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Myroslava Ye. Chernova*, Yaroslav V. Kuntsyak*

STUDY OF THE STRESS STRAIN OF ROCK MASSIF IN THE CROOKED WELLBORE

The main task, which is facing Ukraine, is to increase hydrocarbon production on its own fields. One of the most effective methods of oil and gas production is the construction of a directional (DW) and horizontal wells (HW), and side-hole jamming. In recent years both domestic and foreign scientists are working on developing the methods of controlling the dynamics of BHA operation on the bottom well by applying the fluctuations generators of directed actions, with the purpose of increasing the mechanical drilling speed, improving the quality of treatment of the buttonhole and the withdrawal of the rocks extracted onto the day surface [1]. To determine the effects of energy hydro-acoustic oscillations on the properties of the rock and its strength characteristics it is necessary to carry out theoretical, experimental and industrial research.

The successful construction of DW and HW depends on the completeness of the information on the structure and properties of rocks that are being developed. One of the important data in this direction is the characteristics of the stress-strain state of rock and information on the disturbances, which arise in the process of the dynamic loads.

Rocks are able to destroy the strength in conditions that are close to the one-axial tension (compression), but while approaching the hydrostatic conditions this ability is lost, since in the course of drilling a well its walls are in a complex stress state. Therefore, the assessment of the sustainability of the rock is the most complete in volumetric strain, characteristic of which is the by-pass of maximum Mohr's circles (passport strength), built in the coordinates of the normal and maximum tangential stresses. Using the passport sustainability and Mohr's circles, you can define hour resistance (tensile strength) of rocks to one-axial stretching-compression, to torsion, to the cut with the compression, to the volumetric compression.

By-pass circles of stress have a different shape, in particular, for loose rock it is an inclined line, which starts in the beginning of the coordinates, and for the plastic rocks it is the line which is parallel to the axis of the normal stresses. For the rocks, which are characterized by fragile destruction and significant difference destructive stress in compression and tension, as a by-pas shape we take part of the circle with a line, a parabola, a cycloid, an exponential curve *etc*. [2, 3].

* Ivano-Frankivsk National Technical University of Oil and Gas, Ukraine

The rock, which is being drilled both in DW and in the HW, is not an isotropic medium, therefore the stress-strain state of rocks around the bottom-hole should be taken into account with a view of considering the fissure zone being formed, the task of whose calculation is different from the normal tasks by the fact that the limit of the dividing the range of the elastic deformations and fissuring zones is unknown beforehand. It has to be determined in the decision process. Thus, during the construction of the algorithm for calculating the ranges of elastic stresses and deformations it is expedient to use the methods that are suitable for the ranges of arbitrary shapes. This implies, in particular, the technique which is based on the application of the potential expression of the desired solutions [4, 5, 6]. This enables the building of a definition of the desired circuit elastic zone using the iteration scheme. Under this condition the optimal satisfaction of all the necessary conditions is gradually achieved. A sufficient number of experiments made enables us to detect the presence of the monotonous nature of the operators of the task in question, which provides the convergence of the iteration process over a wide range.

Taking into consideration that the extinction of the disturbances in the bottom hole in the direction perpendicular to its walls corresponds with the exponential law, the zone of stress can be considered as a two-dimensional circular area **Ω**, consisting of the areas of plastic and elastic deformations accordingly Ω_1 and Ω_2. We assume that in the area Ω_1 the Saint-Venant plasticity condition will be satisfied,

$$\left(\sigma_r - \sigma_\theta\right)^2 + 4\tau_{r\theta}^2 = \sigma_T^2 \tag{1}$$

where σ_r, σ_θ, $\tau_{r\theta}$ – are the relevant elements of the stress tensor.

To determine the stress-strain state in Ω_2 we use the system of differential equations:

$$\left.\begin{array}{l} \left(\lambda + \mu\right)\dfrac{\partial \tilde{\epsilon}}{\partial r} - \mu\left(\dfrac{U}{r^2} + \dfrac{2}{r^2}\dfrac{\partial V}{\partial \theta}\right) + \mu\Delta U + X = 0 \\ \left(1 + \mu\right)\dfrac{1}{r}\dfrac{\partial \tilde{\epsilon}}{\partial \theta} - \mu\left(\dfrac{V}{r^2} - \dfrac{2}{r^2}\dfrac{\partial U}{\partial \theta}\right) + \mu\Delta U + Y = 0 \end{array}\right\} \tag{2}$$

where:

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$$

$$\tilde{\epsilon} = \frac{\partial U}{\partial r} + \frac{U}{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}$$

$U = U(r, \theta)$ and $V = V(r, \theta)$ – are the radial and tangential displacement respectively,
$X = X(r, \theta)$ and $Y = Y(r, \theta)$ – are the components of the load volume vector,
λ and μ – Lyame elastic constants.

