

STABILITY ANALYSIS AND H_∞ CONTROL OF DISCRETE T–S FUZZY HYPERBOLIC SYSTEMS

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This paper focuses on the problem of constraint control for a class of discrete-time nonlinear systems. Firstly, a new discrete T–S fuzzy hyperbolic model is proposed to represent a class of discrete-time nonlinear systems. By means of the parallel distributed compensation (PDC) method, a novel asymptotic stabilizing control law with the “soft” constraint property is designed. The main advantage is that the proposed control method may achieve a small control amplitude. Secondly, for an uncertain discrete T–S fuzzy hyperbolic system with external disturbances, by the proposed control method, the robust stability and H_∞ performance are developed by using a Lyapunov function, and some sufficient conditions are established through seeking feasible solutions of some linear matrix inequalities (LMIs) to obtain several positive diagonally dominant (PDD) matrices. Finally, the validity and feasibility of the proposed schemes are demonstrated by a numerical example and a Van de Vusse one, and some comparisons of the discrete T–S fuzzy hyperbolic model with the discrete T–S fuzzy linear one are also given to illustrate the advantage of our approach.

Keywords: discrete T–S fuzzy hyperbolic model, parallel distributed compensation (PDC), positive diagonally dominant (PDD) matrices, robust stability.

1. Introduction

The Takagi–Sugeno (T–S) fuzzy model (Takagi and Sugeno 1985) has been a popular choice in modeling and designing a systematic control for nonlinear systems containing uncertain information which cannot be described accurately by mathematical tools. The T–S fuzzy linear model adopts a linear dynamic model as the consequent part of a fuzzy rule, which makes it possible to apply the classical and mature linear systems theory to nonlinear systems. Thus, it becomes one of the more successful methods for studying nonlinear systems.

There have been many research results for it, such as stability analysis, guaranteed-cost and observer-based control designs (Tanaka and Sugeno, 1992; Jadbabaie *et al.*, 1998; Tanaka and Wang, 2001; Fuan and Chen, 2004; Chen and Liu, 2005; Feng, 2006; Kim *et al.*, 2008; Li *et al.*, 2009; Yan *et al.*, 2010; Zhang *et al.*, 2012; Zhao *et al.*, 2013; Tong *et al.*, 2011; 2012; 2014; Siavash and

Alireza 2014). Especially, considering the uncertainties of the discrete T–S fuzzy linear model, numerous references have proposed different methods, such as the robust control strategy and the adaptive control approach (Cao and Frank, 2000; Cao *et al.*, 2000; Chen *et al.*, 2000; Tong *et al.*, 2009; 2010; Du, 2012; Qi *et al.*, 2012; Wang, 2014). A piecewise static-output-feedback controller and a piecewise Lyapunov function were designed to make the uncertain closed-loop fuzzy system stochastically stable with guaranteed performance (Qiu *et al.*, 2010). The works of Su *et al.* (2013; 2014), Qiu *et al.* (2009) and Li *et al.* (2011) discussed T–S fuzzy systems with time delay. Although there have been many successful applications for the discrete T–S fuzzy linear system, this model for approximation of nonlinear systems still has its structural limitations.

Considering the advantages of bilinear systems (Mohler, 1973; Elliott, 1999) and T–S fuzzy control, fuzzy control based on the T–S fuzzy bilinear model

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was firstly presented by Li and Tsai (2007), and stability conditions of the system were given via LMIs. Li and Tsai (2008) also presented robust fuzzy controllers for a class of discrete-time T-S fuzzy bilinear systems, in which the parallel distributed compensation method was utilized to design a fuzzy controller to ensure robust asymptotic stability of the closed-loop system and to guarantee an H_∞ norm-bound constraint on disturbance attenuation for all admissible uncertainties. Non-fragile guaranteed cost control was designed for the fuzzy bilinear system (Zhang and Li, 2010; Li and Zhang, 2012). Based on the piecewise quadratic Lyapunov function (PQLF), piecewise fuzzy observer-based controllers were designed for discrete T-S fuzzy bilinear systems with an unavailable state (Li et al., 2013).

From the above discussions, it can be seen that the existing literature has faced extensive discussions on the T-S fuzzy model. However, notice that for practical applications any controller for dynamic systems should be designed such that it guarantees systems stability requiring permissible magnitudes of control inputs (Park et al., 2004). In general, the approaches of constrained control include model predictive control (Bemporad et al., 2003), control with saturation nonlinearity (Zhao and Gao, 2012) and probabilistic control (Datta et al., 2012). Unfortunately, for most real-life problems, these methods often change the constraint control into very complex optimization problems. To tackle this issue, based on the fuzzy hyperbolic model (FHM) (Zhang and Quan, 2001; Zhang, 2009) and the T-S fuzzy one, Chen and Li (2012) established a new T-S model, namely, the T-S fuzzy hyperbolic model for complex continuous-time nonlinear systems. The consequent part of the proposed model is a hyperbolic dynamic model. The advantage of the model over its T-S fuzzy linear counterpart is that the control amplitude is much smaller than for the T-S fuzzy linear model.

Recently, the problems of non-fragile guaranteed cost constraint control for continuous-time T-S fuzzy hyperbolic models have been discussed further (Chen and Li, 2015). However, the control method has not been mentioned in discrete-time control systems. As we know, discrete-time systems have come to play a more important role than their continuous-time counterparts in the digital age, and discrete-time fuzzy-model-based control systems have drawn an increasing research interest. Motivated by the above concerns, we focus on constraint control of discrete-time nonlinear systems. Firstly, a novel discrete T-S fuzzy hyperbolic model for discrete-time nonlinear systems is proposed. Secondly, the PDC control is designed given the local control law $u_j(t) = H_j \tanh(Kx(t))$. By fuzzy blending, the overall fuzzy hyperbolic control law is obtained as $u(t) = \sum_{j=1}^r h_j(s(t)) H_j \tanh(Kx(t))$, where the range of each component $\tanh(k_j x_j(t))$, $j = 1, 2, \dots, r$, in vector

$\tanh(Kx(t))$ belongs to $(-1, 1)$. This design approach can deal with the constraint problem via a soft constraint approach. Finally, the robust H_∞ constraint control problem for an uncertain discrete T-S fuzzy hyperbolic system with external disturbance is further investigated.

