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SEQUENTIAL PREVENTIVE MAINTENANCE POLICIES WITH CONSIDERATION OF RANDOM ADJUSTMENT-REDUCTION FEATURES

STRATEGIA SEKWENCYJNEJ KONSERWACJI ZAPOBIEGAWCZEJ Z UWZGLĘDNIENIEM CECH LOSOWEJ KOREKCJI I LOSOWEJ REDUKCJI WIEKU

In existing literature, imperfect maintenance has been widely studied and many studies treat the effectiveness of imperfect maintenance as a fixed constant. In reality, it is more realistic to regard the maintenance efficiency as a random quantity as it may not be precise value due to the lack of sufficient data and/or the variation from system to system. In this paper, a hybrid imperfect maintenance model with random adjustment-reduction parameters is proposed, and a maintenance policy, namely the sequential preventive maintenance in periodic leisure interval, is studied based on the proposed hybrid random imperfect maintenance model, and the corresponding maintenance strategy is optimized by the genetic algorithm (GA). A numerical example and an example of the fuel injection pump of diesel engines are presented to illustrate the proposed method.

Keywords: Maintenance policy; Imperfect maintenance; Preventive maintenance; Random maintenance efficiency.

W literaturze, temat konserwacji niepełnej został szeroko zbadany i wiele z opisywanych badań traktuje wydajność konserwacji niepełnej jako wartość stałą. W rzeczywistości jednak wydajność konserwacji należy traktować jako wielkość losową, ponieważ nie można jej dokładnie określić ze względu na brak wystarczających danych i / lub różnice między poszczególnymi systemami. W niniejszej pracy zaproponowano model hybrydowy konserwacji niepełnej łączący pojęcia parametrów losowej korekcji i losowej redukcji wieku. Na podstawie proponowanego modelu hybrydowego losowej konserwacji niepełnej przebadano strategię sekwencyjnej konserwacji zapobiegawczej przeprowadzanej okresowo w czasie wolnym od pracy; omawianą strategię konserwacji zoptymalizowano za pomocą algorytmu genetycznego (GA). Proponowaną metodę zilustrowano przykładem liczbowym oraz omówiono na przykładzie pompy wtryskowej paliwa do silników wysokoprężnych.

Słowa kluczowe: polityka konserwacji, konserwacja niepełna, konserwacja zapobiegawcza, losowa wydajność konserwacji.

1. Introduction

Systems are suffering deterioration due to aging and unexpected shock damages after launched. Maintenance is executed to retain a system in or restore it to an acceptable operating condition for the fulfillment of requirement. Generally, it involves two major maintenance categories: corrective (unplanned) or preventive (planned). Corrective maintenance (CM) is any maintenance activity performed when the system is failed or breakdown. Preventive maintenance (PM) is all activities performed in an attempt to retain a system in specified condition by providing systematic inspection, detection, and prevention of incipient failure. Commonly, preventive maintenances are undertaken regularly at pre-selected intervals to reduce or eliminate the accumulated deterioration, and corrective maintenances are carried out whenever shocked and unexpected failure happens. Obviously, CM is performed at unpredictable time points because the failure time of products is unknown. CM is typically carried out in three steps:

(1) Diagnosis of the problem, (2) Repair and/or replacement of faulty component(s), and (3) Verification of the repair action. Preventive maintenance (PM) is the maintenance that occurs when the system is still in operating condition.

According to the efficiency, maintenance can be generally classified into five categories as: perfect, minimal, imperfect, worse, and worst [21]. A perfect maintenance action restores the system to “as good as new” condition. In most cases, a replacement can be viewed as a perfect maintenance. A minimal maintenance activity restores a system back to the functioning state without changing its failure intensity. After minimal repair, it has the same failure intensity with when it failed, and it seems “as bad as old”. Imperfect maintenance does not restore the system “as good as new” or “as bad as old” conditions. It assumes the maintenance efficiency is somewhere between the two extreme cases, i.e. perfect and minimal. The imperfect maintenance broadly exists and be more realistic and in practical engineering. The worse maintenance is a negative maintenance action making a system

worse after repair (increases failure intensity) but not break down. Worst maintenance will lead a system to failure or breakdown.

