Jozef Gocál*, Peter Krušinský**, Eva Capková***

Static analysis of historical trusses

Analiza statyczna historycznych więźb dachowych

Słowa kluczowe: historyczne więźby dachowe, analiza geometryczna, analiza statyczna

Key words: Historical trusses, Geometric analysis, Static analysis

1. INTRODUCTION

Geometric and static analysis of the medieval historical trusses are divided into two groups according to the typological classification. The first group represents using of rafter collar-beam construction with longitudinal stiffening frame truss (Bela-Dulice, Okolicne). The second one represents application of simple rafter collar-beam construction without longitudinal stiffening frame truss (Turciansky Peter, Abramova). The analysis examines the relation between the origin geometric concept in designing the truss and its static concept regarding the current criteria. The pairs of truss constructions have been chosen based on their age, span and typological similarity.

2. ANALYSIS OF HISTORICAL TRUSSES IN THE CHURCH OF THE HOLIEST CHRIST'S BODY IN BELA-DULICE AND THE CHURCH OF ST. PETER OF ALCANTARA IN THE FRANCISCAN MONASTERY IN OKOLIČNÉ

2.1. Description of the trusses

The main goal is to compare two geometrically and typologically similar historical trusses regarding the original geometric concepts and static solutions. The first church chosen for the geometric and static analysis is the Church of the Holiest Christ's Body in Bela-Dulice, built in 1409, that is typologically clean with a minimal manipulation during its existence. The second chosen church for analysis is the Church of St. Peter of Alcantara in the Franciscan monastery in Okoličné that was finished in 1500. The second Church is typologically clean with a minimal manipulation with elements during its existence as well.

The sharp roof above the nave of The Roman Catholic Church of the Holiest Christ's Body has a typical rafter collar-beam construction with longitudinal stiffening truss, dated to 1409d. It contains four main trusses (every third truss is the main one) and six secondary trusses. In the main roof truss, the collar-beams cross the central king posts, which are lapped together with the rafters in the vertex. Symmetrical braces stabilize the king post and connect it to the tie beam. The rafters are mortised to the ends of the tie beam. Tall angle braces are used for the transversal bracing. All the joints are secured by wooden dowels. The central longitudinal truss consists of a sill beam lying on the tie beams, four king posts, strutted in the 3/5 of their height by horizontal braces and stabilised in the bottom end by symmetrical braces, and diagonal bracings placed in the upper part of the king posts.

The Roman Catholic Church of St. Peter of Alcantara is an oriented sacral building with a lateral situated high reaching square tower. The Church consists of three main structures – nave hall, sanctuary and chapel. The Church was completed in the ninth decade of the 15th century with monumental roofs and primary historical trusses. The truss above the sanctuary is dated to the years 1499/00d. The truss is ending with triangular hipped, it has a rafter construction with two levels of collar-beams and is linked with

* Ing., PhD., University of Žilina, Slovakia
** Ing. arch., PhD., University of Žilina, Slovakia
*** RNDr., PhD., University of Žilina, Slovakia

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a central truss. The main trusses are stabilized by two kinds of high symmetrical braces: the first is between the tie beams and rafters; the second one is between the tie beams and king posts. The braces at king posts intersect tall spicate braces, which create asymmetrical saltires system.

2.2. Geometrical analysis of the trusses

The essential element in the geometric analysis of main roof truss above the nave is the square ABCD with a length equal to the half of the width of the truss above the nave (Fig. 1). We sign the length of the side of the basic square ABCD as *a*. The height of the truss (point V) was obtained by the circumference ^{1}k with the centre in the point C and passing through the point of intersection of the diagonals of the square ABCD, with the radius equal to the half of the length of the diagonal of the basic square ABCD $a \cdot \sqrt{2}/2$. The relationship between the width and the height of the truss above the nave can be numerically expressed as a ratio $2 : (1 + \sqrt{2}/2)$. The slope of the roof, as well as the rafter AV, is thus defined.

The collar-beam is located at a height that is equal to *a*. The king post (line BV) is divided by the collar-beam in point C in the ratio $\sqrt{2} : 1$. The location of the collar-beam on rafters (points E, F) can be obtained by the construction of the circumference ^{2}k (the centre of the circumference is point C and the radius is equal to $(\sqrt{2} - 1) \cdot$ a) and that cuts the king post in point G. Point G together with the circumference ^{2}k will be the starting point for the analysis of the longitudinal frame truss.

The endings of raking braces on rafters (points H, I) are located in the height equals to 1/2 *a*. In the height equal to 1/4 *a* on the king post is the ending of the raking braces (point J). The points E', F' on the collar-beam have the same distance from the point B as points E, F from the point C, i.e. they are perpendicular projections of the points E, F on the collar-beam. The circumference ^{3}k with the centre E' and the radius equal to 1/4 *a* intersects the collar-beam in the endings of raking braces (points M, N, symmetrically the circumference with the centre in F' intersects the collar-beam in the points O, P).

The geometric analysis of the main truss above the sanctuary of the Church in Okoličné (Fig. 2) has many common elements with geometric analysis of the main truss above the nave of the Church in Bela-Dulice. Again there is a square ABCD as an essential element with the length of its side equal to the half of the width of the truss above the sanctuary and we sign it *a* as well. By the same construction as in previous truss we obtained the height of the truss (with using the circumference ^{1}k). Thus the ratio between the width and the height of the truss we can again express as $2:(1 + \sqrt{2}/2)$ and both trusses have the same slope.

