Vibrations in Physical Systems Vol.26 (2014)

Distributed Dynamic Vibration Absorber in Beam

Waldemar ŁATAS
Cracow University of Technology, Institute of Applied Mechanics
31-864 Cracow, Al. Jana Pawła II 37, Poland
latas@mech.pk.edu.pl

Abstract

The paper deals with forced vibration of Euler-Bernoulli beam with variable cross-section equipped with a distributed dynamic absorber. The beam is subjected to the concentrated and distributed harmonic excitations. The problem is solved using Galerkin's method and Lagrange's equations. Performing time-Laplace transformation the displacement amplitude of arbitrary point of the beam may be written in the frequency domain. The aim of the paper is to find the effectiveness of the distributed vibration absorbers in beams. As an example numerical results of vibration reduction in wind turbine's tower are presented.

Keywords: distributed dynamic vibration absorber, beam vibration, vibration reduction

1. Introduction

As a main application the dynamic vibration absorbers [DVA] (the most common are tuned mass dampers – TMD), correctly attached to vibrating structure and tuned to the frequency of harmonic excitation, can cause to cease the motion at the point of attachment [1-2].

Vibration analysis and the proper choice of the absorbers parameters in beam structures have been very often the subjects of study [3-10]. For continuous structures, such as beams, usually the best location of the vibration absorber is the point of excitation, but it may be difficult due to technical limitations. Depending upon the situation if the local optimization problem (for example minimization of the vibration amplitude at the given point) or global optimization problem (minimization of the kinetic energy of the whole structure) are to be considered, one may obtain different optimal parameters of the single absorber or the system of absorbers and the main issue in optimization is the proper placement of the absorbers.

In many cases there are used systems of tuned mass dampers [MTMD] which may be tuned for several resonant frequencies if broadband excitation is applied or for a single frequency [3,5] [11-14]. To suppress the structural waves in beams there may be used the absorbers distributed continuously along the length of the beam. A special application is the reduction of noise from railway track [15]. Compared with absorber applied at a single point, the distributed absorber is effective in case of arbitrary location of the exciting force and by appropriate tuning may work in a wide frequency band.

In this article a model based on Euler-Bernoulli theory is built for a beam with variable cross-section, subjected to the continuous and concentrated excitation, equipped with a dynamic vibration absorber with distributed parameters. Numerical example presented concerns the problem of passive vibration control in the real-world wind-turbine's tower-nacelle system.

2. Theoretical model

Figure 1 presents a system considered in the paper – a beam with variable cross-section subjected to the distributed and concentrated forces, with a distributed vibration absorber attached. The beam is of length l, the physical and geometrical parameters are functions of the position: mass density $\rho(x)$, cross-section area $A(x)$, area moment of inertia $I(x)$, Young's modulus $E(x)$, viscous damping coefficient $\alpha(x)$ (Voigt-Kelvin rheological model).

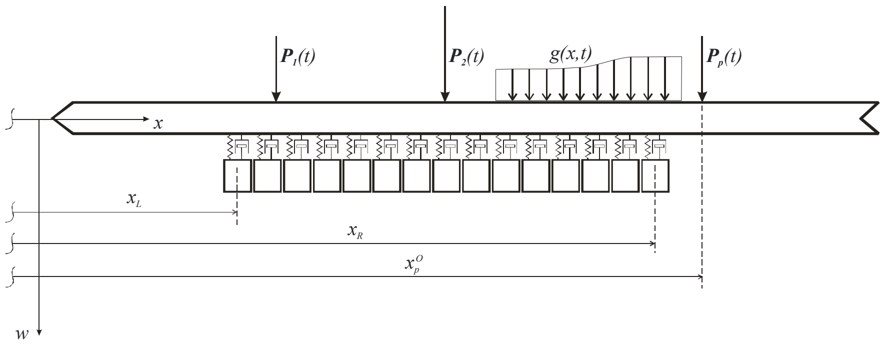

Figure 1. Beam with a distributed dynamic vibration absorber

Assuming Euler-Bernoulli model of the beam deformation and Voigt-Kelvin model of the beam material, the kinetic energy, the elastic potential energy and the dissipative function are given by:

$$T = \frac{1}{2}\int_0^l \rho(x)A(x)\left(\frac{\partial w}{\partial t}\right)^2 dx \tag{1}$$

$$V = \frac{1}{2}\int_0^l E(x)I(x)\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx \tag{2}$$

$$R = \frac{1}{2}\int_0^l E(x)I(x)\alpha(x)\left(\frac{\partial^3 w}{\partial t \partial x^2}\right)^2 dx \tag{3}$$

The transverse displacement is assumed to have the form of the series:

$$w(x,t) = \sum_{i=1}^{n} q_i(t)\varphi_i(x) \tag{4}$$

In the above expression $\varphi_i(x)$ are the basic functions, chosen in calculations as the modes of vibration of the beam with constant cross-section area, without absorbers attached. The functions $q_i(t)$ are time-dependent generalized co-ordinate that should be determined.