Section 2 presents a discrete T-S fuzzy hyperbolic model and analyzes the stability of the closed-loop discrete fuzzy system by utilizing the PDC method to design a fuzzy controller. In Section 3, for the problem of discrete nonlinear system with external disturbance, a robust fuzzy controller is designed and a robust H_∞ stability condition is given in terms of LMIs. Section 4 illustrates the effectiveness of the proposed schemes via some simulations. Some conclusions are included in Section 5.

Notation. The notation used throughout this paper is fairly standard, $A > 0$ ($A \geq 0$, $A \leq 0$, $A \leq 0$, respectively) means that the matrix A is positive definite (positive semi-definite, negative definite, negative semi-definite, respectively). The identity matrix, which is of appropriate dimensions, will be denoted by I . The superscript “ T ” stands for the matrix transpose, R^n denotes the n -dimensional Euclidean space. The symbol “ $*$ ” in a square matrix stands for the transposed elements in the symmetric positions. The shorthand $\text{diag}\{k_1, k_2, \dots, k_n\}$ denotes a block diagonal matrix with diagonal blocks being the matrices k_1, k_2, \dots, k_n . Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Modeling and stability analysis of a discrete T-S fuzzy hyperbolic model

2.1. Modeling of a discrete T-S fuzzy hyperbolic model. The continuous T-S fuzzy hyperbolic model was firstly presented to represent continuous-time nonlinear systems (Chen and Li, 2012). In this subsection, a discrete T-S fuzzy hyperbolic model will be proposed to represent discrete-time nonlinear systems. This novel fuzzy model is still described by fuzzy “IF-THEN” rules, which express local dynamics in a hyperbolic tangent model. Finally, the overall fuzzy system is obtained by fuzzy, smooth “blending” of the local hyperbolic tangent model. The i -th rule of the discrete T-S fuzzy hyperbolic model is described below:

Plant rule i : If $s_1(t)$ is F_{i1} and \dots and $s_g(t)$ is F_{ig} , then

$$x(t+1) = A_i \tanh(Kx(t)) + B_i u(t), \quad i \in S = \{1, 2, \dots, r\}, \quad (1)$$

where r is the number of fuzzy rules and F_{ij} is the fuzzy set, $x(t) \in \mathbb{R}^n$ stands for the state vector, and $u(t) \in \mathbb{R}$ signifies the control input, $s(t) = [s_1(t), s_2(t), \dots, s_g(t)] \in \mathbb{R}^s$ are the known premise

variables. It is assumed that the premise variables do not depend on the control input $u(t)$ or disturbances $\omega(t)$ in this paper. $A_i \in \mathbb{R}^{n \times n}, B_i \in \mathbb{R}^n. \tanh(Kx(t)) = [\tanh(k_1x_1(t)), \dots, \tanh(k_nx_n(t))]$ with

$$\tanh(k_1x_1(t)) = \frac{e^{k_1x_1} - e^{-k_1x_1}}{e^{k_1x_1} + e^{-k_1x_1}}$$

and k_i is a specified constant.

By using the fuzzy inference method with a singleton fuzzifier, product inference and a center average defuzzifier, the overall discrete T-S fuzzy hyperbolic system can be rewritten as

$$x(t+1) = \sum_{i=1}^r h_i(s(t))(A_i \tanh(Kx(t)) + B_i u(t)), \quad (2)$$

where

$$h_i(s(t)) = \frac{\mu_i(s(t))}{\sum_{i=1}^r \mu_i(s(t))}$$

and

$$\mu_i(s(t)) = \prod_{j=1}^g F_{ij}(s_j(t)), \quad i \in S, \quad F_{ij}(s_j(t))$$

is a membership degree of $s_j(t)$ in F_{ij} . In this paper, $\mu_i(s(t))$ are assumed such that $\mu_i(s(t)) \geq 0, i \in S$, and $\sum_{i=1}^r \mu_i(s(t)) > 0$ for all t . From the definition of $h_i(s(t))$, we can see that $h_i(s(t)) \geq 0, i \in S$, and $\sum_{i=1}^r h_i(s(t)) = 1$. We write $h_i(s(t))$ as h_i for a brief description.

Before presenting the main results of this paper, we introduce some lemmas, which will be used in the sequel.

Lemma 1. (Margaliot and Langholz, 2003) *If a square matrix P is positive diagonally dominant (PDD), then for all $x \neq 0$ the following result holds:*

$$\tanh^T(x(t))P \tanh(x(t)) \leq x^T(t)Px(t).$$

Lemma 2. (Zhang and Li, 2010) *Given any matrices M, N , and a symmetric matrix $P > 0$ with appropriate dimensions, for any real scalar $\varepsilon > 0$, the following inequality holds:*

$$M^T P N + N^T P M \leq \varepsilon M^T P M + \varepsilon^{-1} N^T P N.$$

2.2. Fuzzy controller design and stability analysis. Based on the parallel distributed compensation (PDC) method (Tanaka and Wang, 2001), the j -th fuzzy controller of the discrete T-S fuzzy hyperbolic system (2)

is designed as follows:

Control rule j : If $s_1(t)$ is F_{j1} and ... and $s_g(t)$ is F_{jg} , then

$$u_j(t) = -H_j \tanh(Kx(t)), \quad j \in S = \{1, 2, \dots, r\}, \quad (3)$$

where $H_j \in \mathbb{R}^{1 \times n}$ is the controller gain matrix to be determined, $K = \text{diag}(k_1, k_2, \dots, k_n), k_i$ is a positive constant, which has been obtained by system identification.

By using the fuzzy inference method, the overall fuzzy control law is represented by

$$u(t) = - \sum_{j=1}^r h_j(s(t))H_j \tanh(Kx(t)). \quad (4)$$

Remark 1. In (4), each component in vector $\tanh(Kx(t))$ is bounded whose range of is $(-1, 1)$, so it is obvious that the fuzzy hyperbolic controller (4) is also bounded. It can be seen intuitively that when the variation range of the x value is very big, the controller (4) has a constraint control property of compressibility, and can achieve a small control amplitude when H_j is a limited value. This advantage will be illustrated by simulation results.