The location of the main collar-beam is different, here is the collar-beam located in the height equals to 3/4 *a*. The higher collar-beam was obtained by the circumference ^{2}k (with centre in point C and radius $(\sqrt{2} - 1) \cdot$ a), that intersects the line CV (the centre line of king post) in the point F. Braces between the rafter and tie beam have theirs endings on the rafter (point I) in the height equals to 1/2 *a* (same as in the previous truss), and the endings of braces on the tie beam is in 1/2 *a* (point J). Braces between the tie beam and the king post have their endings on the tie beam in 1/3 *a* (point L) and their endings on the king post (point K) was constructed by the circumference ^{3}k with the centre in point B and radius $a \cdot \sqrt{5}/2$.

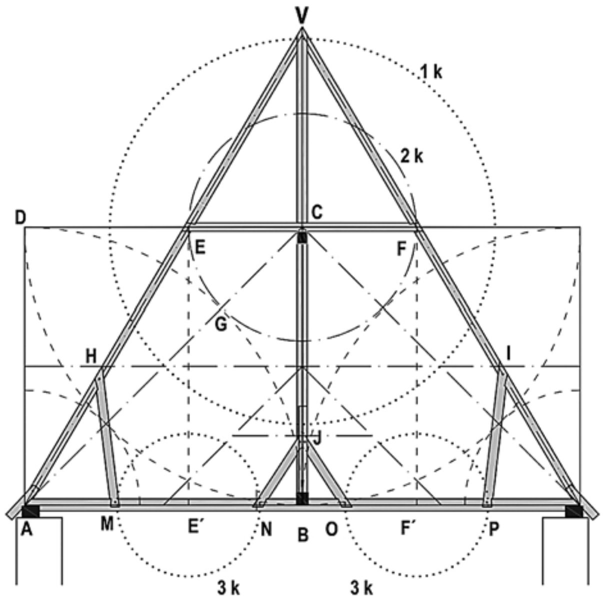

Fig. 1. Geometric analysis of the main truss in Bela-Dulice

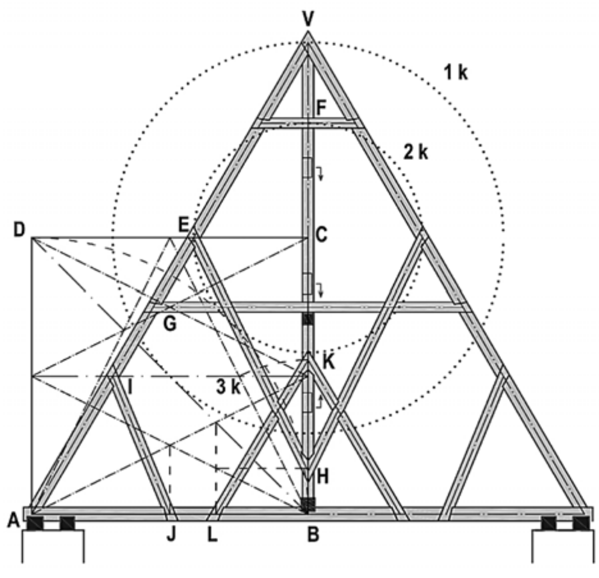

Fig. 2. Geometric analysis of the main truss in Okoličné

The element, absenting in the previous truss, is the spice brace. Its ending on the rafter is defined by the circumference ^{2}k (point E) and its ending on the king post (point H) is located on the height 1/6 *a*.