Imperfect maintenance models have been extensively studied in the past decades as many maintenance actions may realistically not resulting in perfect and minimal situations but in an intermediate one. Many imperfect models have been proposed, for example, Pham et al. [21], Nakagawa [17], Block et al. [3], Kijima [6, 7], Wang [26], Lam [9], Zhao [33], Pham and Wang [22], Wang and Pham [27]. Pham et al. [21] summarize various treatments of imperfect maintenance of binary-state systems. Wu and Zuo [31] studied the commonality and interrelationship between some commonly used imperfect maintenance, and categorized the existing models into two groups, i.e. linear and nonlinear models. Liu et al. [15] proposed a new approach to selecting the most adequate imperfect maintenance model among several candidates based on the collected failure data. The uncertainty associated with imperfect maintenance model selection is also considered in maintenance decision-makings. In most recently, the imperfect maintenance model has been extended to the context of multi-state systems. For example, Liu et al. [14] proposed a new imperfect maintenance model for multi-state components, and jointly optimized the redundancy levels and maintenance strategy for multi-state systems.

According to Brown and Proschan [4], maintenance policies based on planned inspections are “periodic inspection”, and “inspection interval dependent on age”. By periodic inspections, a failed unit is identified (e.g., spare battery, a fire detection device, etc.). With aging of units, the inspection interval may be shortened [23, 28]. These inspection methods are subject to imperfect maintenance caused by randomness in the actual time of inspection in spite of the schedule, imperfect inspection, and cost structure. Therefore, realistic and valid maintenance models must incorporate random features of the inspection and maintenance policy [29].

In this paper, we develop a hybrid PM model considering the random features of both the adjustment factor and age reduction factor, called the random adjustment-reduction maintenance (RAM) model. Throughout this paper, we will call the RAM model for short. This model is an extension to the study by Wu and Clemets-Croome [30] in which we will discuss in details the RAM including the failure rate PM, the age reduction PM, the hybrid PM addressing the random adjustment-reduction factors. It is more realistic to describe the imperfect maintenance efficiency through a random variable and a hybrid model. Later on, a finite-horizon PM decision model is proposed with considering sequential PM policy under the random PM efficiency. We then optimize the sequential PM policy by using the genetic algorithm.

The remainder of this paper is organized as follows: Section 2 derives reliability metrics including the failure intensity function and the reliability function for the RAM model. Section 3 introduces the proposed PM policy model under the features of random maintenance strategy. Section 4 presents the genetic algorithm to obtain the optimal PM sequence T_p^* and PM times N^* . Two studied cases are given to illustrate the proposed maintenance policy in Section 5. A brief conclusion is given in Section 6.

2. Imperfect PM Model

The earliest preventive maintenance models consider that a system after a PM activity is “as good as new” and this kind of PM is called the perfect PM. The replacement of component or system with a new one can be considered to be a perfect one. Sometimes, the system after PM activities cannot be “as good as new”. Barlow et al. [1] introduced a minimal repair model in which PM activities do not change the failure intensity of the system. Later on, Nakagawa [17] studied a failure rate PM model, Malik [16] proposed an age reduction PM model, and Kijima [6, 7] proposed and discussed type I and type II imperfect repair models. Lin et al. [13] introduced a hybrid PM

model by combining the failure rate PM model and the age reduction PM model. Random maintenance quality was studied by Wu et al. [30] and random variables were implemented in failure rate and age reduction models respectively.

In the failure rate model, Nakagawa [17] assumed that when a repairable system launches, its failure intensity will continuously increase if no PM activity intervenes, otherwise its failure intensity will be changed by a PM, that is, after the i^{th} PM action, the failure intensity function can be written as $A_i \lambda_{i-1}(t)$ where $t \in (0, t_{i+1} - t_i)$ and $\lambda_{i-1}(t)$ is the failure intensity function at $t \in (0, t_i - t_{i-1})$. A_i should satisfy $A_i > 1$, and it is considered as an adjustment factor or improvement factor which illuminate although the failure intensity is reset to the value at $t=0$, after PM, its slope will increase in the next repair cycle. The larger A_i is, the higher slope its failure intensity has after a PM.

In age reduction model, Malik [16] suggested that a system's failure intensity is $\lambda_0(t)$ where $t \in (0, t_1)$, and it will monotonously increase without maintenance activity. When PM is taken at t_1 , the failure intensity will be formulated as $\lambda_1(t) = \lambda_0(t + \alpha_0 t_1)$ for $t \in (0, t_2 - t_1)$ and $\alpha_0 \in (0, 1)$. α_0 is defined as the virtual age reduction factor. It means that before performing a PM action, the actual age and virtual age are both equal to T_{p1} , and after the PM action, the actual age is $t + t_1$ while the virtual age $(t + \alpha_0 t_1)$, where virtual age is less than actual age and the health condition becomes better after a PM. Then, the failure intensity of the system is a function with respect to the virtual age, and each PM action reduces the virtual age of the system to a certain extent. Kijima et al. [6] [7] introduced two types of virtual age PM model. In the Kijima's type I model, it assumes that PMs serve only to remove damage created in the last sojourn, the virtual age at the start of working after PM is $v_k = t_{k-1} + \xi_k(t_k - t_{k-1})$, and in the Kijima's type II model, it assumes that the PM actions could remove all damage accumulated up to that point in time and virtual age can be expressed as $v_k = \xi_k(v_{k-1} + (t_k - t_{k-1}))$ where $\xi_k \in (0, 1)$ in both I and II models. Actually, the Kijima's type I model is similar to Malik's model, and type I and II models are both practical in different kinds of system and maintenance activity.