Substituting (4) into (2), the overall closed-loop system can be rewritten as

$$x(t+1) = \sum_{i=1}^r \sum_{j=1}^r h_i h_j (A_i - B_i H_j) \tanh(Kx(t)). \quad (5)$$

2.2.1. Main results.

Theorem 1. *Assume that there exist matrixes $P > 0, Z = Z^T$ and some constant matrixes M_i, M_j , such that the following LMIs are satisfied:*

$$\begin{bmatrix} -Y & * \\ A_i Y - B_i M_i & -K^{-1} Y K^{-T} \end{bmatrix} < 0, \quad 1 \leq i \leq r, \quad (6)$$

$$\begin{bmatrix} -Y & * \\ \frac{A_i Y + A_j Y - B_i M_j - B_j M_i}{2} & -K^{-1} Y K^{-T} \end{bmatrix} < 0, \quad 1 \leq i < j \leq r \quad (7)$$

$$z_{ij} \geq 0, \quad \forall i \neq j, \quad (8)$$

$$y_{ij} + z_{ij} \geq 0, \quad \forall i \neq j, \quad (9)$$

$$y_{ii} - \sum_{i \neq j} (y_{ij} + 2z_{ij}) \geq 0, \quad \forall i. \quad (10)$$

where $Y = P^{-1}, H_i = M_i Y^{-1}, H_j = M_j Y^{-1}, i, j = 1, 2, \dots, r$. Then the closed-loop system (5) is globally asymptotically stable,

Proof. Choose the following Lyapunov function candidate for the system (5):

$$V(t) = x^T(t)K^T PKx(t),$$

where K is defined in (5), $P > 0$.

Along the trajectories of the system (5), the corresponding time difference of $V(t)$ is given by

$$\begin{aligned} \Delta V &= V(t+1) - V(t) \\ &= x^T(t+1)K^T PKx(t+1) - x^T(t)K^T PKx(t) \\ &= \left\{ \sum_{i=1}^r \sum_{j=1}^r h_i h_j (A_i - B_i H_j) \tanh(Kx) \right\}^T K^T PK \\ &\quad \times \left\{ \sum_{n=1}^r \sum_{l=1}^r h_n h_l (A_n - B_n H_l) \tanh(Kx) \right\} \\ &\quad - x^T K^T PKx \\ &\leq \sum_{i=1}^r \sum_{j=1}^r \sum_{n=1}^r \sum_{l=1}^r h_i h_j h_n h_l \tanh^T(Kx) (A_i - B_i H_j)^T \\ &\quad \times K^T PK (A_n - B_n H_l) \tanh(Kx) - \tanh^T(Kx) \\ &\quad \times P \tanh(Kx) \\ &= \frac{1}{4} \sum_{i=1}^r \sum_{j=1}^r \sum_{n=1}^r \sum_{l=1}^r h_i h_j h_n h_l \tanh^T(Kx) (G_{ij} + G_{ji})^T \\ &\quad \times K^T PK (G_{nl} + G_{ln}) \tanh(Kx) \\ &\quad - \tanh^T(Kx) \\ &\quad \times P \tanh(Kx) \\ &\leq \sum_{i=1}^r \sum_{j=1}^r h_i h_j \left\{ \tanh^T(Kx) \left(\frac{G_{ij} + G_{ji}}{2} \right)^T K^T PK \right. \\ &\quad \times \left. \left(\frac{G_{ij} + G_{ji}}{2} \right) \tanh(Kx) \right. \\ &\quad \left. - \tanh^T(Kx) P \tanh(Kx) \right\} \\ &= \sum_{i=1}^r h_i^2 \tanh^T(Kx) (G_{ii}^T K^T PK G_{ii}) \tanh(Kx) \\ &\quad + 2 \sum_{i=1}^r \sum_{i < j}^r h_i h_j \tanh^T(Kx) \left(\frac{G_{ij} + G_{ji}}{2} \right)^T K^T PK \\ &\quad \times \left(\frac{G_{ij} + G_{ji}}{2} \right) \tanh(Kx) - \tanh^T(Kx) P \tanh(Kx), \end{aligned}$$

where $G_{ij} = A_i - B_i H_j$, $H_{ij} = G_{ij} + G_{ji}$.

If

$$G_{ii}^T K^T PK G_{ii} - P < 0, \quad (11)$$

$$\left(\frac{G_{ij} + G_{ji}}{2} \right)^T K^T PK \left(\frac{G_{ij} + G_{ji}}{2} \right) - P < 0, \quad (12)$$

we can obtain $V(t+1) - V(t) < 0$, which implies that the closed-loop system (5) is asymptotically stable at the equilibrium point $x = 0$. Pre-multiplying and post-multiplying (11) and (12) by Y , and applying the Schur complement (Li et al., 2009), we obtain (6) and (7). Moreover, since

$$\begin{aligned} y_{ii} &\geq \sum_{j \neq i} (y_{ij} + 2z_{ij}) = \sum_{j \neq i} (|y_{ij} + z_{ij}| + |-z_{ij}|) \\ &\geq \sum_{j \neq i} |y_{ij}|, \end{aligned}$$

the matrix Y is positive diagonally dominant. This completes the proof. ■

3. Robust H_∞ control of the discrete T-S fuzzy hyperbolic model

3.1. Problem formulation and preliminaries. In this section, we will deal with the robust stability and H_∞ control problem of discrete T-S fuzzy hyperbolic systems with external disturbance. The i -th rule of the uncertain discrete T-S fuzzy hyperbolic system is designed as follows:

If $s_1(t)$ is F_{i1} and ... and $s_g(t)$ is F_{ig} , then

$$\begin{aligned} x(t+1) &= A_i \tanh(Kx(t)) + B_i u(t) + N_i \omega(t) \\ i \in S &= \{1, 2, \dots, r\}, \quad (13) \end{aligned}$$

where $\omega(t) \in \mathbb{R}^m$ stands for the external disturbance inputs which are assumed to belong to $L_\infty[0, \infty)$, $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^n$, $N_i \in \mathbb{R}^n$.

By using the fuzzy inference method, the overall uncertain T-S fuzzy hyperbolic system is represented by

$$\begin{aligned} x(t+1) &= \sum_{i=1}^r h_i(s(t)) [A_i \tanh(Kx(t)) \\ &\quad + B_i u(t) + N_i \omega(t)] \end{aligned} \quad (14)$$

Next, a fuzzy state-feedback controller with a small amplitude will be designed to robustly asymptotically stabilize the discrete T-S fuzzy hyperbolic system (14) and to make this fuzzy system satisfy the H_∞ performance index

$$\begin{aligned} J &= \sum_{t=0}^{\infty} \tanh^T(x(t)) \tanh(x(t)) < x(0)^T P x(0) \\ &\quad + \gamma^2 \sum_{t=0}^{\infty} \omega^T(t) \omega(t), \end{aligned} \quad (15)$$

where $x(0)$ is the initial value of the state vector, γ represents a prescribed disturbance attenuation constant, $P > 0$ is positive diagonally dominant (PDD).