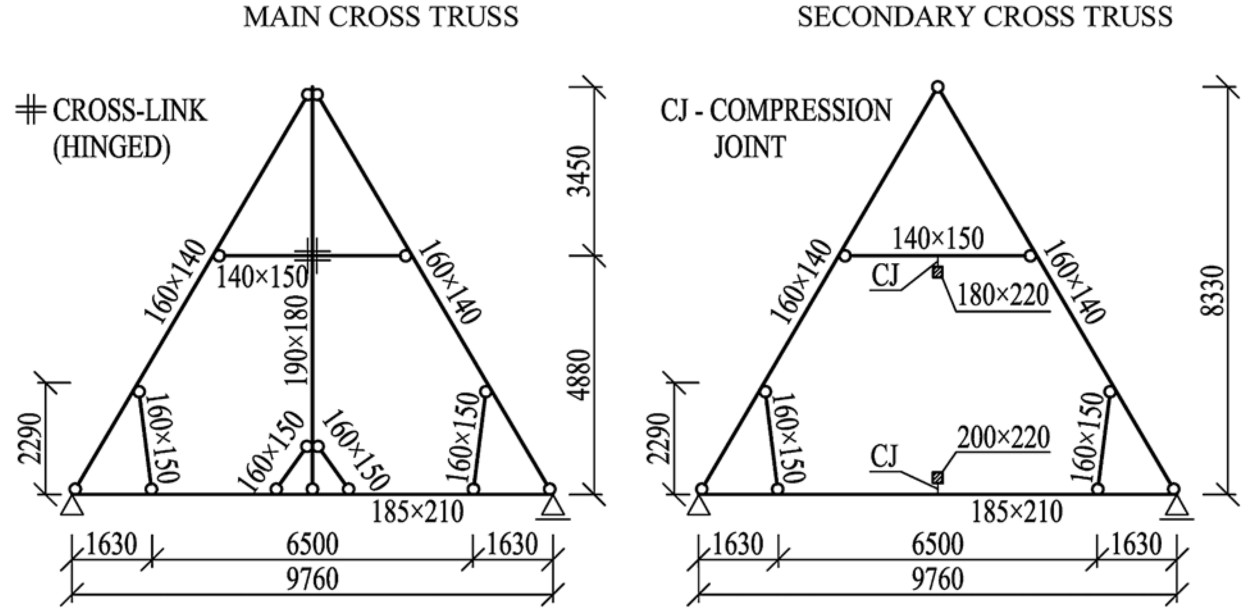

Fig. 3. Numerical model of cross trusses of the roof structure in Bela-Dulice

2.3. Numerical analysis of the trusses

Numerical models of the roof structures were developed in the finite element analysis (FEA) software SCIA Engineer [4]. The roof structures are modelled as three dimensional structures with beam elements. The geometry of numerical models are related to the roof structure's geometrical analyses presented hereinbefore. The basic geometric parameters of the numerical models are shown in Fig. 3 and Fig. 4.

The cross sections of members are designated in the form b×h [mm], where "b" is the width and "h" is the height of the cross section in millimetres. Mechanical properties of the wood are taken into account applying the strength class of C24 according to [5]. All the member connections are modelled as hinge joints with axial rigid connection and with capability of initial slip of 1 mm in the axial direction of member in order to consider theoretical influence of gaps, cracks and geometry imperfections, occurring in historical carpentry joints. The collar beams and tie beams of the secondary trusses are connected to the upper and bottom chords of the longitudinal trusses by joints, which are able to transfer only compression forces (designated as CJ – compression joints – in Fig. 3 and Fig. 4) [12].

The roof superstructures were loaded according to European standards [6, 7, 8] by permanent load (self-weight and weight of roofing) and variable load (wind actions). With regard to the roof pitch angle (about 60°), the snow load was not applied on the roof. The combinations of load cases were generated according to the standard STN EN 1990 [6].

The results of numerical analysis of the both roof superstructures are presented by the values of maximum tensile (+) and compression (–) stresses

