Numerical modeling of dynamic heating processes

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The article deals with the numerical modeling and simulation of dynamic heating processes. The mathematical models, based on classical Fourier's law and Vernotte equation were described. Both models can be used for analysis of the electro – thermal processes. The numerical model of exemplary heating process was created and implemented in the Mathcad environment. The calculation results of two different models of heat transfer were compared. The modeling accuracy and the range of applicability of the described models were determined.

1. Introduction

The specific quantities of electrical heating devices enables, in many cases, for achieving of high volumetric power values within the external surfaces. This fact enables for realization of huge number of electro-thermal technologies characterized by high speed of heating, as volumetric as surface. From a range of such applications, the following issues can be listed:

- analysis and synthesis of phenomenon during shock wave propagation;
- nanotechnology;
- the surface processes of high - energetic laser impulses;
- the medical applications.

All applications above require usage of advanced methematical describtion of heat transfer phenomenon within heated elements. Unlike a typical description of heat transfer, based on the Fourier equation, such problems require the Cattaneo – Vernotte approach. In most calculating systems for modeling and simulation, the Cattaneo – Vernotte equations are not implemented. The fact can result a very large simulation errors, especially in modern electrothermal techniques. In the article the mathematical and numerical model for the simulation of dynamic heating processes was described. The model and the classical approach were used for multi-variant calculations. The comparison between calculation results in both cases allow evaluation of accuracy and determination the limits of utility of both heat transfer models.

2. Heat transfer in solids

The basic problems connected with the electrical heating enable application of classical differential equations of heat transfer. The Fouriers law constitute that heat flux density is proportional to the temperature gradient in every point of space and every time step (1) [3; 4]. The classical equation of transient heat transfer can be presented as (2) for space without heat sources [6]. The parabolic equation contains some simplifications resulting from establishment of infinite speed of heat waves propagation. Such models can be used in most techniques of electro – thermal energy conversions. But in general, the assumption of finite heat propagation speed is true [1; 5; 6]. Basing on this assumption, Cattaneo and Vernotte formulated a modified transient heat transfer equation. The equation for the case of 1D heat transfer can be written as (3).

$$q(x,\tau) = -\lambda \nabla t(x,\tau) \tag{1}$$

where: q- heat flux density, λ - thermal conductivity, t- temperature, τ - time.

$$\frac{\partial t}{\partial \tau} = a\nabla^2 t \tag{2}$$

where: a- thermal diffusivity defined as a ratio of thermal conductivity and a product of density and the specific heat $\left(a = \lambda / \left(c_p \cdot \rho\right)\right)$.

$$q(x, t+\tau_q) = -\lambda \frac{\partial t(x,\tau)}{\partial x} \tag{3}$$

Where: x- spatial coordinate, τ_q- the relaxation time resulting from the velocity of heat propagation.

$$\tau_q = a / C^2 \tag{4}$$

After the development of equation (3) into the Taylor's series for heat flux density, it was obtained:

$$q(x,t) + \tau_q \frac{\partial q(x,\tau)}{\partial \tau} = -\lambda \frac{\partial T(x,\tau)}{\partial x} \tag{5}$$

In general, the equation of heat transfer in micro scale in layers of heated body, requires the usage of the model taking into account delay of temperature gradients from heat fluxes [3; 4; 6]. In the cases where the normalization time can be skipped, the equation (5) can be written as classical Cattaneo – Vernotte equation (6) [6].

$$c_p\left[\frac{\partial t\,(x,\tau)}{\partial \tau} + \tau_q \frac{\partial^2 t\,(x,\tau)}{\partial \tau^2}\right] = \lambda \nabla^2 t(x,\tau) + p_V(x,\tau) + \tau_q \frac{\partial p_V(x,\tau)}{\partial \tau} \tag{6}$$

where: p_V - volumetric power density.

In regions without heat sources, the equation (7) is valid [6].

$$c_p\left[\frac{\partial T(x,\tau)}{\partial \tau}+\tau_q\frac{\partial^2 T(x,\tau)}{\partial \tau^2}\right]=\nabla\left[\lambda\,\nabla T(x,\tau)\right] \tag{7}$$

The equations above were used to modeling of heating processes to compare the simulation results with classical model of heat transfer phenomenon. During the calculations, the axially symmetric models were used. So that, in the axis of symmetry the zero normal heat flux was assumed. The equations were supplemented with required third type boundary conditions in external surfaces.