Then, substituting (4) into (14), the overall closed-loop system can be rewritten as

$$x(t+1) = \sum_{i=1}^r \sum_{j=1}^r h_i h_j [(A_i - B_i H_j) \tanh(Kx(t)) + N_i \omega(t)]. \tag{16}$$

3.2. Main results.

Theorem 2. *Given some scalars ε, ς and $\gamma > 0$, assume that there exist some matrixes $P > 0, Z = Z^T$ and constant matrices M_i, M_j , such that*

$$\Phi_{ij} < 0, \quad 1 \leq i, j \leq r, \tag{17}$$

$$z_{ij} \geq 0, \quad \forall i \neq j, \tag{18}$$

$$y_{ij} + z_{ij} \geq 0, \quad \forall i \neq j, \tag{19}$$

$$y_{ii} - \sum_{i \neq j} (y_{ij} + 2z_{ij}) \geq 0, \quad \forall i, \tag{20}$$

where

$$\Phi_{ij} = \begin{bmatrix} -Y + I & 0 & * \\ 0 & -\gamma^2 I & * \\ \Xi_1 & 0 & -K^{-1} Y K^{-T} \\ A_i Y - B_i M_j & 0 & 0 \\ Y & 0 & 0 \\ 0 & N_i & 0 \\ 0 & N_j & 0 \\ * & * & * \\ * & * & * \\ * & * & * \\ -\varepsilon^{-1} K^{-1} Y K^{-T} & * & * \\ 0 & -I & * \\ 0 & 0 & \Xi_2 \\ 0 & 0 & 0 & -\frac{\varsigma}{2} K^{-1} Y K^{-T} \end{bmatrix},$$

$$\Xi_1 = \frac{A_i Y - B_i M_j + A_j Y - B_j M_i}{2},$$

$$\Xi_2 = -\left(\frac{\varsigma}{2} + \varepsilon^{-1}\right)^{-1} K^{-1} Y K^{-T},$$

where $H_i = M_i Y^{-1}, H_j = M_j Y^{-1}, Y = P^{-1}, i, j = 1, 2, \dots, r$. Then the uncertain discrete closed-loop system (16) is robust asymptotically stable and satisfies H_∞ performance index for all $\omega(t) \in L_2[0, \infty)$.

Proof. Choose the following Lyapunov function candidate for the system (16):

$$V(t) = x^T(t) K^T P K x(t), \tag{21}$$

where K is defined in (14), $P > 0$.

Along the trajectories of the system (16), the corresponding time difference of $V(t)$ is given by

$$\begin{aligned} \Delta V &= V(t+1) - V(t) \\ &= x^T(t+1) K^T P K x(t+1) - x^T(t) K^T P K x(t) \\ &= \left\{ \sum_{i=1}^r \sum_{j=1}^r h_i h_j [(A_i - B_i H_j) \tanh(Kx) + N_i \omega(t)] \right\}^T \\ &\quad \times K^T P K \left\{ \sum_{n=1}^r \sum_{l=1}^r h_n h_l [(A_n - B_n H_l) \tanh(Kx) + N_n \omega(t)] \right\} - x^T K^T P K x \\ &\leq \sum_{i=1}^r \sum_{j=1}^r \sum_{n=1}^r \sum_{l=1}^r h_i h_j h_n h_l [\tanh^T(Kx) (A_i - B_i H_j)^T \\ &\quad + \omega(t)^T N_i^T] K^T P K [(A_n - B_n H_l) \tanh(Kx) + N_n \omega(t)] - \tanh^T(Kx) P \tanh(Kx) \\ &\leq \sum_{i=1}^r \sum_{j=1}^r \sum_{n=1}^r \sum_{l=1}^r h_i h_j h_n h_l \left\{ \tanh^T(Kx) (A_i - B_i H_j)^T \right. \\ &\quad \times K^T P K (A_n - B_n H_l) \tanh(Kx) \\ &\quad \left. + \tanh^T(Kx) \right. \\ &\quad \times (A_i - B_i H_j)^T K^T P K N_n \omega(t) + (N_i \omega(t))^T \\ &\quad \times K^T P K (A_n - B_n H_l) \tanh(Kx) + (N_i \omega(t))^T \\ &\quad \left. \times K^T P K N_n \omega(t) \right\} - \tanh^T(Kx) P \tanh(Kx) \\ &= \sum_{i=1}^r \sum_{j=1}^r \sum_{n=1}^r \sum_{l=1}^r h_i h_j h_n h_l \left\{ \tanh^T(Kx) (A_i - B_i H_j)^T \right. \\ &\quad \times K^T P K (A_n - B_n H_l) \tanh(Kx) \left. \right\} \\ &\quad + \sum_{i=1}^r \sum_{j=1}^r \sum_{n=1}^r \sum_{l=1}^r h_i h_j h_n h_l \tanh^T(Kx) (A_i - B_i H_j)^T \\ &\quad \times K^T P K N_n \omega(t) \\ &\quad + \sum_{i=1}^r \sum_{j=1}^r \sum_{n=1}^r \sum_{l=1}^r h_i h_j h_n h_l (N_n \omega(t))^T K^T P K (A_i \\ &\quad - B_i H_j) \tanh(Kx) \\ &\quad + \sum_{i=1}^r \sum_{j=1}^r \sum_{n=1}^r \sum_{l=1}^r h_i h_j h_n h_l (N_i \omega(t))^T K^T P K N_n \omega(t) \\ &\quad - \tanh^T(Kx) P \tanh(Kx) \\ &= \frac{1}{4} \sum_{i=1}^r \sum_{j=1}^r \sum_{n=1}^r \sum_{l=1}^r h_i h_j h_n h_l \tanh^T(Kx) (G_{ij} + G_{ji})^T \\ &\quad \times K^T P K (G_{nl} + G_{ln}) \tanh(Kx) \\ &\quad + \sum_{i=1}^r \sum_{j=1}^r \sum_{n=1}^r \sum_{l=1}^r h_i h_j h_n h_l [\varepsilon \tanh^T(Kx) (A_i \end{aligned}$$