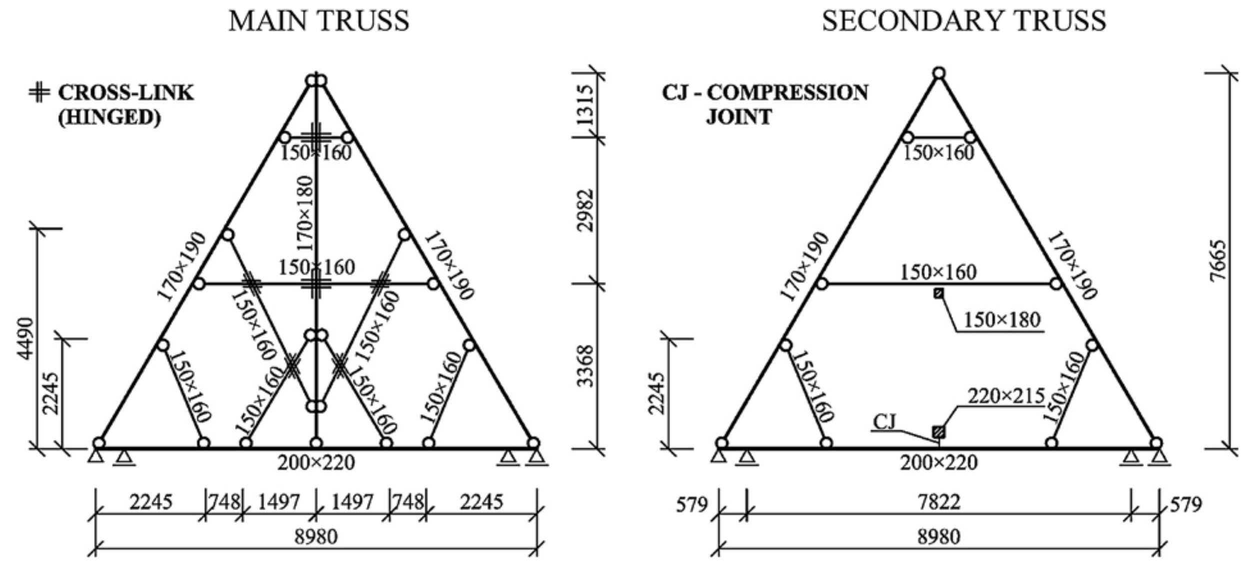

Fig. 4. Numerical model cross trusses of the roof structure in Okolicne

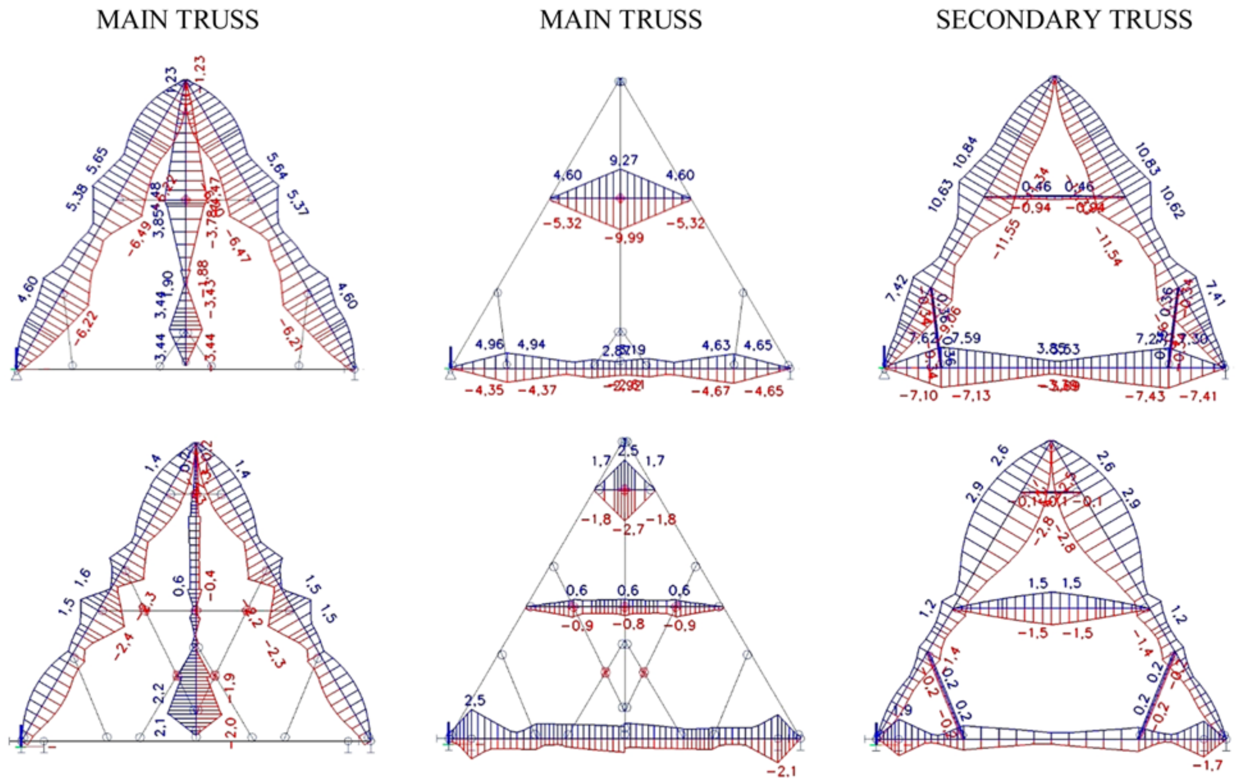

Fig. 5. Normal stresses (MPa) in the major truss members in a) Bela-Dulice, b) Okolicne

and displacements in the major structural members of trusses, calculated for the decisive load combinations. The envelopes of maximum and minimum normal stresses in the major truss members are presented in Fig. 5. The presented values do not include possible buckling effects, which are negligible because of low share of compression forces (caused mainly by permanent loads). The weakening of cross sections by carpentry joints is neglected as well. The envelopes of deformations are presented in Fig. 6 [12].