Substituting the series (4) into discretization (1)–(3) leads to:

$$T = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij}\dot{q}_i\dot{q}_j \tag{5}$$

$$V = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}q_i q_j \tag{6}$$

$$R = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\dot{q}_i\dot{q}_j \tag{7}$$

The terms m_{ij}, k_{ij}, b_{ij} are given by:

$$m_{ij} = \int_0^l \rho(x)A(x)\varphi_i(x)\varphi_j(x)dx \tag{8}$$

$$k_{ij} = \int_0^l E(x)I(x)\varphi_i''(x)\varphi_j''(x)dx \tag{9}$$

$$b_{ij} = \int_0^l E(x)I(x)\alpha(x)\varphi_i''(x)\varphi_j''(x)dx \tag{10}$$

For any given loading of the beam $H(x,t)$ the generalized force is obtained from formula:

$$H_i(t) = \int_0^l H(x,t)\varphi_i(x)dx \tag{11}$$

Using Lagrange's equations leads to a system of ordinary second order differential equations in the time domain with the unknown generalized co-ordinates $q_i(t)$:

$$\sum_{j=1}^{n} m_{ij}\ddot{q}_j + \sum_{j=1}^{n} b_{ij}\dot{q}_j + \sum_{j=1}^{n} k_{ij}q_j = H_i(t), \quad i = 1\ldots n \tag{12}$$

Performing time-Laplace transform (with initial conditions equal to zero) the system of differential equations (12) may be written in the form of the system of linear algebraic equations:

$$\sum_{j=1}^{n} m_{ij}s^2Q_j(s) + \sum_{j=1}^{n} b_{ij}sQ_j(s) + \sum_{j=1}^{n} k_{ij}Q_j(s) = H_i(s), \quad i = 1\ldots n \tag{13}$$

where $Q_i(s)$, $H_i(s)$ denote Laplace transforms of $q_i(t)$, $H_i(t)$.

Having calculated from the system (13) the transforms $Q_i(s)$ the transforms of the beam deflection may be obtained:

$$W(x,s)=\sum_{i=1}^{n}Q_i(s)\varphi_i(x) \tag{14}$$

The loading $H(x,t)$ depends on the distributed force $g(x,t)$ and p concentrated forces $P_j(t)$ applied to the beam at the points of coordinate x_j^0, additionally it depends on the distributed force $f(x,t)$ applied to the beam from the distributed vibration absorber:

$$H(x,t)=q(x,t)+\sum_{j=1}^{p}P_j(t)\delta(x-x_j^O)+f(x,t) \tag{15}$$

Generalized force $H_i(t)$ for the i-th generalized coordinate $q_i(t)$ is obtained from (11):

$$H_i(t)=\sum_{j=1}^{p}P_j(t)\varphi_i(x_j^O)+\int_0^L g(x,t)\varphi_i(x)dx+\int_{x_L}^{x_R} f(x,t)\varphi_i(x)dx \tag{16}$$

where x_L, x_R are the limits of the distributed dynamic absorber (Figure 1).

The Laplace transform of the i-th generalized force may be expressed as:

$$H_i(s)=\sum_{j=1}^{p}P_j(s)\varphi_i(x_j^O)+G_i(s)+F_i(s) \tag{17}$$

where it is introduced notations:

$$G_i(s)=\int_0^L g(x,s)\varphi_i(x)dx\,;\,F_i(s)=\int_{x_L}^{x_R} f(x,s)\varphi_i(x)dx \tag{18}$$

In the above expressions $g(x,s)$, $f(x,s)$ are Laplace transforms of the $g(x,t)$, $f(x,t)$.

The Laplace transform of the distributed force applied to the beam from the vibration absorber is given by [10]:

$$f(x,s)=-\frac{(\tilde{c}(x)s+\tilde{k}(x))\tilde{m}(x)s^2}{\tilde{m}(x)s^2+\tilde{c}(x)s+\tilde{k}(x)}W(x,s)=-\frac{(\tilde{c}(x)s+\tilde{k}(x))\tilde{m}(x)s^2}{\tilde{m}(x)s^2+\tilde{c}(x)s+\tilde{k}(x)}\sum_{j=1}^{n}Q_j(s)\varphi_j(x) \tag{19}$$

where: $\tilde{m}(x)$, $\tilde{c}(x)$, $\tilde{k}(x)$ – linear mass density, linear damping and stiffness coefficients densities of the distributed vibration absorber.