$$\begin{aligned}
 & -B_i H_j)^T K^T P K (A_i - B_i H_j) \tanh(Kx) \\
 & + \varepsilon^{-1} (N_n \omega(t))^T K^T P K N_n \omega(t) \\
 & + \sum_{i=1}^r \sum_{j=1}^r \sum_{n=1}^r \sum_{l=1}^r h_i h_j h_n h_l (N_i \omega(t))^T K^T P K N_n \omega(t) \\
 & - \tanh^T(Kx) P \tanh(Kx) \\
 \leq & \sum_{i=1}^r \sum_{j=1}^r h_i h_j \left\{ \tanh^T(Kx) \left(\frac{G_{ij} + G_{ji}}{2} \right)^T K^T P K \right. \\
 & \times \left(\frac{G_{ij} + G_{ji}}{2} \right) \tanh(Kx) + \varepsilon \tanh^T(Kx) \\
 & \times (A_i - B_i H_j)^T K^T P K (A_i - B_i H_j) \tanh(Kx) \\
 & + \varepsilon^{-1} (N_i \omega(t))^T K^T P K N_i \omega(t) + \frac{\zeta}{2} (N_i \omega(t))^T \\
 & \times K^T P K N_i \omega(t) + \frac{\zeta^{-1}}{2} (N_j \omega(t))^T K^T P K N_j \omega(t) \\
 & - \tanh^T(Kx) P \tanh(Kx) + \tanh^T(Kx) \tanh(Kx) \\
 & - \gamma^2 \omega^T(t) \omega(t) - \tanh^T(Kx) \tanh(Kx) \\
 & \left. + \gamma^2 \omega^T(t) \omega(t) \right\} \\
 = & \sum_{i=1}^r \sum_{j=1}^r h_i h_j \eta^T \Omega \eta - \tanh^T(Kx) \tanh(Kx) \\
 & + \gamma^2 \omega^T(t) \omega(t),
 \end{aligned}$$

where

$$\begin{aligned}
 \eta^T &= \left[\tanh^T(Kx) \omega^T(t) \right], \\
 G_{ij} &= A_i - B_i H_j, H_{ij} = G_{ij} + G_{ji}, \\
 \Omega &= \begin{bmatrix} \Omega_{11} & 0 \\ 0 & \Omega_{22} \end{bmatrix}, \\
 \Omega_{11} &= \left(\frac{G_{ij} + G_{ji}}{2} \right)^T K^T P K \left(\frac{G_{ij} + G_{ji}}{2} \right) \\
 & + \varepsilon (A_i - B_i H_j)^T K^T P K (A_i - B_i H_j) - P + I, \\
 \Omega_{22} &= \left(\frac{\zeta}{2} + \varepsilon^{-1} \right) N_i^T K^T P K N_i + \frac{\zeta^{-1}}{2} N_j^T K^T P K N_j \\
 & - \gamma^2 I.
 \end{aligned}$$

If $\Omega < 0$, we can obtain

$$\begin{aligned}
 V(t+1) - V(t) &< -\tanh^T(Kx) \tanh(Kx) \\
 & + \gamma^2 \omega^T(t) \omega(t). \tag{22}
 \end{aligned}$$

Based on the accumulated result of (22) from $t = 0$ to $t = \infty$, we have the following inequality:

$$\begin{aligned}
 V(x(\infty)) - V(x(0)) &< - \sum_{t=0}^{\infty} \tanh^T(Kx) \tanh(Kx) \\
 & + \gamma^2 \sum_{t=0}^{\infty} \omega^T(t) \omega(t).
 \end{aligned}$$

That is to say,

$$\begin{aligned}
 \sum_{t=0}^{\infty} \tanh^T(Kx) \tanh(Kx) \\
 < V(x(0)) + \gamma^2 \sum_{t=0}^{\infty} \omega^T(t) \omega(t). \tag{23}
 \end{aligned}$$

Thus, the H_∞ performance index is satisfied.

Let $Y = P^{-1}$ and $M_i = H_i Y, M_j = H_j Y$. Pre- and post-multiplying both the sides of Ω by $\text{diag}\{Y, I\}$, using the Schur complements (Li et al., 2009), we will obtain the LMI (17). Finally, since

$$\begin{aligned}
 y_{ii} &\geq \sum_{j \neq i} (y_{ij} + 2z_{ij}) \\
 &= \sum_{j \neq i} (|y_{ij} + z_{ij}| + |-z_{ij}|) \geq \sum_{j \neq i} |y_{ij}|,
 \end{aligned}$$

the matrix Y is positive diagonally dominant. This completes the proof. ■

4. Simulation examples

In this section, a discretization of the Van de Vusse system and a mathematical constructive example will be presented to illustrate the effectiveness of the proposed method. Some comparisons with the results in recent publications are given to clarify the superiority of our approach.

Example 1. Consider the dynamics of an isothermal continuous stirred-tank reactor (CSTR) for the Van de Vusse example (Li et al., 2008) of the following form:

$$\begin{aligned}
 \dot{x}_1 &= F_1(x_1, x_2, u) \\
 &= -k_1 x_1 - k_3 x_1^2 + u(C_{A0} - x_1), \tag{24}
 \end{aligned}$$

$$\begin{aligned}
 \dot{x}_2 &= F_2(x_1, x_2, u) \\
 &= k_1 x_1 - k_2 x_2 + u(-x_2) \tag{25}
 \end{aligned}$$

$$y = x_2, \tag{26}$$

where the state x_1 [mol/L] represents the concentration of the reactant inside the reactor, the state x_2 [mol/L] is the concentration of the product in the output stream of the CSTR, the output $y = x_2$ determines the grade of the final product, the input-feed stream to the CSTR consists of a reactant with concentration C_{A0} and the controlled input is the dilution rate $u = F/V$ [h⁻¹], F the input flow rate to the reactor [L/h] and V is the constant volume of the CSTR in liters.

In the following, in the system (23)–(25), the parameters are chosen as $k_1 = 5h^{-1}, k_2 = 1h^{-1}, k_3 = 1$ [L/(mol h)], $C_{A0} = 5$ [mol/L], and $V = 1$ [L].