Lin et al. [13] introduced a hybrid PM model with combining the failure rate model and the age reduction model. The failure intensity $\lambda_k(t)$ after the k^{th} PM becomes to $a_k \lambda_{k-1}(b_k t_k + t)$, where t_k is the interval between $(k-1)^{\text{th}}$ and k^{th} PM activities.

Actually, in previous literature, adjustment factor and age reduction factor directly affect system's failure intensity when PM actions are performed and they represent the maintenance efficiency. Gasmil et al. [5] proposed a statistical method to estimate the maintenance efficiency according to failure data, and unknown parameters were estimated using the maximum likelihood estimate (MLE) method with 1% and 5% lower and upper s -confidence bounds. It is, however, impossible to obtain a fixed precise value unless sufficient data can be collected. Liu et al. [15] found that the uncertainty associate parameters estimation and model selection cannot ignored in decision-making, especially in the case of lack of sufficient data. Wu et al. [30] introduced random maintenance quality in both the failure rate model and the age reduction model respectively and in which adjustment factor A_i and age reduction factor α_i were considered as random variables respectively, then two maintenance policies model were discussed separately. It is obviously more realistic than previous models which treat parameters in maintenance models as fixed constants corresponding to operational time.

As an extension of Wu's model, we consider the hybrid PM model with random PM efficiency. It is of course more useful and applicable to practical analysis and modeling. The recursive relationship of failure intensity $\lambda_i(t)$ at time t , before the i^{th} PM can be expressed as follows:

$$\begin{aligned} \lambda_1(t) &= \lambda(t) \\ \lambda_2(t) &= \int_0^\infty A_1 \int_{-\infty}^{+\infty} \lambda_1(t + \alpha_1 T_{P1}) dF_1(\alpha_1) dG_1(A_1) \\ \lambda_3(t) &= \int_0^\infty A_2 \int_{-\infty}^{+\infty} \lambda_2(t + \alpha_2 T_{P2}) dF_2(\alpha_2) dG_2(A_2) \\ &\vdots \\ \lambda_i(t) &= \int_0^\infty A_{i-1} \int_{-\infty}^{+\infty} \lambda_{i-1}(t + \alpha_{i-1} T_{P(i-1)}) dF_{i-1}(\alpha_{i-1}) dG_{i-1}(A_{i-1}) \end{aligned}$$

where A_i is the adjustment factor and α_i is the age reduction factor of i^{th} PM activity, and they are both random quantities with distribution functions $G_i(A_i)$ and $F_i(\alpha_i)$ respectively. $T_{P(i-1)}$ represents the interval time between $(i-1)^{\text{th}}$ and i^{th} PM activities, and there are N PM cycles. The failure intensity function can be rewritten using iterative operation as:

$$\lambda_i(t) = \prod_{k=1}^{i-1} \left(\int_0^\infty A_k dG_k(A_k) \right) \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} \lambda \left(t + \sum_{j=1}^{j=i-1} (\alpha_j T_{Pj}) \right) dF_j(\alpha_{i-1}) \dots dF_1(\alpha_1) \quad (1)$$

$$\lambda_{N+1}(t) = \prod_{k=1}^N \left(\int_0^\infty A_k dG_k(A_k) \right) \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} \lambda \left(t + \sum_{j=1}^{j=i-1} (\alpha_j T_{Pj}) \right) dF_j(\alpha_{i-1}) \dots dF_1(\alpha_1) \quad (2)$$

It is worth noting that if $\int_0^\infty A_i \lambda_i(t) dG_i(A_i) < \lambda_i(t)$, then random variable A_i should satisfy $0 < \int_0^\infty A_i dG_i(A_i) < 1$, it means the slope of failure intensity function will decrease after PM actions. On the other hand, when $\int_0^\infty A_i \lambda_i(t) dG_i(A_i) > \lambda_i(t)$, $\int_0^\infty A_i dG_i(A_i) > 1$ should be satisfied, and the slope will increase. $\int_0^\infty A_i dG_i(A_i) = 1$ denoting no change to the slope after PMs. Meanwhile, the increment of the virtual age is $\int_{-\infty}^{+\infty} \alpha_{i-1} T_{P(i-1)} dF_{i-1}(\alpha_{i-1})$ after PM actions.

