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# Soft computing based prediction of friction angle of clay

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#### ABSTRACT

**Purpose:** This article uses soft computing-based techniques to elaborate a study on the prediction of the friction angle of clay.

**Design/methodology/approach:** A total of 30 data points were collected from the literature to predict the friction angle of the clay. To achieve the friction angle, the independent parameters sand content, silt content, plastic limit and liquid limit were used in the soft computing techniques such as artificial neural networks, M5P model tree and multi regression analysis.

**Findings:** The major findings from this study are that the artificial neural networks are predicting the friction angle of the clay accurately than the M5P model and multi regression analysis. The sensitivity analysis reveals that the clay content is the major influencing independent parameter to predict the friction angle of the clay followed by sand content, liquid limit and plastic limit.

**Research limitations/implications:** The proposed expressions can used to predict the friction angle of the clay accurately but can be further improved using large data for a wider range of applications.

**Practical implications:** The proposed equations can be used to calculate the friction angle of the clay based on sand content, silt content, plastic limit and liquid limit.

**Originality/value:** There is no such expression available in the literature based on soft computing techniques to calculate the friction angle of the clay.

**Keywords:** Friction angle of the clay, Artificial neural network, Sensitivity analysis, M5P model tree, Multi regression analysis

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METHODOLOGY OF RESEARCH, ANALYSIS AND MODELLING

# **1. Introduction**

Friction angle is measured as an important geotechnical parameter to find shear strength of a soil. It can also be used for determining slope stability and designing of various engineering structures like pavements, retaining walls etc. Numerous experimental methods are employed for estimation of shear strength parameters like triaxial and direct shear test but this method consumes more time and is costly. Therefore, various soft computing techniques developed can be used for predicting the friction angle of the soil with more accuracy. One of such technique is artificial neural network (ANN) which solves many complex problems and produce accurate results. It comprises of input, hidden and output layer which are interconnected with each other through a set of connection weights. Using this technique variety of problems related to geotechnical engineering can be solved and hence is commonly used nowadays [1-10]. This study includes the prediction of friction angle of the clay using inputs such as sand content (SC), clay content (CC), plastic limit (PL) and liquid limit (LL) in percentage. Finally, a mathematical model equation is proposed using weights and biases. This proposed mathematical equation is compared to MRA and M5P models for the prediction of output friction angle. The prediction made using ANN model was also compared with the empirical correlations available in literature.

# 2. Background

Shear strength is considered as a vital parameter of soil for evaluating stability and measures to prevent failure of slopes. Various studies have been carried out to relate the engineering properties (bearing ratio, optimum moisture content and maximum dry unit weight) of the stabilised and un-stabilised soil with the index properties of the respective soil [1-12]. Deviator stress of sand reinforced with waste plastic strips was predicted using ANN by [13]. Bearing capacity of the regular shaped footings [14-16], multi-edge footings resting on sand was determined by using artificial neural networks by [17]. Similarly, the settlement of the footing was predicted by [18]. Shear wave velocity of the soil [19], free swell index [20] hydraulic conductivity in the clay liners [21], pile driving records reanalysed [22], bearing capacity of the piles [23], in situ soil properties at any halfspace point at a site [24], uplift capacity of suction caissons [25], modelling soil collapse [26], pre-consolidation pressure [27], cyclic swelling pressure of mud rock [28], undrained lateral load capacity of piles in clay [29], effective

stress parameter of unsaturated soils [30], soil and subsurface sediments distribution in dam [31], stability analyses of slopes [32], swelling pressures of expansive soils [33], compression index of soils [34], strength of reinforced lightweight soil [35], permeability coefficient of soils [36], soil specific surface area [37]. Further, in the recent past, different soft computing techniques have been used for the prediction of different parameters in the civil engineering [38-47]. From the above literature, it is evident that the application of soft computing techniques such as ANN, MRA and M5P for the prediction of friction angle of the clay in the field of the geotechnical engineering has not yet been carried out.