For the study of a discrete T-S fuzzy hyperbolic system, based on the work of (Li *et al.*, 2013), we can obtain the following discrete Van de Vusse model:

$$x_1(t+1) = x_1(t) + T(-k_1x_1(t) - k_3x_1^2(t)) + T(u(t)(C_{A0} - x_1(t))), \quad (27)$$

$$x_2(t+1) = x_2(t) + T(k_1x_1(t) - k_2x_2(t)) + T(u(t)(-x_2(t))), \quad (28)$$

$$y(t) = x_2(t), \quad (29)$$

where $T = 0.05$ ms is the sampling time. Then, some equilibrium points of (26)–(28) are tabulated in Table 1. Under these equilibrium points $[x_e \ u_e]$, which are also chosen as the desired operating points $[x_d \ u_d]$, we can use the T-S fuzzy-model-based modeling method of Hsiao *et al.* (2010) to construct all system matrices when we represent the system (26)–(28) by using the T-S fuzzy linear model (Tanaka and Wang, 2001).

Based on the modeling method by Hsiao *et al.* (2010), we can obtain

$$\begin{aligned} f_1(x) &= T(-k_1x_1(t) - k_3x_1^2(t)), \\ g_1(x) &= T(u(t)(C_{A0} - x_1(t))), \\ f_2(x) &= T(k_1x_1(t) - k_2x_2(t)), \\ g_2(x) &= T(u(t)(-x_2(t))). \end{aligned}$$

Applying the results of Hsiao *et al.* (2010) to the system (26)–(28) at operating points, the system matrices $A_i, B_i, i = 1, 2, 3, 4$ can be obtained. These matrices are the same as those in the T-S fuzzy linear model when we represent the system (26)–(28) by utilizing T-S fuzzy hyperbolic model. Accordingly, we can obtain the following discrete fuzzy hyperbolic control laws:

R^1 : If x_1 is about 0.6835, then

$$\begin{aligned} x_\delta(t+1) &= A_1 \tanh(Kx_\delta(t)) + B_1u_\delta(t), \\ u_\delta(t) &= H_1 \tanh(Kx_\delta(t)). \end{aligned}$$

R^2 : If x_1 is about 1.1343, then

$$\begin{aligned} x_\delta(t+1) &= A_2 \tanh(Kx_\delta(t)) + B_2u_\delta(t), \\ u_\delta(t) &= H_2 \tanh(Kx_\delta(t)). \end{aligned}$$

Table 1. Data for equilibrium points.

x_{e1}	x_{e2}	u_e
0.6835	1.7987	0.9
1.1343	2.0256	1.8
1.2949	2.0233	2.2
2.0711	1.7259	5

R^3 : If x_3 is about 1.2949, then

$$\begin{aligned} x_\delta(t+1) &= A_3 \tanh(Kx_\delta(t)) + B_3u_\delta(t), \\ u_\delta(t) &= H_3 \tanh(Kx_\delta(t)). \end{aligned}$$

R^4 : If x_4 is about 2.0711, then

$$\begin{aligned} x_\delta(t+1) &= A_4 \tanh(Kx_\delta(t)) + B_4u_\delta(t), \\ u_\delta(t) &= H_4 \tanh(Kx_\delta(t)), \end{aligned}$$

where

$$A_1 = \begin{bmatrix} 0.2509 & -1.0151 \\ 0.2649 & 0.9443 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.0673 & -1.0963 \\ 0.2841 & 0.9208 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -0.2296 & -1.1564 \\ 0.2999 & 0.9180 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} -1.5134 & -1.5052 \\ 0.3730 & 0.8025 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.2158 \\ -0.0899 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.1933 \\ -0.1013 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0.1853 \\ -0.1012 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 0.1464 \\ -0.0863 \end{bmatrix},$$

$$x_\delta(t) = x(t) - x_d, \quad u_\delta(t) = u(t) - u_d.$$

By solving the LMIs (6)–(10), the positive diagonally dominant matrix can be calculated as

$$P = \begin{bmatrix} 0.6741 & 0 \\ 0 & 0.6114 \end{bmatrix},$$

$$Z = \begin{bmatrix} 0 & 0.0100 \\ 0.0100 & 0 \end{bmatrix},$$

$$M_1 = [0.9716 \quad -8.4223],$$

$$M_2 = [2.2301 \quad -9.2796],$$

$$M_3 = [-2.5465 \quad -10.7967],$$

$$M_4 = [-11.0728 \quad -14.7438],$$

and the following controller gain matrices are obtained:

$$H_1 = [0.6550 \quad -5.1495],$$

$$H_2 = [1.5034 \quad -5.6737],$$

$$H_3 = [-1.7116 \quad -6.6013],$$

$$H_4 = \begin{bmatrix} -7.4644 & -9.0146 \end{bmatrix}.$$

Based on the stable fuzzy controller design approach for a discrete T-S fuzzy linear system (Tanaka and Wang, 2001), we can figure out the positive symmetric matrix

$$P = \begin{bmatrix} 0.0512 & 0.0034 \\ 0.0034 & 0.0509 \end{bmatrix}$$

and controller gain matrices:

$$F_1 = \begin{bmatrix} 0.5169 & -5.5961 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} -0.2418 & -6.2432 \end{bmatrix},$$

$$F_3 = \begin{bmatrix} -1.7310 & -6.8761 \end{bmatrix},$$

$$F_4 = \begin{bmatrix} -6.8668 & -9.3380 \end{bmatrix}.$$

The membership function of state x_1 is shown in Fig. 1, and

$$\tanh(Kx) = \begin{bmatrix} \tanh(0.5x_1) & \tanh(0.7x_2) \end{bmatrix}^T.$$

Thus, the whole discrete fuzzy hyperbolic control law is

$$u = (h_1H_1 + h_2H_2 + h_3H_3 + h_4H_4) \times \tanh(Kx_\delta)u_d, \quad (30)$$

where h_1, h_2 and h_3 satisfy $h_1 + h_2 + h_3 = 1$.

To illustrate the advantage of the proposed control, here, the system response curves under the conditions of different initial values are studied. Figures 2–5 denote respectively the simulation results of applying the discrete fuzzy hyperbolic controller (29) and the discrete fuzzy linear controller (Tanaka and Wang, 2001) to the discrete Van de Vusse model (26)–(28), with the operating point $x_d^T = [1.2949 \ 2.0233]$ and $u_d = 2.2000$ under the initial conditions $x(0) = [1 \ 0.55]^T$ and $x(0) = [15 \ -2.5]^T$.