Insertion of (19) into (18) gives the system of linear equations (13) written in the form:

$$\sum_{j=1}^{n}\left(m_{ij}s^2+b_{ij}s+k_{ij}+\int_{x_L}^{x_R}\frac{\left(\tilde{c}(x)s+\tilde{k}(x)\right)\tilde{m}(x)s^2}{\left(\tilde{m}(x)s^2+\tilde{c}(x)s+\tilde{k}(x)\right)}\varphi_i(x)\varphi_j(x)dx\right)Q_j(s)=\sum_{j=1}^{p}P_j(s)\varphi_i(x_j^O)+G_i(s),\quad i=1\ldots n \tag{20}$$

Having solved the system (20) the transform of the beam deflection may be obtained from series (14). Assuming steady state vibration, after substituting $s = j\omega$ ($j = \sqrt{-1}$), it may be obtained the deflection of the beam in the frequency domain.

3. Numerical results: tuned distributed vibration absorber – wind-turbine's tower-nacelle system

It has been built a numerical algorithm which determines in *s*-domain the transform of the deflection of the beam for any set of functions describing its physical and geometrical characteristics: $A(x)$, $I(x)$, $E(x)$, $\alpha(x)$, $\rho(x)$, and for arbitrary boundary conditions at the ends of the beam. When harmonic excitation is considered the algorithm allows to obtain the amplitude-frequency characteristics of the beam deflection and allows for further calculations of the similar frequency characteristics of the slope, bending moment, transverse force and the time-averaged kinetic energy.

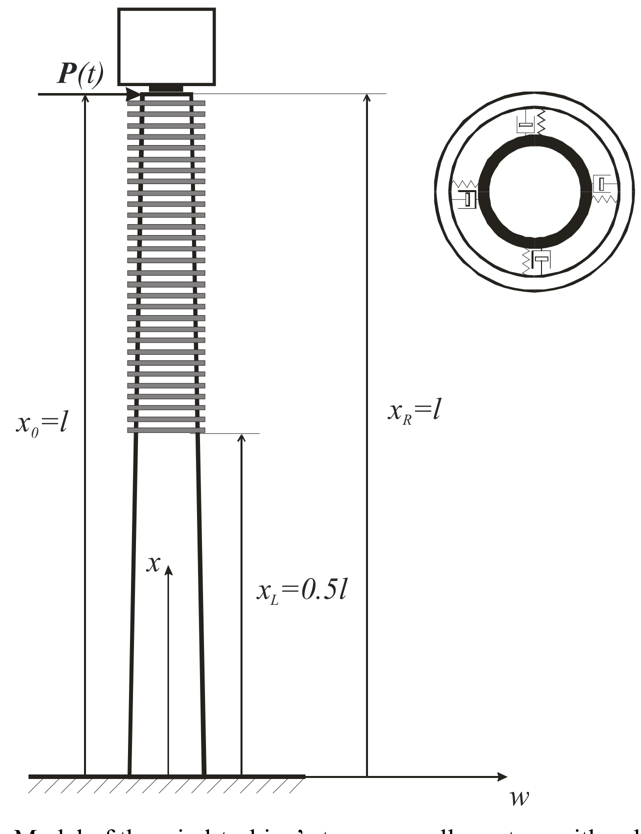

Figure 2. Model of the wind-turbine's tower-nacelle system with a distributed vibration absorber attached

Wind turbine towers are slender structures built usually as steel pipes with a diameter decreasing with altitude. Because of the simplicity of the tower geometry it is modeled as a vertically oriented beam, fixed to the ground at the bottom and with a solid mass, modeling a nacelle, attached to the upper end of the beam. Due to the low intrinsic damping, steel slender structures are prone to low frequency vibration (caused by wind flow, seismic motions) and for this reason are provided with damping devices, such as pitch actuators and vibration absorbers, tuned usually to the very first natural frequencies. The Figure 2 presents a model of the wind-turbine's tower-nacelle system with a distributed vibration absorber attached along half the length of the tower. The following parameters of the real-world full-scale Vensys 82 wind tower are taken in calculations [16–17]:

- length of the tower: 85.0 m;
- mass of the tower: 169000 kg;
- mass of the nacelle: 90000 kg;
- mass density $\rho = 7800$ kg/m^3;
- Young's modulus $E = 2.1\cdot 10^{11}$ N/m^2.