# 3. Collection of the data

Database used for construction of ANN, MRA and M5P models were taken from previous literatures. Total 60 data have been used out of which 42 data were used for training and 18 were used for the testing of the model. Hence, the same was followed in the present study. 30 data were taken from [48], 12 data were collected from [49], 3 data from [50], 3 data from [51], 5 data from [52], 3 data from [53], 2 data from [54] and 1 data from [55]. The data was divided for the training and the testing purpose as per literature [13-19] where it is suggested that the 70% data for testing and 30% data for the training is sufficient to produce best results. The total data used for modelling were tabulated in Table 1.

#### 4. Soft computing techniques

# 4.1. Artificial neural networks

ANN, initially introduced by [56] is the branch of the artificial intelligence. ANN tries to imitate the nervous system and the function of the human brain. ANN modelling has the ability to differentiate complex nonlinear connections amongst the input and the output variables without any preceding expectations. Further, ANN can use raw data (input) without any need of manipulation or preprocessing which makes the ANN more useful and less costly in comparison to the conservative techniques. An ANN is required to be trained before making any interpretation of the new information for which many algorithms were available in literature. Among them, a feed forward back propagation algorithm is the utmost versatile [13-20] and effective for the multilayer neural network.

Table 1.
Data for the modelling

S No	SC,	CC,	PI,	LL,	Friction angle,	S. No.	SC,	CC,	PI,	LL,	Friction angle,
5. NO.	%	%	%	%	deg.		%	%	%	%	deg.
1	0	52	23	48	33	31	14	53	67.98	30.66	15
2	4	66	27	53	30	32	12	82	78.58	31.5	17.4
3	2	57	16	37	25	33	26	42	71	30	21
4	17	51	20	32	36	34	12	55	51.92	9.35	13.6
5	13	56	26	55	31	35	2	80	34.8	85.1	14.5
6	28	47	17	45	29	36	31.5	27.5	31.54	71.75	12.75
7	36	33	21	41	37	37	11	60	74.84	38.1	12.5
8	33	36	21	47	35	38	8	45	41	65	41
9	44	31	21	43	34	39	56	31	26	45	36
10	42	29	14	41	38	40	46.3	11.4	41.03	24.23	33
11	43	22	45	54	40	41	34.8	13.8	44.45	30.18	34
12	44	21	26	54	32	42	11	59	57.46	20.61	12
13	52	25	42	52	41	43	25	43	15	39	33
14	10	23	19.8	32.7	26	44	17	48	20	46	30
15	49	40	36	58	42	45	22	39	24	43	29
16	11	78	65	27.6	27.3	46	20	38	16	31	31
17	39	33	48	68	39	47	38	19	25.29	11.08	26
18	2.5	85	23.23	47.79	24.6	48	43.4	13.2	36.97	22.57	35
19	15	75	32.78	84.8	8.5	49	61	8	22	28	32
20	40.8	14.4	42	31.06	34.8	50	56	13	38	46	40
21	7	14	56	35	10	51	6	19	53	33	12
22	43.8	13.1	41.58	28.86	26.1	52	5	15	50	31	10
23	11	61	75.05	33.01	11	53	55	11	42	54	41
24	12	48	67	28	22	54	55	18	34	42	37
25	51.4	15.3	41.24	28.57	30.6	55	60	8	26	37	34
26	13	53	51.95	9.66	13.4	56	58	13	31	41	39
27	11	80	72	29.9	20.3	57	44	21	23	35	31
28	28	31.5	28.5	46.5	31.5	58	1	74	38	88	26
29	47	17.5	44	29	27.5	59	32	50	35	56	36
30	41.5	13	26.5	41	28.5	60	25	51	50	66	38

Back-propagation (BP) algorithm contains interconnected layers (input, hidden and output). The output of the neuron or the node of the input layer were sent to node in the hidden layer as an input, and the output of the neuron or the node of the hidden layer was sent finally to the output layer. The number of neurons in the hidden layers and the number of hidden layers is dependent on the problem in hand. Hence researchers were resort to a cumbersome trial and error procedure. All nodes (excluding the input layer) in the BP network were having an activation function and a bias node. The bias contains a constant input. The activation function filters the summed output. Activation functions in ANN were used based on the objective. Computed vectors of the output corresponding to the solution of the problem, were created by the output layer.