From these simulations, we can find that the state of the discrete nonlinear system (26)–(28) under the discrete fuzzy hyperbolic controller (29) can converge to the operating point faster than that under the discrete fuzzy linear controller (Tanaka and Wang, 2004). Furthermore, the amplitude of the fuzzy hyperbolic controller is also smaller than that of the fuzzy linear controller. ♦

Example 2. An uncertain discrete T-S fuzzy hyperbolic model is given as

R^i : If x_i is $L_i, i = 1, 2, 3$, then

$$\begin{aligned} x(t+1) &= A_i \tanh(Kx(t)) + B_i u(t) + N_i \omega(t), \\ u &= -H_i \tanh(Kx(t)), \end{aligned}$$

where

$$A_1 = \begin{bmatrix} 0.09 & -0.19 \\ 0.07 & -0.24 \end{bmatrix},$$

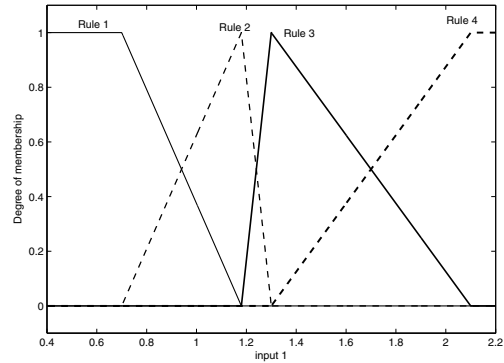


Fig. 1. Membership functions of x_1 .

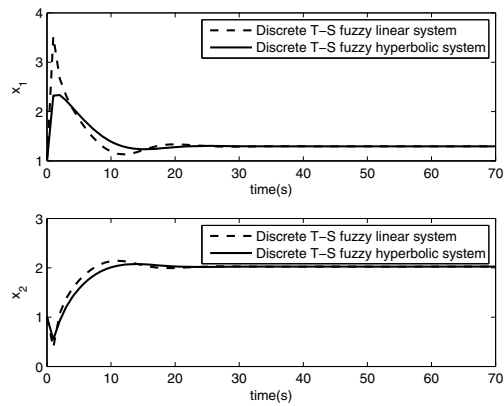


Fig. 2. State response curves and comparisons under the initial condition $x(0) = [1 \ 0.55]$. (DTSFSL: T-S fuzzy linear system, DTSFHS: T-S fuzzy hyperbolic system).

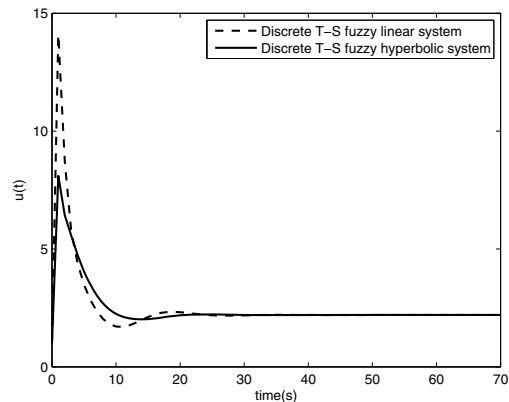


Fig. 3. Control curves and comparisons under the initial condition $x(0) = [1 \ 0.55]$.

$$A_2 = \begin{bmatrix} -0.07 & 0.28 \\ -0.18 & 0.15 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0.17 & -0.05 \\ 0.6 & -0.06 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.01 \\ 0.03 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.02 \\ 0.01 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0.01 \\ 0.03 \end{bmatrix}, \quad N_1 = \begin{bmatrix} 0.02 \\ 0.04 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} 0.03 \\ 0.05 \end{bmatrix}, \quad N_3 = \begin{bmatrix} 0.01 \\ 0.02 \end{bmatrix}.$$

Let $\gamma=2, \varepsilon=0.25, \varsigma=4$. By solving the LMI (17) in Theorem 2, the positive diagonally dominant matrix and controller gain matrices are obtained as

$$P = \begin{bmatrix} 94.4987 & -0.0622 \\ -0.0622 & 391.1528 \end{bmatrix},$$

$$Z = \begin{bmatrix} 0 & 0.001 \\ 0.001 & 0 \end{bmatrix},$$

$$M_1 = [0.0487 \quad -0.0169],$$

$$M_2 = [0.0100 \quad 0.0115],$$

$$M_3 = [0.1632 \quad -0.0045],$$

and the controller gain matrices are

$$H_1 = [4.6030 \quad -6.6193],$$

$$H_2 = [0.9440 \quad 4.4871],$$

$$H_3 = [15.4247 \quad -1.7716].$$

Let $\tanh(Kx) = [\tanh(0.1x_1) \quad \tanh(0.2x_2)]$, and choose membership functions and external disturbance as follows:

$$\mu L_1 = \frac{1}{15(1 + 5\exp^2(-x_1))},$$

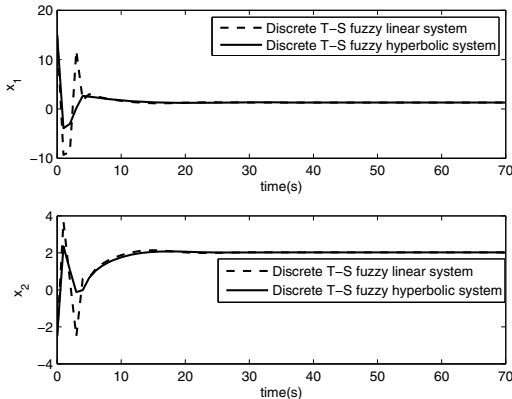


Fig. 4. State response curves and comparisons under the initial condition $x(0) = [15 \quad -2.5]$.