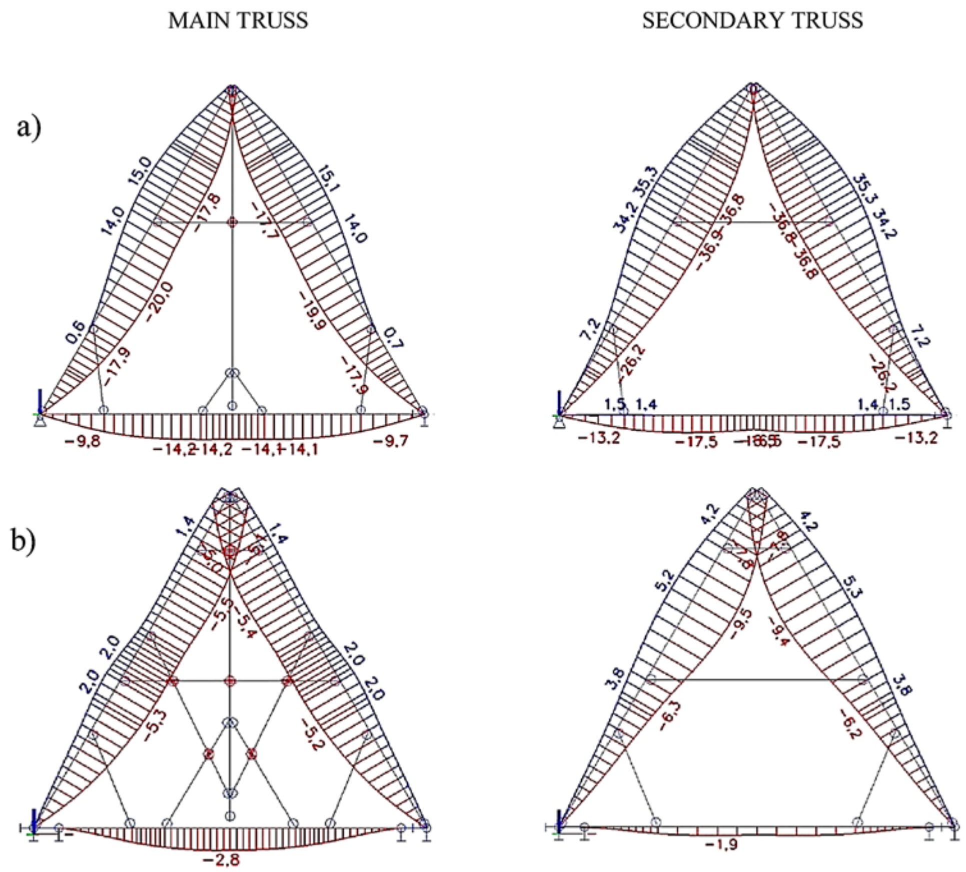

Fig. 6. Vertical deformations (mm) in the major truss members in a) Bela-Dulice, b) Okolicne

Based on the results of numerical analysis it can be stated out that in both investigated cases the normal stresses in all truss members do not exceed the design values of bending and compression strength equal to 16.60 MPa and 14.53 MPa, respectively, determined according to STN EN 1995–1-1 [9], assuming the timber strength class C24, modification factor k_{mod} = 0.9 and partial safety factor γ_M = 1.3. The maximum normal stresses in the secondary truss members are generally much higher than in the main truss members. There can be observed very significant differences between normal stresses in the major truss members (rafters, collar beams, tie beams, king posts) of the two investigated roof structures. The normal stresses in the truss members in Bela-Dulice are approximately 3-times higher than in the case of truss in Okolicne. These differences are partially caused by a little longer truss span as well as by larger and more appropriate oriented cross sections of the major truss members. Another reason is connected with the different geometrical concept applied in the original design. The bearing structure of the roof in monastery church in Okolicne is obviously more over dimensioned, when the construction of main cross truss is supplemented by the spike struts and in all cross trusses there are applied additional collar beams at the roof ridge. The different geometric concept of the proposal applied in Okolicne has even more reflected in the deformations of the major truss members, which are approximately 4–7-times lower than in the case of truss in Bela-Dulice.

3. ANALYSIS OF HISTORICAL TRUSSES IN THE CHURCH OF ST. KOZMA AND DAMIAN IN THE ABRAMOVA AND THE CHURCH OF ST. PETER IN TURCIANSKY PETER

3.1. Description of the trusses

The Roman-Catholic Church of the Holy Kozma and Damian in Abramova is located in the periphery of the village of Abramova, in the middle of the cemetery. The Church was built in 1375 and is the construction with one nave with the rectangular floor plan. The sanctuary is on the north side with a separate room for the sacristy and the tower. The truss above the nave has a rafter collar-beam construction without longitudinal stiffening frame truss. It consists of ten constructional same trusses, which are made up of a pair of rafters. In the top the rafters they are joined by top pin. The collar-beam and the rafters are joined together by a dovetail joints. All of the original carpentry joints are secured with wooden dowels; secondary elements are fixed with the original ones by metal-tipped nails.

The Roman Catholic Church of St. Peter is an oriented sacral building, which was built on hillock with cemetery in central part of the village Turciansky Peter. Today´s church was built in 1368 (dendrochronology dated). The roof construction has two parts. The Eastern part, above the presbytery is dated to the years 1505/1506d with original design. The West part is dated to the year 1475d (significantly rebuilt). For this reason the roof construction above the sanctuary was analysed. The truss above the sanctuary has a traditional rafter collar-beam construction currently without longitudinal stiffening frame truss. All trusses have the same design scheme. The rafters are pinned on the ends of the tie beams and in the top the rafters they are joined together by top pin. The collar-beams are joined with rafters by dovetail joints. All the elements were originally fixed using wooden dowels.

3.2. Geometric analysis of the trusses

3.2.1. Geometric analysis of the truss of the Roman-Catholic Church of the St. Peter in Turciansky Peter

The floor plan of the whole truss is rectangular and the ratio between its width and length is 2:5. The truss above the sanctuary has, like the truss in Abramova, a simple collar-beam structure with raking braces.