During the calculations of dynamic heating processes, the commercial calculating systems were not used. In most popular programs there is a less of models and calculating procedures enables to account the effects of delay of temperature gradients from heat fluxes [6]. Therefore, the authors algorithm was created during the calculations. The numerical algorithm was based on finite difference method and was implemented in the Mathcad system. Two meshes were used for geometrical coordinates (Δx) and the time steps $(\Delta\tau)$. The equation of heat transfer, after derivatives approximation with differential quotients can be written as (8).

$$\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)_{i,\tau}=\frac{T_{i+1,\tau}-T_{i,\tau}}{W_{i+1,\tau-1}}\psi_{i+1}+\frac{T_{i-1,\tau}-T_{i,\tau}}{W_{i-1,\tau-1}}\psi_{i-1} \tag{8}$$

where: for regular mesh: $\psi_{i+1}=\psi_{i-1}=1/\Delta x$. The denominator of equation above contains the thermal resistances.

3. Calculations

Calculation work was begin on the analysis of general induction heating cases, also typical for the most of the classic cases. The case of cylindrical geometry surface heated charge measuring 30 mm in outer diameter and 300 mm high have been tested. The realization of surface heating consisted in forcing the operating frequency of the current in the inductor at the level of 150 kHz. With this value the skin depth was about 2 mm (9). The entire power secreted in heated material was about 3 kW. Results of the heating process simulation with the usage of the basic (1) and hyperbolic (3) heat conducting equation were achieved for a 20 second heating time. Relative differences between results received with the usage of various heat propagation were determined to illustrate the differences in results. There is data obtained shown in the form of relative temperature differences vs. time curves in the figure 1.

$$\delta_g=\sqrt{\frac{2\cdot\rho}{\omega\cdot\mu}} \tag{9}$$

where: ρ - resistivity, ω - pulse, μ - magnetic permeability.

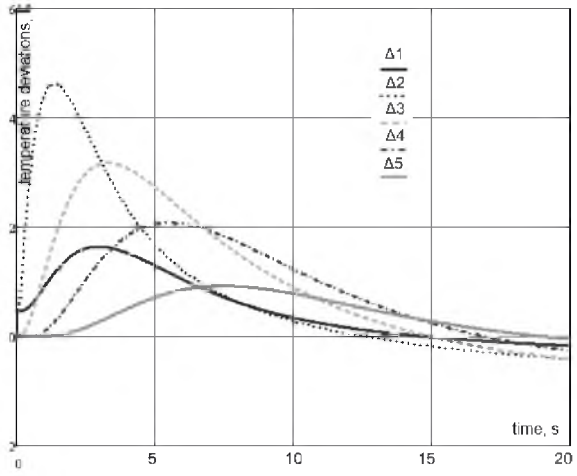

Fig. 1. Describtions of the relative temperature differences between the results achieved on the basis of two different numerical models. Δ1 – Δ5 – differences between results for corresponding temperatures

Results presented help to pronounce relatively small differences obtained with the usage of the heat propagation models described. Therefore, it is possible to find the lack of functionality of utility of hyperbolic heat transfer equation in classic applications, with relatively long heat time.

Subsequent calculations were designed to demonstrate the impact of heating power values on the dynamic characteristics. Moreover, the designation of deviations between results obtained on the basis of the analyzed equations of heatpropagation was performed. Calculations were made for the case of surface heating of the non-magnetic steel charge. The charge in the form of cylinder had a diameter of 3 and height of 30 mm. Depending on the heating power dissipated by the surface of charge, different heating characteristics and temperature values were obtained. The values of heating power were changed in the range of 100- 200000 W, which resulted in charge analyzed heating up to temperature limits from 200 to 2000°C within 0.01-2s.

In the Figure 2 characteristics of the time-temperature coordinates relative to the three cases of power amounting to 200, 2000 and 20000 Watts is presented. The characteristics were prepared as dependences of relative temperatures, defined as percentage deviations from maximal values , from the relative heating times. It was observed that with increasing power values, the heating times were reduced and the maximal temperatures have reached a higher values.