The system reliability in the i^{th} PM cycle can be expressed as:

$$R_i(t) = e^{-\int_0^{t+T_v} \lambda_i(t) dt} \quad (3)$$

where $T_v = \sum_{j=1}^{j=i-1} (\int_{-\infty}^{+\infty} \alpha_j T_{Pj} dF_j(\alpha_{i-1}))$.

The random maintenance efficiency could be more reasonable to meet realistic system requirements in practice due to many uncertainties in the field environments. Based on this random PM efficiency, a random sequential maintenance policy model will be discussed in the next section.

3. Sequential Maintenance Policy and Formulation

Optimal maintenance policies have been investigated in the past several decades with the purpose of providing maximum system reliability and/or availability and safety performance with the lowest maintenance costs and the highest profit per unit time. Barlow et al. [1] and Osaki et al. [20] proposed the basic age replacement model from the renewal reward theorem, and the expected cost per unit time in the steady state was discussed. Barlow et al. [2] studied block replacement model and compared it with age replacement model. The models extended from these two basic models were proposed in later literature [16, 30]. Furthermore, some models studied in recently

years are worth mentioning. Nakagawa [18] introduced two kinds of imperfect PM models and computed the optimal PM sequences for Weibull distribution. Policy N, based on the failure number of the system for multi-state repairable system was studied to maximize the long-run expected profit per unit time and geometric process had been employed by Zhang et al. [32]. Lam [8] studied a maintenance model for two-unit redundant system with one repairman, and the long-run average cost per unit time for each kind of replacement policy was derived. Satow et al. [24] represented a two-component system of which components suffer shock damage interaction, and the minimum expected cost per unit of time for infinite time operation was expressed and optimized. Zhou et al. [34] integrated sequential imperfect maintenance policy into condition-based predictive maintenance, and a reliability-centered predictive maintenance policy was proposed for a continuously monitored system subjected to degradation due to the imperfect maintenance. The preventive maintenance strategy has been applied to a vehicle fleet [19].

In this section, we consider such a maintenance policy that a system is suffering deterioration process with operation aging and the time for the system to be replaced by a new one in a finite time. PM activities need to be performed in replacement cycle in order to reduce the system deterioration [12] and restore it to a better state. According to the practical requirement and convenience, PM actions are usually scheduled at the weekend or leisure periods since such actions would not interrupt producing and working in these periods. During each PM cycle, failures may occur which will make the system breakdown, and minimal repairs will be done immediately to restore the system to working state. The possible replacement cycle is illustrated in Fig. 1. In this figure, there are N PM cycles in finite operational time T_o with the intervals T_{P_i} respectively, and T_{P_i} has different interval according to the system state, but must be in the leisure periods such as weekend and shut down time. This policy can be considered as "sequence maintenance in periodical leisure intervals". Failures are corrected by minimal repairs during each PM cycle.

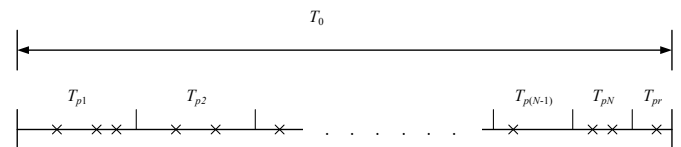


Fig. 1. Finite time replacement under PM policies

The hazard function in each PM cycle can be written as:

$$H_i(t) = \int_0^{T_{Pi}} \lambda_i(t) dt \quad (4)$$

where $\lambda_i(t)$ is the failure intensity function during the i^{th} PM cycle, and T_{P_i} is the pre-specified interval between i^{th} and $(i+1)^{\text{th}}$ PM actions. From Eq.(1), the hazard function is given by:

$$\begin{aligned} H_i(t) &= \int_0^{T_{Pi}} \lambda_i(t) dt \\ &= \int_0^{T_{Pi}} \left(\prod_{k=1}^{i-1} \left(\int_0^\infty A_k dG_k(A_k) \right) \right) \left(\int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} \lambda \left(t + \sum_{j=1}^{j=i-1} (\alpha_j T_{Pj}) \right) dF_j(\alpha_{i-1}) \dots dF_1(\alpha_1) \right) dt \end{aligned} \quad (5)$$

where $\bar{T}_P = [T_{P1}, T_{P2}, T_{P3}, \dots, T_{Pi}, \dots, T_{PN}]$ is a vector of PM sequential intervals, and N is the PM times.