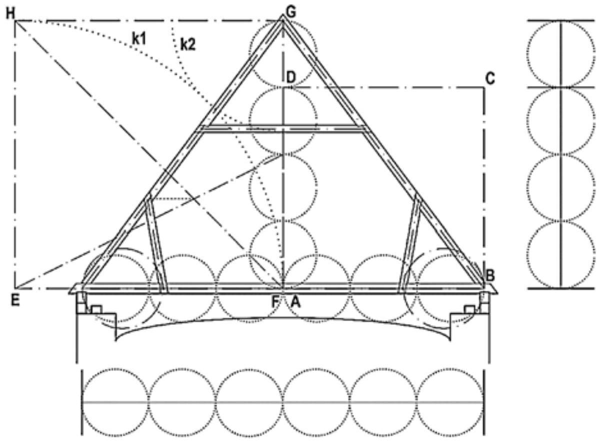

Fig. 7. Geometric analysis of the main truss in Turčiansky Peter

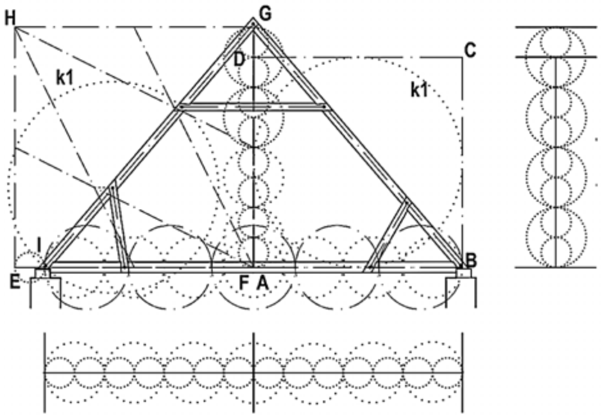

Fig. 8. Geometric analysis of the main truss in Abramova

The relationship between the width and height of the truss can be quantified by the ratio 6:4 = 3:2, but we assume that the height of the truss was obtained by adding the third to the half of the width, we sign this principle as *n plus one* (n is equal to 3 in this case). If we use a basic square ABCD whose side is equal to one half of the width and we sign its length as *a*, then the height of the truss above the sanctuary has a length ($a + a/3$). The collar-beam is located at a height, which divides the total height of the truss in the ratio of $\sqrt{2} : 1$ (in the picture, it is constructed using circles ^1k and ^2k). The ending of the raking brace on the rafter is located in the one third of the height of the truss and the ending of the raking brace on the tie beam is located in one fifth of the overall width of the truss (Fig. 7).

3.2.2. Geometric analysis of the truss of the Roman-Catholic Church of the Holy Kozma and Damian in Abramova

All geometric constructions are based on the basic square ABCD, whose side length is equal to the half of the truss above the nave. The floor plan of the truss above the nave is a square shaped and its dimensions are twice the square ABCD, i.e. the width of the truss above the nave has a length of 2*a*. The relationship between the width and height of the truss can we quantify by the ratio 14:8 = 7:4, but we assume that the height of the truss was obtained by adding the seventh to the half of the width, thus again was used the principle *n plus one*, here n is equal 7. The width of the roof above the nave has length of 2*a* and the height of the truss has length of ($a + a/7$). The collar-beam is located in two thirds of the height of the truss and the ending of the raking brace on the rafter is located in one third of the height of the truss. The ending of raking braces on the tie beam is located in the one fifth of the overall width of truss (Fig. 8).

3.3. Static analysis of trusses in Abramova and Turciansky Peter

Since these roof structures have much simpler construction consisting only of uniform cross-trusses, only planar numerical models were created for their analysis, again by means of the SCIA Engineer software [4]. As regards the material properties as well as modelling the joints of members, there have been applied the same conditions and principles like in the previous couple of trusses. The geometrical schemes of these models are shown in Fig. 9.

Both the roof superstructures were loaded as in the previous case. In addition, with regard to the roof's angles, they were loaded also by snow according to STN EN 1991–1-3 [10]. The results of numerical analysis of the both roof superstructures are again presented in Fig. 10 by the values of maximum tensile (+) and compression (–) stresses and displacements in the major structural members of trusses, calculated for the decisive load combinations, neglecting the buckling effects as well as the weakening of cross sections by carpentry joints. Based on the results of numerical analysis it can be stated out that in both investigated cases the normal stresses in all truss members are strongly lower than the design values of bending and compression strength (see part 2.3). Thus, the both roof structures are much over dimensioned even from the viewpoint of the current European standards.

4. CONCLUSIONS

In all of analysed trusses the essential concept in geometric design is the concept of the basic square that becomes from the proportion of the floor plan. The basic square was the base for defining the height in spite of the difference between the date of object's creation object and the date of realisation of the truss. We managed to identify two principles for defining height of the truss. In the case of structures with a central stiffening there was used the second square root of two for defining the height and in the case of simpler collar-beam structures there was used the principle *n plus one*, which were used at the turn of the 19th and 20th century. From previous research it is clear that steeper trusses were designed using the square root and in case of the trusses with the slope less than 50° there were used the principle *n plus one*.

To conclude, we can say that the compared Gothic trusses are geometrically and proportionately very consistently designed, particularly the roof in Okolicne. The roof above the nave of the church in Bela-Dulice is designed very economically, using from its basic square only its height and the square root with supporting the rafters at two points. This results in greater deformations and lower degree of reliability (but still sufficient) than

