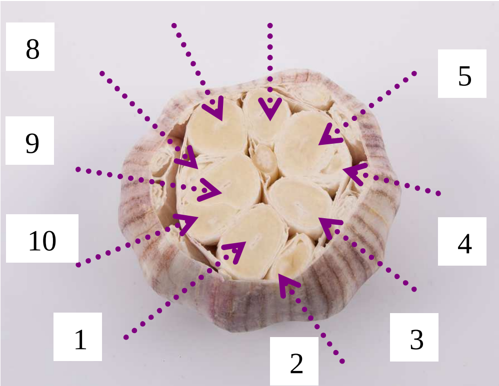

Fig. 9a. Garlic, cross-section, number of seeds 10=2·**5**
Photo: G.P. Skorny, J. Śledziowski

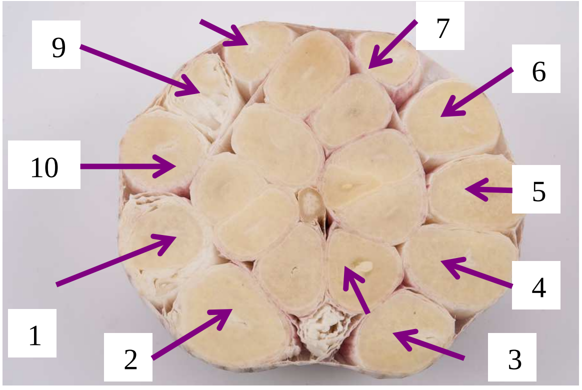

Fig. 9b. Garelic, cross-section, number of seeds 10=2·**5**
Photo: G.P. Skorny, J. Śledziowski

Garlic is easy to grow and can be grown year-round in mild climates.

A decagonal symmetry *(i.e. decaradial)*, with rotation angle 36° about central axis occurs in the front view of a pumpkin (Fig. 10a). There is double of Fibonacci number 13 in the certain cross-sections of the pumpkin, i.e 26 = 2·**13**.

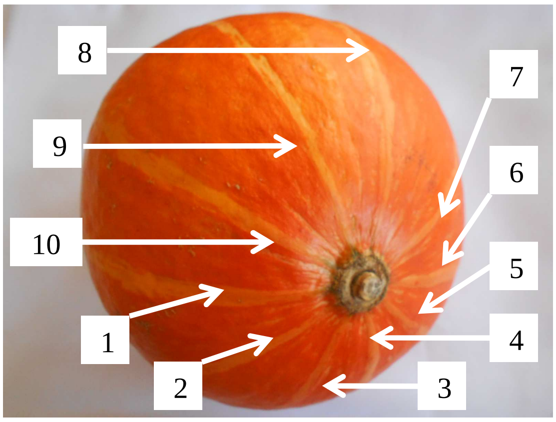

Fig. 10a. Pumpkin, front view, number of seeds 10=2·**5**
Photo: G.P. Skorny, J. Śledziowski

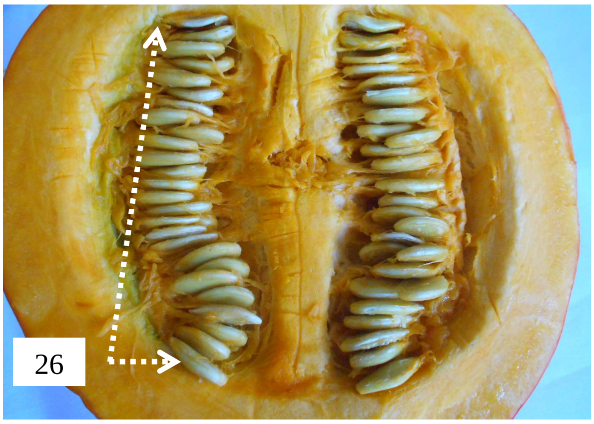

Fig. 10b. Pumpkin, cross-section, number of seeds 26=2·**13**
Photo: G.P. Skorny, J. Śledziowski

4. Symmetry in leafage of vegetables

4.1. Tetragonal symmetry

A tetragonal *(i.e. tetraradial)* symmetry in the leafage pattern is shown in a Chinese cabbage (Fig. 11a). Here we see Fibonacci number 3 which can be read on illustration with three leaves (Fig. 11b).

Chinese cabbage is now commonly found in markets throughout the world, catering both to the Chinese diaspora and to northern markets who appreciate its resistance to cold.

Fig. 11a. Chinese cabbage, general view
Photo: G.P. Skorny, J. Śledziowski

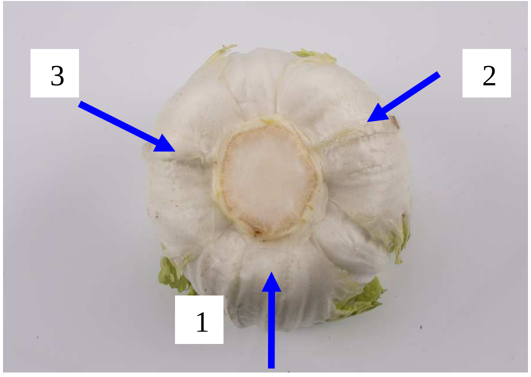

Fig. 11b. Chinese cabbage, number of leaves 5
Photo: G.P. Skorny, J. Śledziowski

4.2. Pentagonal symmetry

A pentagonal symmetry *(i.e. pentaradial)* we can find in the ordinary cabbage (Fig. 12a). The variant of pentagonal symmetry with rotation angle of 72° about the central axis show some leaves pattern of ordinary cabbage (Fig. 12b). Here we can find Fibonacci number 5.