The functions approximating cross-section area $A(x)$ and area moment of inertia $I(x)$ are determined based on the actual dimensions of the tower cross-section, where maximal values are as follows: $A_{MAX} = 0.2949$ m^2, $I_{MAX} = 0.746$ m^4. The internal damping of the tower is neglected.

The basic functions in formula (4) are chosen as the modes of vibration of the beam with constant cross-section area and moment of inertia, equal the average values for the tower, with the bottom end fixed and with a solid mass, equal the mass of a nacelle, attached to the upper end.

The proposed distributed vibration absorber may be an alternative to the absorber applied at a single point near the nacelle, because it can be easier attachment of a number of smaller masses along the tower instead of one large mass at the top.

The total weight of the absorber is 4225 kg, 2.5 percent of the weight of the turbine's tower. Parameters of the distributed absorber are taken to be constant along the length of the beam: $\tilde{m}(x) = \text{const}$, $\tilde{c}(x) = \text{const}$, $\tilde{k}(x) = \text{const}$. The first three natural frequencies of the presented tower-nacelle system are: $f_1 = 0.352$ Hz, $f_2 = 2.721$ Hz, $f_3 = 8.132$ Hz. In numerical calculations presented it is assumed that the tower is excited by a concentrated harmonic force applied at the top (Figure 2).

As the first mode of vibration is the most important, as the easiest excited, it will be presented the results of tuning of the distributed absorber to the first natural frequency $f_1 = 0.352$ Hz. The calculated dimensionless displacement amplitude of the top of the beam, referenced to the static deflection, is shown in Figure 2 for a few sets of the distributed absorber physical parameters.

The graphs show the amplitude as a function of frequency for the case without the absorber attached, for the absorber with optimal values of stiffness $\tilde{k}(x)$ and damping $\tilde{c}(x)$ coefficients densities (calculated for a given absorber mass distribution along the length of the beam) and additionally for other values of parameters.

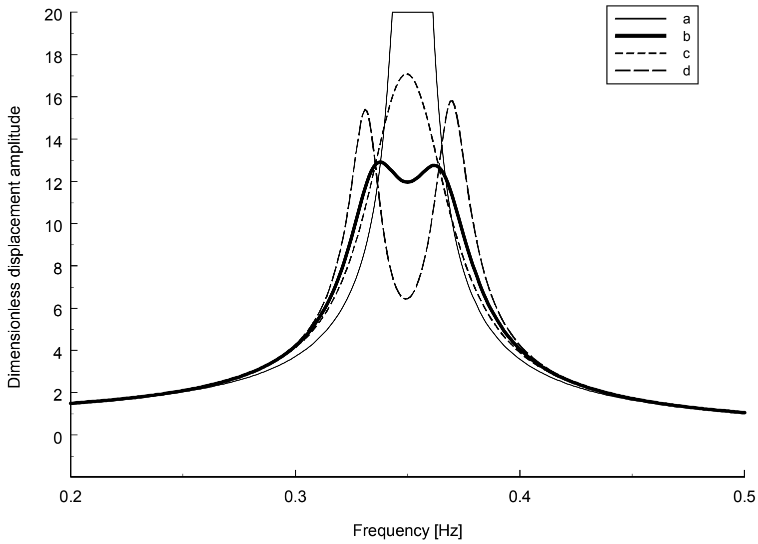

Figure 3. Dimensionless displacement amplitude of the top of the beam:
a) without absorber attached;
b) $\tilde{c}(x) \cdot l = 40370$ N/m , $\tilde{k}(x) \cdot l = 2800$ Ns/m ;
c) $\tilde{c}(x) \cdot l = 40370$ N/m , $\tilde{k}(x) \cdot l = 4000$ Ns/m ;
d) $\tilde{c}(x) \cdot l = 40370$ N/m , $\tilde{k}(x) \cdot l = 1500$ Ns/m

4. Conclusions

The computational model presented can be used in local and global problems of optimal choice of the distributed vibration absorber parameters in Euler-Bernoulli beam with variable cross-section. Theoretical calculations are illustrated by an example of the possible use of the distributed vibration absorber in wind turbine's tower vibration passive control. Distributed absorbers can be effective in those cases when it is not precisely defined a position of the concentrated force applied and in a case of the distributed load.

The model presented in the paper can be further used in investigation of the optimal location of the absorber band on the beam and various tuning methods, in particular studying of the tunable absorber and with variable parameters along its length.

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