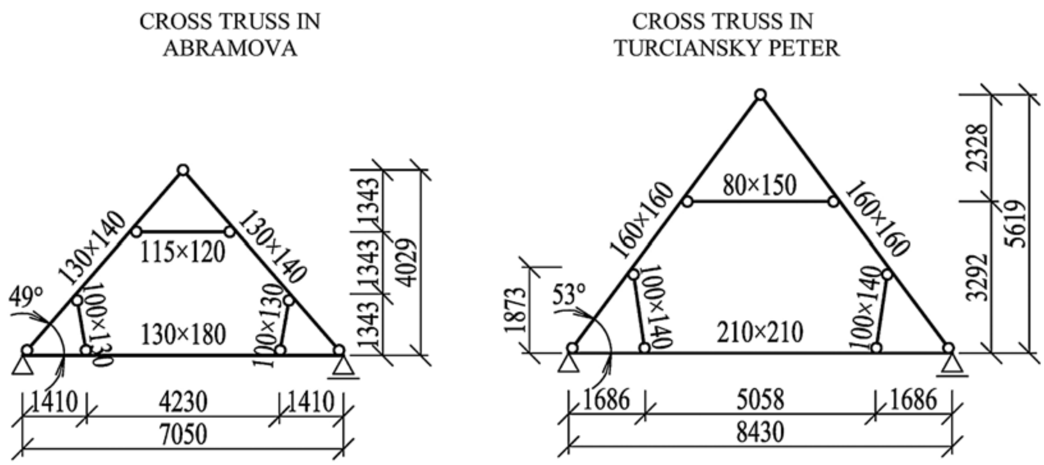

Fig. 9. Numerical model of the roof structures in Abramova and Turciansky Peter

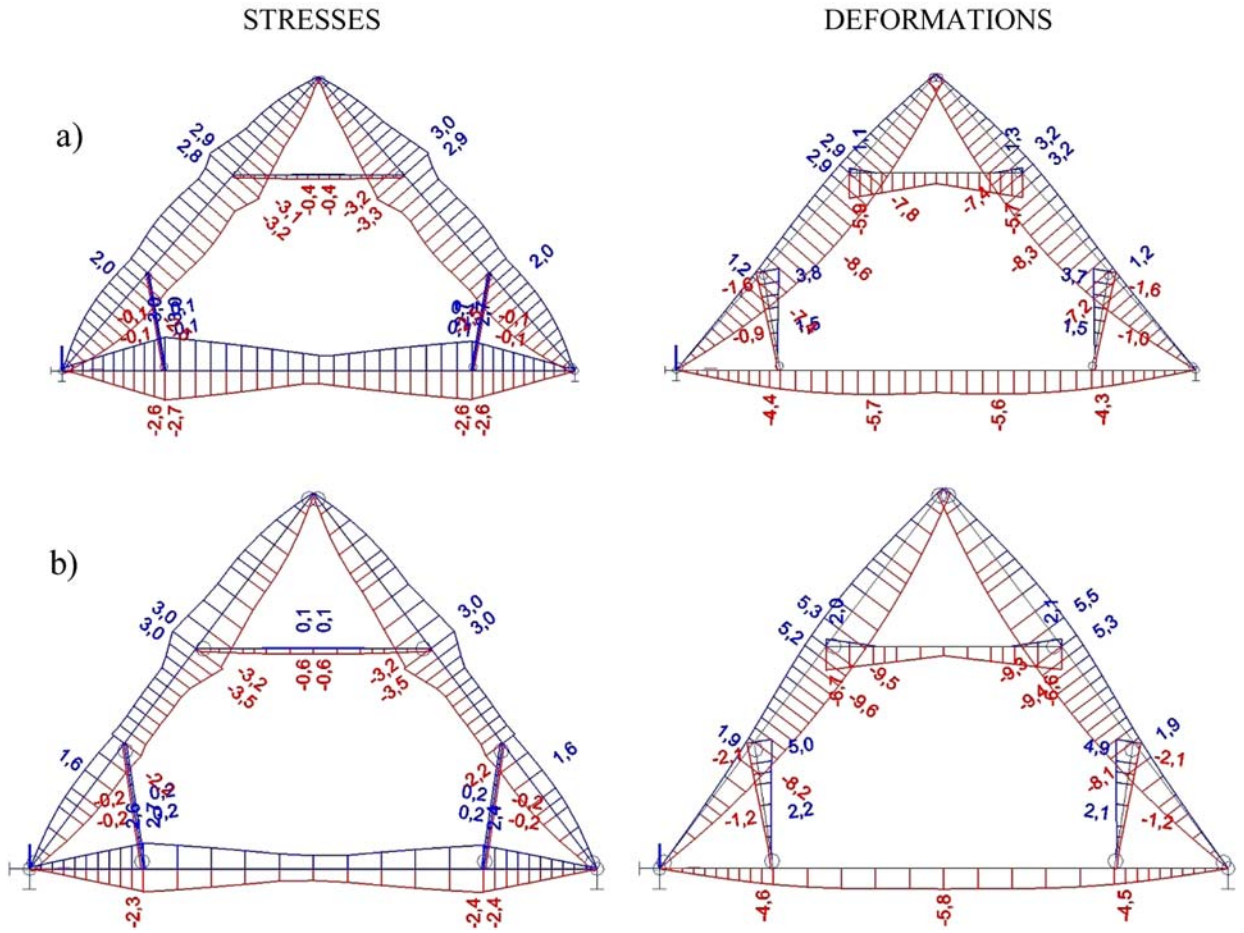

Fig. 10. Normal stresses (MPa) and vertical deformations (mm) in the cross truss members in a) Abramova and b) Turciansky Peter

in the case of the roof in Okolicne. The truss above the sanctuary of the church in Okolicne is geometrically very precise designed with an emphasis on the significantly greater reliability. Unlike the truss in Bela-Dulice it has a lot more elements, which results from the maximum use of proportional possibilities of the basic square. Consequently, the rafters are supported at four points and the distances of nodes in the tie beams are more optimal. Those facts have an impact on higher reliability of the roof structure. It can be assumed that with regard to greater importance of the church of St. Peter of Alcantara in Okolicne there were high demands on the reliability and durability of the roof structure above the sanctuary. Of course, it had to comply with adequate financial resources for realisation of the building.