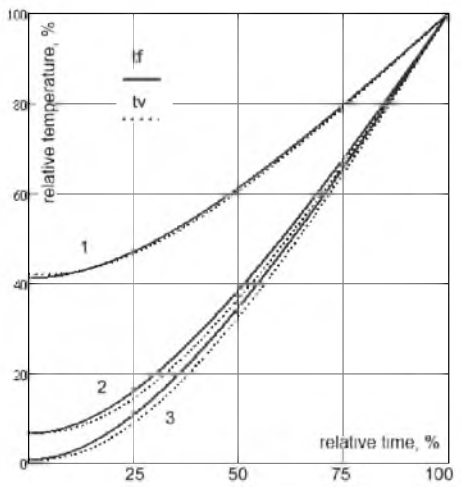

Fig. 2. Relative temperatures characteristics during the heating time for different power values. *tf*- temperatures from the Fourier equation, *tv*- temperatures from Vernotte equation. 1- the power value of 200 W; 2- the power value of 2000 W; 3- the power value of 20 000 W

Basing on the calculation results, it was shown that deviations characteristics, resulting from differences of algorithms used is quite similar for every power values. Maximal percentage differences between results were observed in the time of 25% of analysed range. The deviations, despite similar percentage values, are quite different in respect to the absolute values. With increasing heating power values, the maximal divergences between results reached high values. The characteristics were shown in Fig. 3.

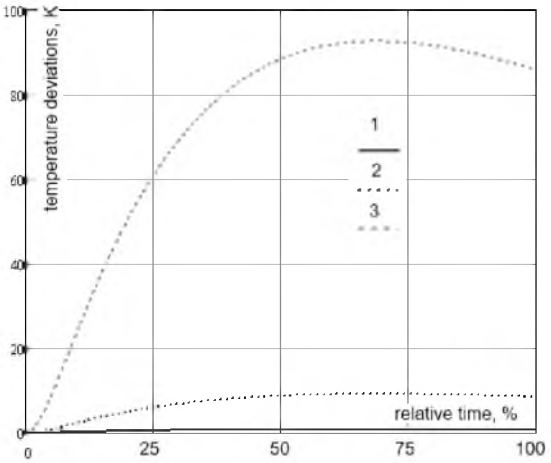

Fig. 3. Characteristics of absolute temperature divergences between calculating results from two models of heat transfer. 1- the power value of 200 W; 2- the power value of 2000 W; 3- the power value of 20 000 W

It was demonstrated that in cases of relative small heating power values and process times larger than 10 milliseconds, the differences between results from classical (1) and Vernotte (3) equations has reached the values of 10 K. Maximal divergences has been reached for the case of the largest analyzed heating power (the power density was $95x10^6$ W/m^3). In this case the calculating results divergences were on the level of 90 K. In heating systems characterized by small dimensions and short heating times, the utility of close to reality Vernotte equation is very necessary. The dynamic of processes requires a great precision by dosing a heating power.

Increasingly, the processes realized in induction heating systems are used in semiconductor industry and wide – known nanotechnology industry. As manufacturing of modern semiconducting materials and elements as production of nanostructures requires a high accuracy of operating characteristics of heating devices. In this example, the heat transfer phenomenon during heating of the most popular semiconducting materials was discussed. The basic subject of calculations was examination of temperature characteristics, obtained from classical Fourier (1) and Vernotte (3) heat transfer equation. The model of cylindrical workpiece was used. The power value was determined to guarantee warming of the workpiece to the temperature of 1900°C in the time of 400 ms. The heating characteristics of Si, Ge, GaAs and GaP were shown in figure 4. Additionally, in this figure, the results from Fourier's model (the same for all analyzed materials) were shown.

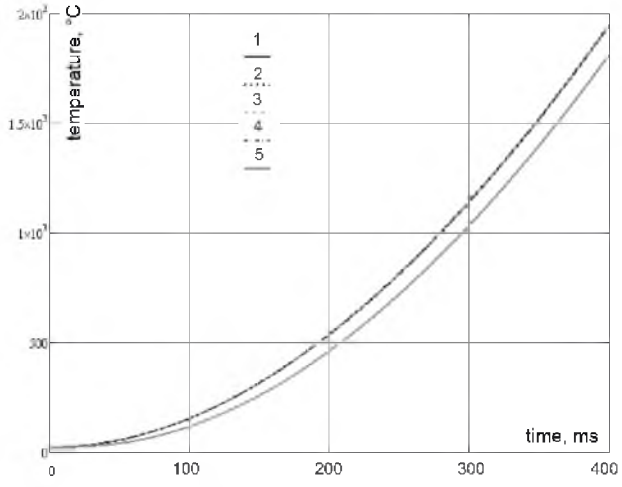

Fig. 4. Temperatures characteristics in the centre of the workpieces geometry for different semiconducting materials. 1- results from Fourier equation; 2- results for Si; 3- results for Ge; 4- results for GaAs; 5- results for GaP

Taking into consideration the relaxation time, only GaP was heated quite differently from classical model of heat transfer. The characteristics of deviations from classical approach for GaP were shown in Fig. 5.