# 4.2. Multiple regression analysis

Multiple regression analysis refers to a group of techniques to analyze straight-line relationships between two variables or more. Multiple regression estimates the  $\beta$ 's in the equation

$$y_j = \beta_0 + \beta_1 x_{1j} + \beta_2 x_{2j} + \dots + \beta_p x_{pj} + \varepsilon_j \tag{1}$$

X's and Y are the independent and the dependent variables respectively and the  $\beta$ 's are the unknown regression

coefficients. The most widely used approach to solve regression problem is the least squares in which, the  $\beta$ 's were selected so as to minimize the sum of the squared residuals ( $\varepsilon_j$ ). The intercept,  $\beta_0$ , is the point at which the regression plane intersects the Y axis. Once the  $\beta$ 's were estimated, correlation coefficient (ranges from -1 to 1) was used to assess the reliability of such estimate. Stronger or no relationship exists when the value of coefficient of correlation is nearing  $\pm 1$  or close to zero respectively. The regression equation can only measure the relationships linearly, or straight-line. The data are a random sample from a larger data and are independent and normally distributed; a set of statistical tests (t and F tests) may be applied to the  $\beta$ 's and the coefficient of correlation.

#### 4.3. M5P model tree

M5P was a conventional decision tree. This tree was developed using the technique of divide and conquer. This technique was used for the estimation of numeric continuous parameters. The terminal node has a linear function for these numeric parameters. Generation of the tree was done in two steps. A splitting criterion for the formation of a decision tree was introduced in the first step. This splitting criterion was based on the standard deviation of the class values.

This standard deviation enters the node as a indicator of that node's error, and measures the predicted error reduction. To make the node purer, the splitting method causes the child node to have lower value of the standard deviation relative to the parent node. After evaluating all possible splits and optimizing the predicted reduction of errors, the split is chosen by M5 model tree. The data division generates a large tree that can cause the test data to be overfitted and managed by using the pruning method to replenish the overgrown tree that was accomplished by replacing a sub tree with a linear regression function in the second step.

#### 5. Development of ANN model

To achieve optimum architecture, the optimal number of nodes in hidden layer and number of hidden layers were settled properly. For the best use of ANN model, the main issue arises when to suspend training. Excess training of the model may result in noise leading to over fitting effect. The number of iterations was optimized by trial and error [13]. The RMSE and target values for different iterations were estimated. The iteration with minimum value of RMSE is selected for determining optimal architecture of ANN. From the Figure 1 it can be summarised that the iteration having less value of RMSE is opted for optimization of neural network model [17].



Fig. 1. Variation between RMSE,  $R^2$  and number of iterations

Keeping this in view, with the topology of 4-6-1, the iterations were selected as 2500 as evident from the Figure 1. The architecture of ANN model through the illustration is shown in Figure 2.

The mean square error is defined as:

$$MSE = \frac{\sum_{i=1}^{n} \left(\phi_i - \phi_F\right)^2}{n}$$
(2)

$$R^2 = \frac{A_1 - A_2}{A_1}$$
(3)

where

$$A_1 = \sum_{i=1}^n \left(\phi_i - \overline{\phi}\right)^2 \tag{4}$$

$$A_{2} = \sum_{i=1}^{n} \left( \phi_{F} - \phi_{i} \right)^{2}$$
(5)

where  $\phi_i$ ,  $\overline{\phi}$  and  $\phi_F$  are the experimental, average of the experimental and predicted friction angle of the clay respectively; and n is the number of training data.



Fig. 2. Revised architecture of ANN for friction angle

Table 2.	
Performance measures of the ANN model	
D C	

Performance measures	r	$\mathbb{R}^2$	MSE	RMSE	MAE	MAPE
Training	0.93	0.93	14.30	3.78	3.00	14.13
Testing	0.96	0.96	5.57	2.36	1.61	5.61





The friction angle of the clay obtained from the neural network was compared with the actual friction angle of the clay in order to verify the prediction accuracy of the ANN model. The comparison between the friction angle of the clay obtained from the ANN and the actual friction angle of the clay for the training and the testing data were presented in Figure 3. Study of Figure 3 reveals that the calculated values of the coefficient of determination (R<sup>2</sup>) were found to be 0.93 and 0.96, respectively, for the training

and the testing data. Further, the accuracy of the developed model was assessed with other statistical parameters (correlation coefficient (r), MSE, RMSE, MAE and MAPE) for the training and the testing data which were tabulated in Table 2.