On the internal L_1 and external L_2 borders of the area under consideration Ω (being concentric circles radii $R_1 < R_2$) we assume the following:

$$\sigma_r \mid_{L_1} = \varphi_1(\theta); \quad \tau_{r\theta} \mid_{L_1} = \varphi_1(\theta) \tag{3}$$

$$U \mid_{L_2} = \psi_1(\theta); \quad V \mid_{L_2} = \psi_2(\theta) \tag{4}$$

Along the border L of the section of areas Ω_1 and Ω_2 the following conditions should be observed:

$$\sigma_r^{(1)} \mid_L = \sigma_r^{(2)} \mid_L ; \ \tau_{r\theta}^{(1)} \mid_L = \tau_{r\theta}^{(2)} \mid_L ; \quad \sigma_\theta^{(1)} \mid_L = \sigma_\theta^{(2)} \mid_L \tag{5}$$

where $\sigma_r^{(1)}$, $\sigma_\theta^{(1)}$, $\tau_{r\theta}^{(1)}$ and $\sigma_r^{(2)}$, $\sigma_\theta^{(2)}$, $\tau_{r\theta}^{(2)}$ – the voltage in the areas Ω_1 and Ω_2 respectively.

In the plastic zone using the voltage function $\Phi = \Phi(r, \theta)$, which is determined by the following formulae:

$$\sigma_r = \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2}; \quad \sigma_\theta = \frac{\partial^2 \Phi}{\partial r \partial \theta}; \ \tau_{r\theta} = \frac{1}{r^2}\frac{\partial \Phi}{\partial \theta} - \frac{1}{r}\frac{\partial^2 \Phi}{\partial r \partial \theta},$$

satisfying the equilibrium conditions:

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} = 0; \qquad \frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} + \frac{2\tau_{r\theta}}{r} = 0.$$

Then the plasticity condition (1) will have the form:

$$\left(\frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2} - \frac{\partial^2 \Phi}{\partial r^2}\right)^2 + 4\left(\frac{1}{r^2}\frac{\partial \Phi}{\partial \theta} - \frac{1}{r}\frac{\partial^2 \Phi}{\partial r \partial \theta}\right)^2 = \sigma_T^2 \tag{6}$$

Thus, taking into account the conditions (3) on the circuit L_1 should be as follows:

$$\left.\begin{aligned} &\left(\frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2}\right)\Big|_{r = R_1} = \varphi_1(\theta); \\ &\left(\frac{1}{r^2}\frac{\partial \Phi}{\partial \theta} - \frac{1}{r}\frac{\partial^2 \Phi}{\partial r \partial \theta}\right)\Big|_{r = R_1} = \varphi_2(\theta). \end{aligned}\right\} \tag{7}$$

Thus, the function $\Phi(r, \theta)$, which is determined by the Cauchy problem (7) for equation (6) can be found regardless of the other terms and conditions.

The following conditions (4) in the external circuit L_2 are regular boundary conditions. Along the border L it is necessary to perform a large number of conditions, than in the uniquely solvable boundary problems. This fact is used to determine the form of the border L which is being determined in the course of solution L.

In accordance with the principles on the use of the potential of expressions [4, 5], the solution of the problem:

$$\left.\begin{aligned} &\left(\lambda\epsilon + 2\mu\frac{\partial U}{\partial r}\right)\Big|_L = \sigma_r^{(1)}\Big|_L \\ &\mu\left(\frac{1}{r}\frac{\partial U}{\partial\theta} + \frac{\partial V}{\partial r} - \frac{V}{r}\right)\Big|_L = \tau_{r\theta}^{(1)}\Big|_L \end{aligned}\right\} \tag{12}$$

$$U\,|_{L_2} = 0; \quad V\,|_{L_2} = 0 \tag{13}$$

for the system of equations (2) it is appropriate to be sought in the form of contour capacity:

$$\begin{pmatrix} U(r,\theta) \\ V(r,\theta) \end{pmatrix} = \int_{\tilde{L}} G(r,\theta,\rho,\varphi)\mu(\rho,\varphi)d_{\rho,\varphi}\tilde{L} \tag{14}$$

Taking into account the experience of work [6] as the contour $\tilde{L}$ as the contour $\tilde{R} > R_1$ of with the idea that it will completely belong to the plastic zone.

Taking into account that $r = r(\theta)$ – contour equation L, and substituting the expression (14) in the condition (12), we obtain (15):

$$\int_{\tilde{L}} G\left[r(\theta),\theta,\rho,\varphi\right]\mu(\rho,\varphi)d_{\rho,\varphi}\tilde{L} = q(\theta) \tag{15}$$

where:

$$q(\theta) = \begin{pmatrix} \sigma_\theta^{(1)}\,|_L \\ \tau_{r\theta}^{(1)}\,|_L \end{pmatrix}, \qquad \mu(\rho,\varphi) = \begin{pmatrix} \mu_1(\rho,\varphi) \\ \mu_2(\rho,\varphi) \end{pmatrix}.$$

To solve the equations (2), (4), (5) the following condition should be satisfied (at the expense of the proper choice of a loop L):

$$\left[\lambda\epsilon + 2\mu\left(\frac{U}{r} + \frac{1}{r}\frac{\partial V}{\partial\theta}\right)\right]\Big|_L = \sigma_\theta^{(1)}(r)\,|_L \tag{16}$$

For this purpose, it is convenient to use the iteration scheme.