Also, pentagonal symmetry, with rotation angle 72° about the central axis, there is not only the number of tomatoes momentum (Fig. 13a), but also in tomato foliage (Fig. 13b).

Fig. 12a. Ordinary cabbage, general view
Photo: G.P. Skorny, J. Śledziowski

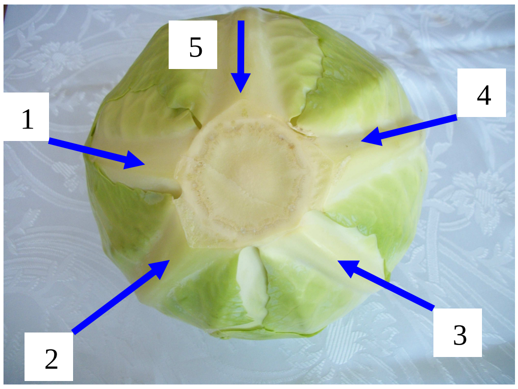

Fig. 12b. Ordinary cabbage, number of leaves 5
Photo: G.P. Skorny, J. Śledziowski

Cabbage is generally grown for its densely leaved heads, produced during the first year of its biennial cycle. Plants perform best when grown in well-drained soil in a location that receives full sun.

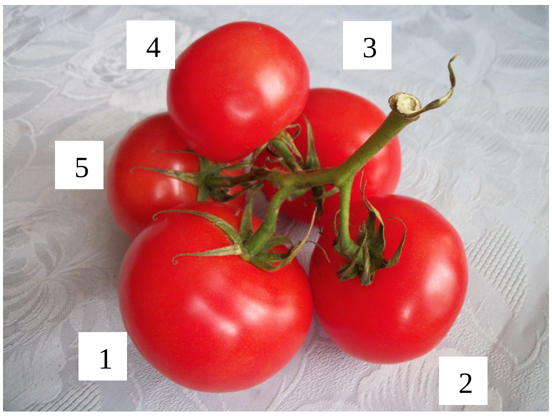

Fig. 13a. Tomatoes, general view, 5 tomatoes
Photo: G.P. Skorny, J. Śledziowski

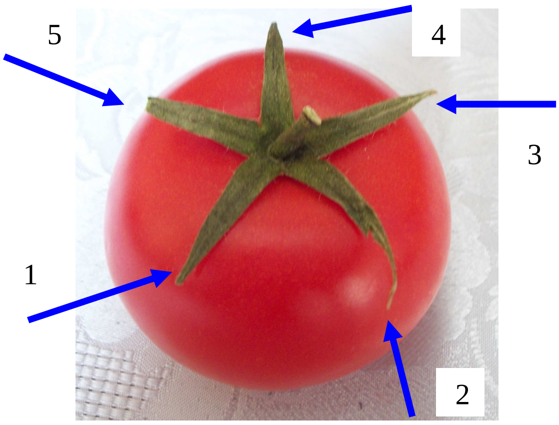

Fig. 13b. Tomato, number of leaves 5
Photo: G.P. Skorny, J. Śledziowski

The tomato is now grown worldwide for its edible fruits, with thousands of cultivars having been selected with varying fruit types, and for optimum growth in differing growing conditions. Moreover a pentagonal symmetry, with rotation angle 72° about the central axis appears in the foliage of cucumber long (Fig. 14a). We have Fibonacci number 5 (Fig. 14b).

Fig. 14a. Cucumber long, general view

Photo: G.P. Skorny, J. Śledziowski

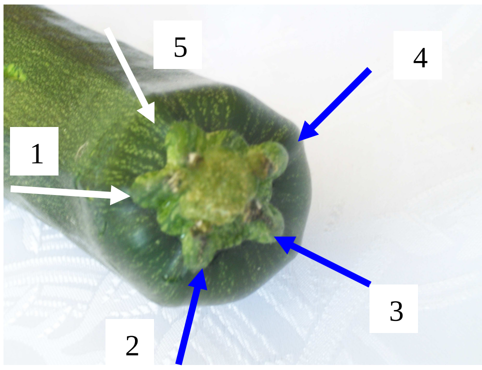

Fig. 14b. Cucumber long, number of leaves 5

Photo: G.P. Skorny, J. Śledziowski

Cucumber long green is a fleshy, easy to grow vegetable. Excellent for slicing. Yields sweet, crisp, emerald green fruit.

5. Conclusions

- Fibonacci numbers in botany are interpreted in the cross-sections of various vegetables. In some cross-sections of vegetables can be observed some dual, triangular, tetragonal, pentagonal, hexagonal and even decagonal symmetry.
- The interpretation of Fibonacci numbers may be used to supplement the classification of vegetables plants.

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