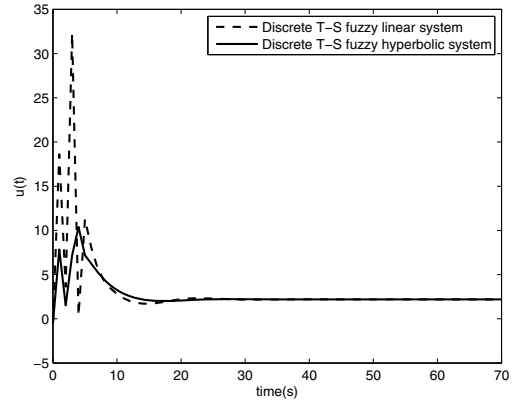


Fig. 5. Control curves and comparisons under the initial condition $x(0) = [15 \quad -2.5]$.

$$\mu L_2 = \frac{1}{15(1 + 2\exp^2(-x_1))},$$

$$\mu L_3 = 1 - \mu L_1 - \mu L_2.$$

and $\omega(t) = 2 \sin(10t) \exp(-0.5t)$. In order to get a better comparative result, here, let the two systems have the same $A_i, B_i, N_i, i = 1, 2, 3$, initial conditions, membership functions and external disturbances. Two different the initial values are used to illustrate the advantage of the proposed control; the initial conditions are respectively

$$x(0) = [4 \quad -1]^T$$

and

$$x(0) = [-3 \quad 10]^T.$$

For different initial conditions, the simulation results of the comparisons between the discrete T-S fuzzy hyperbolic system and the discrete T-S fuzzy linear system are shown respectively in Figs. 6–9.

The simulation results show that the uncertain discrete closed-loop T-S fuzzy hyperbolic system (16) is robust asymptotically stable and satisfies the H_∞ performance index (15). Moreover, the uncertain discrete T-S fuzzy hyperbolic system has a smaller control amplitude than the uncertain discrete T-S fuzzy linear system. ♦

Remark 2. For different initial conditions, many comparisons are done. From Figs. 3–9, it is clearly seen that the controller design approach to the fuzzy hyperbolic controller based on a T-S fuzzy hyperbolic model needs a much smaller control amplitude than a T-S fuzzy linear model. This illustrated effectively the advantages of the proposed method.

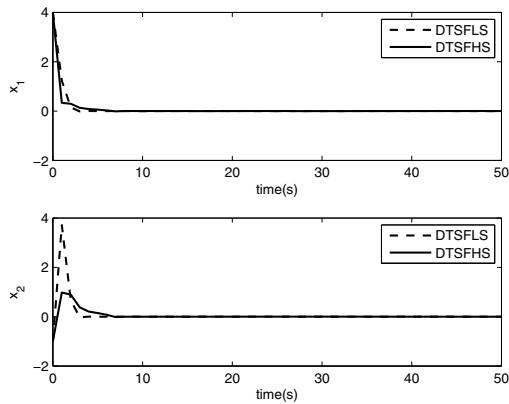


Fig. 6. State response curves and comparisons under the initial condition $x(0) = [4 \ -1]$.

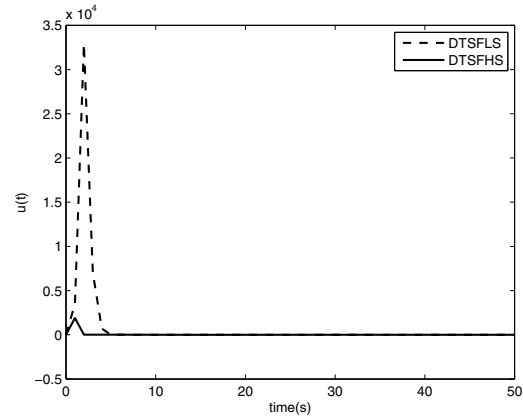


Fig. 9. Control curves and comparisons under the initial condition $x(0) = [-3 \ 10]$.

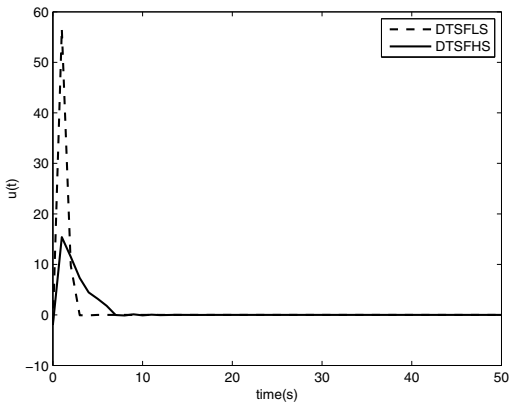


Fig. 7. Control curves and comparisons under the initial condition $x(0) = [4 \ -1]$.

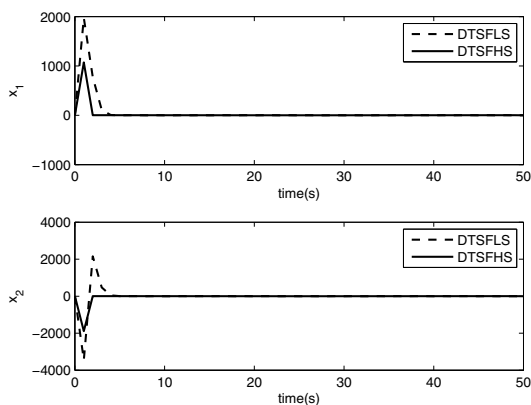


Fig. 8. State response curves and comparisons under the initial condition $x(0) = [-3 \ 10]$.

5. Conclusion

In this paper, a discrete T–S fuzzy hyperbolic model for a class of discrete nonlinear systems was proposed. The parallel distributed compensation method was utilized to design a fuzzy hyperbolic controller. Sufficient conditions for the asymptotic stability of the closed-loop system were formulated by LMIs. In addition, for the discrete T–S fuzzy hyperbolic system with external disturbance, the global robust stability and H_∞ performance were developed by designing a robust H_∞ constraint controller. Finally, we presented some simulation examples to illustrate the validity and feasibility of the proposed schemes. From these simulation results, the control input requirements of the discrete T–S fuzzy hyperbolic system are much lower than for the discrete T–S fuzzy linear system, while the state stabilization time of the two systems is almost the same. In a future work, based on fuzzy piecewise Lyapunov functions, the proposed hyperbolic controller can be employed to control discrete-time nonlinear systems with time delays.

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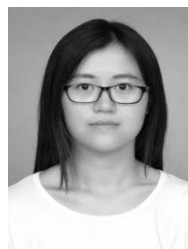


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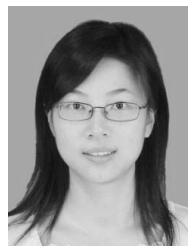


control systems.

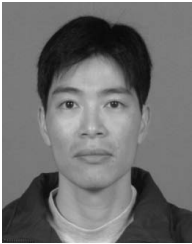
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