Such a mathematical model gives the opportunity to get a picture of the distribution of stresses in the bottom-hole zone when drilling in depending on the direction of the layers of the rocks. The solution of the system of equations for this task was carried out on an advanced computer with the help of the program MathCAD Prime 2.0.

The results of the theoretical investigations of the stress-strain state of a rock for the Donetsk-Dnieper basin (DRS) in the bottom-hole zone of the wells with the specified values of the characteristics of the breed are shown in Figure 1 (solid lines).

According to the figures, we can conclude that within a certain angle the circular stress reaches the maximum value, and the radial voltages, and at the same values of the angle, do not show any sort of abrupt changes. Consequently, at certain values of the angle of the meeting of the rock cutting tool and the layers of the rocks reaches the maximum values of the circular stresses that can cause the destruction of the rock. The convergence of iterative method corresponds quite well with the experimental results which were carried out on the test stands with the use of stabilometrical device. The results of the experimental investigations of the stress-strain state of the rock are shown in Figure 1 (dotted lines).

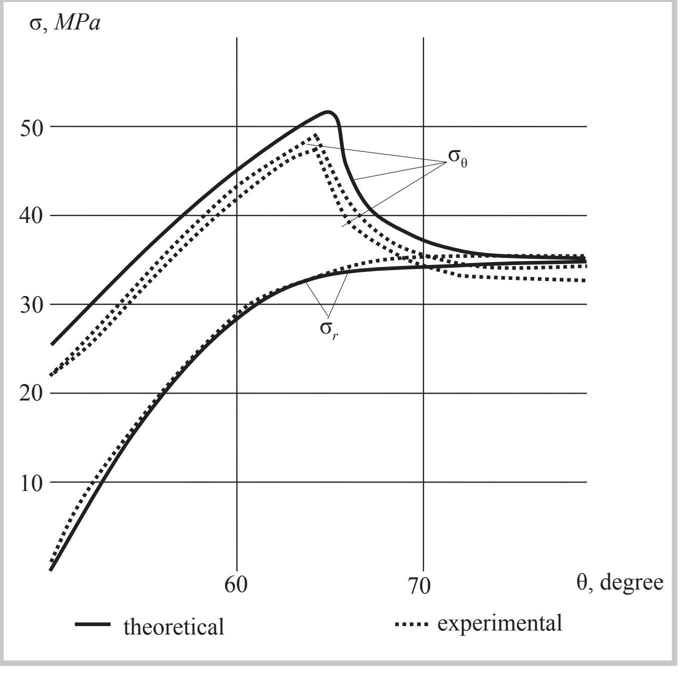

Fig. 1. The distribution of the circular and radial stresses in the wellbore depending on the angle between the direction of the layers of the rocks and the direction of the action of the load from the rock cutting tool

Used for the experimental studies were the cylindrical samples, carved out of the core, at various angles of rocks layers. Three-axial stress states under the influence of volumetric compression was studied by the method of S.I. Alekseyev [7] in stabilometrical device, but the transformer oil was used as a working fluid. Strain-gage sensor glued according to the known method, gave the opportunity to receive at the same time the value of radial and circular stresses that arise in the samples under the pressure, created in the system. During the experiment every three samples with the same direction of the rock layers were studied. The first of them was subjected to the load of up to the moment of destruction, which took place either in the form of "chipped", or in the form of the "barrel". The following two samples were run under the loads less than the maximum of 10 and 15%, respectively, as the important thing was not the process of destruction but the distribution of stresses, which appeared in the rock under the load. The slope of the graph characterizes the average value of the voltage, resulting in the samples according to the load cell, which in each sample was fixed, on the well-known scheme of 12 pieces.

The obtained picture of the distribution of stresses depending on the angle between the direction of the load and the direction of rock layers, made it possible to estimate the value of the critical angle, at which the tensions that arise in the rock under the influence of the load, and for the real conditions of the contact angle of the rock cutting tool with the rock, are the highest. This, in its turn, gives the possibility to foresee at what contact angles of the rock cutting tool with the rock, with the given direction of the rock layer, rock scree or rock fall in the unsustainable formations may become possible. In the light of the above mentioned, it is possible, taking into account the critical area when drilling the DW and HW, it is possible to formulate a scientific and reasonable approach to the well profile design during drilling in the unstable horizons of the mountain massif in the fields of the Dnieper-Donetsk Depression and on the Black sea shelf and to use the directional sonar power generators with adjustable modes parameters, thereby to prevent emergency situations, which arise in the unstable horizons.

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