In the case of the second couple of trusses a different geometrical concept led to design of very simple structures with relatively small height. The smaller slope of the rafters results in lower wind load, although at the cost of increased snow loads, which is still very small. Unlike the previous pair of trusses, in which the safety and durability of the roof superstructure is secondarily increased by using longitudinal trusses ensuring possible load redistribution in the case of local failure of main support members (e.g. rafters) due to degradation of material, in this case the required level of reliability is reached by using more massive cross sections of the main roof members.

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REFERENCES

[1] Struhár A. Geometric harmony of historical architecture in Slovakia. Pallas, Bratislava, 1977.

[2] Suchý L., Krušinský P., Babjaková Z., Ďurian K. Historic trusses of sacral buildings of Turiec. University of Zilina, 2008.

[3] The Complete Pythagoras. Internet version of Kenneth Sylvan Guthrie's Pythagorean Sourcebook and Library (1922), ed. Patrick Rousell. [Online] [Cited: 03 22, 2011.] http://www.completepythagoras.net
[4] SciaEngineer 2010. Software for structural analysis. http://nemetschek-scia.com/sk
[5] STN EN 338 Structural timber. Strength classes. SÚTN, Bratislava, 2004.
[6] STN EN 1990 Eurocode. Basis of structural design. SÚTN, Bratislava, 2009.
[7] STN EN 1991–1-1 Eurocode 1. Actions on structures. Part 1–1: General actions. Densities, self-weight, imposed loads for buildings. SÚTN, Bratislava, 2007.
[8] STN EN 1991–1-4 Eurocode 1. Actions on structures. Part 1–4: General actions – Wind actions. SÚTN, Bratislava, 2007.
[9] STN EN 1995–1-1 + A1 Eurocode 5. Design of timber structures. Part 1–1: General. Common rules and rules for buildings. SÚTN, Bratislava, 2008.
[10] STN EN 1991–1-3 Eurocode 1. Actions on structures. Part 1-: General actions – Snow loads. SÚTN, Bratislava, 2007.
[11] Jasieńko J., Nowak T.P., Bednarz Ł. The baroque structural ceiling over the Leopoldinum Auditorium in Wrocław University – tests, conservation and a strengthening concept. International Journal Architectural Heritage 2014;8(2):269–289.
[12] Jasieńko J., Nowak T., Karolak A. Historical carpentry joints. Wiadomosci Konserwatorskie – Journal of Heritage Conservation 2014;40:58–82.
[13] Branco J., Piazza M., Cruz P. Structural analysis of two King-post timber trusses: non-destructive evaluation and load-carrying tests. Construction and Building Materials 2010;24(3):371–383.

Abstract

The subject of article is a comparison of four load bearing structures of medieval trusses in terms of their original design, based on the geometrical concept, as well as from the perspective of their current static analysis, based on the modern standardized calculation procedures. The first two trusses are the roof above the sanctuary of the Roman Catholic Church in the village of Bela-Dulice from 1409d and the roof above the sanctuary of the Church of St. Peter of Alcantara in the Franciscan monastery in Okolicne from 1499d. These trusses have a similar rafter collar-beam construction with longitudinal stiffening frame truss and with approximately the same span. The other two trusses are the roof of the Roman Catholic Church in Turciansky Peter from 1505/06d and the roof of the Roman Catholic Church in Abramova from 1470/71d. These two trusses have only basic rafter collar-beam construction without longitudinal stiffening frame truss. From the geometric analysis of the observed trusses there are obvious different geometric proportional relationships that were applied in the original design. The aim of static analysis of the load bearing structures using current advanced computing resources is to clarify and compare the static behaviour of the four historical roof structures.

Streszczenie

Przedmiotem artykułu jest porównanie czterech przenoszących obciążenia konstrukcji średniowiecznych więźb dachowych, w oparciu o koncepcję geometryczną, jak również z perspektywy ich obecnej analizy statycznej przeprowadzonej na podstawie współczesnych standardowych procedur obliczeniowych. Pierwsze dwa ustroje dachowe to dach nad kościołem rzymskokatolickim we wsi Bela-Dulice pochodzący z 1409 roku oraz dach nad kościołem pod wezwaniem św. Piotra z Alcantary w klasztorze franciszkanów w miejcowości Okolicne, pochodzący z roku 1499. Obydwie więźby dachowe mają podobną konstrukcję jętkową z wzdłużną ramą usztywniającą i są podobnej rozpiętości. Pozostałe dwie więźby dachowe to dach kościoła rzymskokatolickiego w miejscowości Truciansky Peter, pochodzący z lat 1505–1506, oraz dach kościoła rzymskokatolickiego w Abramovej, pochodzący z lat 1470–71. Te dwa ustroje zbudowane są jako proste konstrukcje jętkowe, bez wzdłużnej ramy usztywniającej więźbę. Z analizy geometrycznej powyższych ustrojów wynikają różne geometryczne relacje proporcjonalne, które były użyte w oryginalnych projektach. Celem analizy statycznej obciążonych konstrukcji, przeprowadzonej z wykorzystaniem współcześnie dostępnych technik komputerowych, była identyfikacja i porównanie pracy statycznej tych czterech historycznych ustrojów dachowych.