Table 2 reveals that all the statistical parameters were within the permissible range (readers may refer to [13] for the range of statistical parameters). After simulating the model for the optimal conditions, matrix of the connection weights between the input layers to hidden layer  $[x_{ji}]$ , hidden layer to the output layer  $[y_{jk}]$ , input bias  $[z_i]$  and the output bias  $[z_0]$  were presented in matrix form as below.

$$x_{ji} = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \\ x_{51} & x_{52} & x_{53} & x_{54} \\ x_{61} & x_{62} & x_{63} & x_{64} \end{bmatrix}$$
$$= \begin{bmatrix} 0.69 & 0.69 & 0.16 & 1.55 \\ 1.43 & 0.45 & -2.13 & 1.94 \\ 3.28 & 1.19 & 0.31 & -2.19 \\ 0.41 & 1.79 & -2.31 & 4.19 \\ -1.17 & -1.38 & 1.14 & -0.84 \\ 0.64 & 1.12 & -0.93 & 0.64 \end{bmatrix}$$

(6)

$$y_{jk} = \begin{bmatrix} y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \\ y_{16} \end{bmatrix} = \begin{bmatrix} 0.59 \\ -3.27 \\ 3.05 \\ 4.19 \\ -2.11 \\ -1.64 \end{bmatrix}$$
(7)
$$z_{j} = \begin{bmatrix} z_{1} \\ z_{2} \\ z_{3} \\ z_{4} \\ z_{5} \\ z_{6} \end{bmatrix} = \begin{bmatrix} 1.89 \\ -0.24 \\ 0.18 \\ 0.19 \\ -0.37 \\ 1.03 \end{bmatrix}$$
(8)

$$z_0 = \begin{bmatrix} 2.33 \end{bmatrix} \tag{9}$$

where:

 $[x_{ji}]$  = weight between j<sup>th</sup> neuron of the hidden layer and i<sup>th</sup> neuron in the input layer;

 $[y_{jk}]$  = weight between the k<sup>th</sup> layer of output neuron and j<sup>th</sup> neuron in the hidden layer;

 $[z_j] = j^{\text{th}}$  neuron of the hidden layer bias;

 $[z_0] =$  output layer bias.

#### 5.1. Sensitivity analysis

Sensitivity analysis is performed to evaluate influence of each parameter on the prediction of the friction angle of clay using a method reported by [36] which depends on weight configuration. But the given method had its own drawbacks as it evaluates absolute weights. So, in order to minimize the drawbacks a method given by [37] was adopted. Therefore, the sensitivity analysis was performed using a method reported by [37]. This method determines the sum and product of the weights that was finalized from the input nodes to the hidden nodes and from the hidden nodes to the final output nodes for all the inputs. The equation showing contribution of the input parameters is shown in Eq. (10).

$$RI_{j} = \sum_{k=1}^{h} w_{jk} \times w_{k} \tag{10}$$

where  $w_k$  denotes connection weight between  $k^{th}$  hidden nodes and output nodes,  $w_{jk}$  stands for connection weights between the j<sup>th</sup> input variables and the k<sup>th</sup> nodes in hidden layer, RIj denotes relative importance of j<sup>th</sup> input layer nodes and h is no. of nodes in hidden layer.

This work comprises of 4 input variables and their influence on the friction angle of the clay depends upon the

weights which were estimated in feed forward ANN model. The final weights between the nodes in an input - hidden layer and the nodes of hidden layer -output layer generated using Gaussian function is shown in the matrix form in the Eq. 6-9. The relative importance of each of the parameters considered in the Gaussian function-based model was shown in Figure 4. From Figure 4 it is evident that the percentage clay fraction was the most vital variable. It contributes about 37% for the estimation of friction angle and other parameters such as sand content, liquid limit and plastic limit contributes about 29%, 26% and -8% respectively. Further, from Figure 4, it is again evident that input parameters like clay, sand and liquid limit have positive influence on the prediction of friction angle whereas plastic limit has negative influence on the output. Therefore, from the above it was concluded that clay content, sand content and liquid limit were directly influencing whereas plastic limit was inversely influencing the friction angle of the clay.