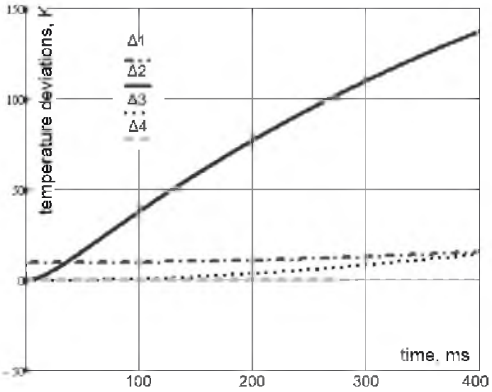

Fig. 5. Characteristics of temperature deviations of the hyperbolic heat transfer equation from the Fourier equation for GaP. Δ1 – Δ4 – deviations of subsequent layers deep into the workpiece

Maximal divergences between results for two models exceeds in this case the values of 130 K. This fact indicates to the inadequacy of classical model to the practical issues. However, for other semiconducting materials (Si, Ge, GaAs), the calculating errors resulting from rejection of relaxation times did not exceed the value of 5 K.

In many modern technologies, giant energetic impulses with short remaining times are being used. The general case of heating propagation in surface heated material was tested by interacting with a power impulse with remaining time 100 μs. Because of the occurrence of the impulse with nominal power 100 000 W, outer layers of the material were heated up to 900°C. Results of the simulation for the case of Fourier and Vernotte equation utility were shown in the Figure 6.

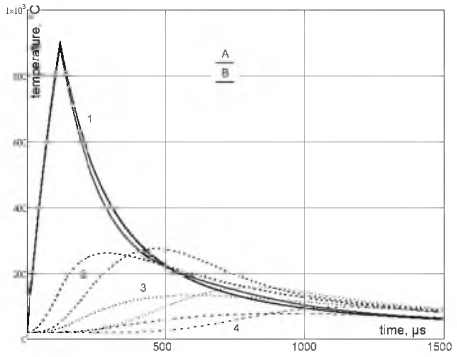

Fig. 6. Temperature curve in time function achieved as a result of simulation of the system reaction on a power impulse with remaining time 100 μs. A- results for Fourier equation; B- results for Vernotte equation in the first node. 1-5 – temperature curves in next nodes investigating the charge material

Both temperatures in successive layers of heated body and dynamic of heating and cooling process were quite different. Utility of classical heat transfer equation has led to obtain a higher temperature values with shorter latency times in relation to the Vernotte model. The temperature divergences between the models in time domain were shown in Figure 7.

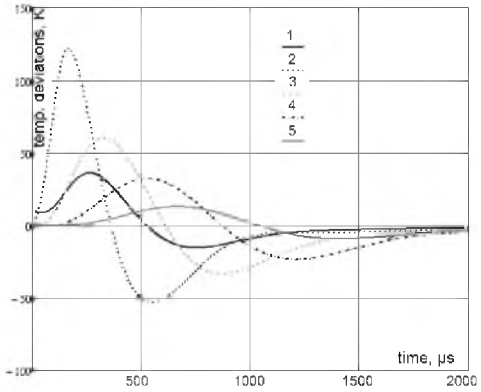

Fig. 7. Characteristics of temperature deviations between parabolic and hyperbolic heat equations as the functions of time. 1-5 - curves in next nodes investigating the charge material (as in the Fig. 6)

4. Conclusions

The subject of this paper consisted of the analysis of widely understood fast heat modeling accuracy with the usage of two different heat propagation models. Basic factors affecting the divergence between results achieved were tested. Apart from parameters and charge geometry, heat power value has a fundamental impact on the heating speed. Calculations, carried out while accomplishing the project, proved the necessity of utilizing the Vernotte equation in issues concerning short heating processes with heating time not exceeding 1 second. A numeric procedure allowing precise analysis of this kind of systems, used particularly for highly specialized heating devices calculations was drew out.

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