Fig. 4. Relative importance of each parameter for the output friction angle

#### 5.2. ANN based proposed model equation

After obtaining the final trained weights, a model equation was proposed in this section as per [50]. Taking into account the weights and biases given in the matrix form in the Eq. 6-9, the ANN model takes the following form:

$$\boldsymbol{\phi} = f_n \left\{ z_0 + \sum_{i=1}^h \left[ y_{jk} f_n \left( \sum_{i=1}^n x_{ji} E_i \right) \right] \right\}$$
(11)

where:

h = number of neurons in hidden layer which is equal to 8 in this case,

 $E_i$  = normalized inputs in the range of 0 to 1,

 $f_n = Activation function,$ 

n = number of input variables.

The following steps  $[E_1-E_8 \text{ and } F_1-F_6 \text{ using Equations} (12-17)$  and (18-23) respectively] were carried out for the development of the model equation using ANN. The final expression obtained was as per equation (24). This equation (24) provides a normalized friction angle of the clay. Equations (25) and (26) provide the output friction angle of the clay in the de-normalized form.

$$E_{1} = x_{11} \times SC + x_{12} \times CC + x_{13} \times PL + x_{14} \times LL + z_{1}$$
(12)

$$E_{2} = x_{21} \times SC + x_{22} \times CC + x_{23} \times PL + x_{24} \times LL + z_{2}$$
(13)

$$E_{3} = x_{31} \times SC + x_{32} \times CC + x_{33} \times PL + x_{34} \times LL + z_{3}$$
(14)

$$E_4 = x_{41} \times SC + x_{42} \times CC + x_{43} \times PL + x_{44} \times LL + z_4$$
(15)

$$E_5 = x_{51} \times SC + x_{52} \times CC + x_{53} \times PL + x_{54} \times LL + z_5$$
(16)

$$E_6 = x_{61} \times SC + x_{62} \times CL + x_{63} \times PL + x_{64} \times LL + z_6$$
(17)

$$F_1 = y_{11} \times e^{-E_1} \tag{18}$$

$$F_2 = y_{12} \times e^{-E_2} \tag{19}$$

$$F_2 = y_{21} \times e^{-E_3}$$
 (20)

$$F_3 = y_{31} \times e^{-3}$$
(20)

$$F_4 = y_{41} \times e^{-E_4} \tag{21}$$

$$F_5 = y_{51} \times e^{-E_5} \tag{22}$$

$$F_6 = y_{61} \times e^{-E_6} \tag{23}$$

$$R_1 = \phi = F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + z_0 \tag{24}$$

$$\phi = e^{-R_1} \tag{25}$$

After de-normalization

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$$\phi = 0.5(\phi + 1)(\phi_{\max} - \phi_{\min}) + \phi_{\min}$$
(26)

Based on the data collected from the literature, a model equation using ANN was proposed (Eq. 26). This equation can be used to determine the friction angle of the clay. The graph between the predicted Vs targeted friction angle of the clay was shown in Figure 3. Therefore, from the Figure 3, ANN can be effectively used for the prediction of the friction angle of the clay.

# 6. Comparison of the developed model with MRA and M5P

Using total dataset used in construction of neural network, multiple regression analysis was performed. The input data used were sand content, clay content, plastic limit and liquid limit and the output was the friction angle of the clay. Multiple regression model obtained from the total data is presented through Eq. (27).

$$\phi = 27.21 + 0.28 \times SC + 0.19 \times CC - 0.215 \times PL + 0.0023 \times LL \quad (27)$$

where  $\phi$  is friction angle of clay, SC is sand content in percentage, CC is clay content in percentage, PL is plastic limit and LL is liquid limit.

Comparison among the proposed ANN model was made with the one obtained using MRA and M5P was attempted in terms of performance measures as shown in Table 3. Comparison of the performance measures was carried out separately for the training and the testing as shown in the Table 3.

From examination of Table 3 reveals that the values of correlation coefficient (r) and the value of coefficient of efficiency ( $R^2$ ) for training and testing data improved by 25.8% and 47.3% and 3.1% and 10.7% in comparison to the MRA model. The values for the MAE, MAPE, RMSE and MSE in case of ANN model for training and testing data were decreased by 45.6%, 50.0%, 46.6% and 71.5% and 41.3%, 39.9%, 23.4% and 41.3% respectively.

Table 3.

Comparison of performance measures of neural network model with the MRA and M5P models

Doutoursonoo	Prediction model								
Periormance	AN	N	MR	A	M5P				
measures	Training	Testing	Training	Testing	Training	Testing			
r	0.93	0.96	0.69	0.93	0.69	0.92			
$\mathbb{R}^2$	0.93	0.96	0.49	0.86	0.50	0.83			
MSE	14.30	5.57	50.17	9.49	50.25	9.86			
RMSE	3.78	2.36	7.08	3.08	7.08	3.14			
MAE	3.00	1.62	5.52	2.76	5.56	2.75			
MAPE	14.13	5.62	28.30	9.36	28.52	8.83			

Further study of Table 3 reveals that the r,  $R^2$  for training and testing data improved by 34.8% and 86.0% and 4.35% and 15.6% in comparison to the M5P model. The values of MAE, MAPE, RMSE and MSE in case of ANN model for training and testing data were decreased by 46.0%, 50.4%, 46.6% and 71.5% and 41.0%, 36.3%, 24.8% and 42.1% respectively. The comparison between ANN model and the models developed using MRA and M5P clearly shows the superiority of the ANN model in terms of performance measures.

#### Comparison with literature

The comparison of the ANN model is also attempted with the correlation proposed by [57]. The correlation proposed by [57] is given below in equation (28).

$$\phi^{ip} = 43 - 10 \times \log(PI) \tag{28}$$

The curve between estimated friction angle from the correlation reported by [57] and measured friction angle is as shown in Figure 5. From Figure 5 it can be concluded that coefficient of determination ( $R^2$ ) attained for the correlation reported by [57] was 0.79. This coefficient of determination represents weak relationship exists between input and output variables. The coefficients of determination ( $R^2$ ) for the training and the testing data obtained using ANN were 0.93 and 0.96 respectively. Therefore, the prediction made using artificial neural network was superior to the one obtained using empirical correlation available in literature.



Fig. 5. Curve between measured friction angle and estimated friction angle (from Equation (28))

# 7. Conclusions

The present work aims to develop the model equation for the prediction of clay friction angle using techniques ANN, MRA, and M5P. The independent variables used to obtain the model equations were sand content, clay content, plastic limit and liquid limit. The following conclusions are put forward.

- 1. The model was developed for friction angle data of 60 records. The findings suggest that neural network architecture's 4-6-1 topology is relatively capable of predicting the clay's friction angle with reasonable precision.
- 2. The sensitivity analysis reveals that the sand content, clay content, liquid limit and plastic limit contribute respectively 37%, 29%, 26% and -8%, indicating a relationship between inputs and output using a connection weight approach.
- 3. The values of correlation coefficient (r) and the value of coefficient of efficiency (R<sup>2</sup>) for the training and the testing data improved by 25.8% and 47.3% and 3.1% and 10.7% in comparison to the MRA model whereas the values for the MAE, MAPE, RMSE and MSE in case of ANN model for training and testing data were decreased by 45.6%, 50.0%, 46.6% and 71.5% and 41.3%, 39.9%, 23.4% and 41.3% respectively in comparison to the MRA model.
- 4. The r, R<sup>2</sup> for the training and the testing data improved by 34.8% and 86.0% and 4.35% and 15.6% respectively in comparison to the M5P model whereas the values of MAE, MAPE, RMSE and MSE in case of ANN model for training and testing data were decreased by 46.0%, 50.4%, 46.6% and 71.5% and 41.0%, 36.3%, 24.8% and 42.1% respectively in comparison to the M5P model.
- 5. The values of the statistical parameters (r, R<sup>2</sup>, MSE, RMSE, MAE and MAPE) indicate that the developed ANN model was superior to the one obtained using MRA and M5P model tree technique.
- 6. Model equation was developed for the friction angle of the clay using artificial neural networks. This developed equation outperformed the empirical equation reported in literature.

On the whole the paper has attempted to provide the insight into the application of the soft computing techniques to predict the friction angle of the clay. This will help for the calculation of the friction angle which otherwise require expensive experimentation. In general, the models of the neural network have the drawback of giving reasons and reasoning behind the model thus obtained. However, alternative approaches can also be explored in the future, such as support vector machine, particle swarm optimization or genetic programming.

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