After the last PM action, the hazard function between N^{th} PM and replacement is given by:

$$H_{N+1}(t) = \int_0^{T_0 - \sum_{k=1}^N T_{P1}} \left(\prod_{k=1}^N A_k dG_k(A_k) \right) \left(\int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} \lambda(t + \sum_{j=1}^{j=N} (\alpha_j T_{Pj})) dF_j(\alpha_{j-1}) \dots dF_1(\alpha_1) \right) dt \quad (6)$$

The expected total maintenance cost in one replacement cycle is given by:

$$C_{Total}(N, \bar{T}_P) = c_r \sum_{k=1}^{N+1} H_k(t) + Nc_p + c_{new} \\ = c_r \sum_{k=1}^{N+1} \int_0^{T_{P1}} \left(\prod_{k=1}^{k-1} A_k dG_k(A_k) \right) \left(\int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} \lambda(t + \sum_{j=1}^{j=k-1} (\alpha_j T_{Pj})) dF_j(\alpha_{j-1}) \dots dF_1(\alpha_1) \right) dt + Nc_p + c_{new} \quad (7)$$

where c_r , c_p and c_{new} are the minimal repair cost, preventive maintenance cost, and replacement cost respectively with $c_{new} > c_r > c_p$, and $C_{Total}(N, \bar{T}_P)$ denotes the expected total maintenance cost under N and \bar{T}_P policies. The optimal N^* and \bar{T}_P^* can be obtained by solving the optimization cost function $C_{Total}(N, \bar{T}_P)$. That is:

$$C_{Total}(N^*, \bar{T}_P^*) = \min C_{Total}(N, \bar{T}_P) \quad (8)$$

The existence of optimum N^* and \bar{T}_P^* is discussed as follows: Assuming that when $N \rightarrow 0$, there is no PM actions during the replacement cycle, then PM cost tends to be zero:

$$\lim_{N \rightarrow 0} C_{Total}(N, \bar{T}_P) = c_r H_1(T_0) + c_{new} = c_r \int_0^{T_0} \lambda(t) dt + c_{new} \quad (9)$$

when $N \rightarrow \infty$, PM can be regarded to be continuously performed, and then, the hazard rate could be considered as zero, and we can conclude:

$$\lim_{N \rightarrow \infty} C_{Total}(N, \bar{T}_P) = Nc_p + c_{new} \rightarrow \infty \quad (10)$$

If we arrange one PM in life time, the expected total maintenance cost could be reduced if and only if

$$c_r(H_0(T_0) - H_0(0)) + c_{new} < c_r(H_0(T_{P1}) - H_0(0)) + c_p + c_r(H_1(T_0 - T_{P1}) - H_1(0)) + c_{new} \quad (11)$$

then

$$c_r((H_0(T_0) - H_0(T_{P1})) - (H_1(T_0) - H_1(T_{P1}))) > c_p \quad (12)$$

$$c_r \left(\int_{T_{P1}}^{T_0} \lambda_0(t) dt - \int_{T_{P1}}^{T_0} \lambda_1(t) dt \right) > c_p \quad (13)$$

$$c_r \left(\int_{T_{P1}}^{T_0} \lambda_0(t) dt - \int_0^{T_0 - T_{P1}} \int_0^{+\infty} A_1 \lambda_0(t + \alpha_1 T_{P1}) dF_1(\alpha_1) dG_1(A_1) \right) > c_p \quad (14)$$

$$\int_{T_{P1}}^{T_0} \lambda_0(t) dt - \int_0^{T_0 - T_{P1}} \int_0^{+\infty} A_1 \lambda_0(t + \alpha_1 T_{P1}) dF_1(\alpha_1) dG_1(A_1) > \frac{c_p}{c_r} \quad (15)$$

$$\int_{T_{P1}}^{T_0} \left(\lambda_0(t) - \int_0^{+\infty} A_1 \lambda_0(t + (\alpha_1 - 1)T_{P1}) dF_1(\alpha_1) dG_1(A_1) \right) dt > \frac{c_p}{c_r} \quad (16)$$

As we know $c_p / c_r < 1$, then if $\lambda_0(t) - \int_0^{+\infty} A_1 \lambda_0(t + (\alpha_1 - 1)T_{P1}) dF_1(\alpha_1) dG_1(A_1)$ is less than zero during $[T_{P1}, T_0]$, there is no suitable T_{P1} that satisfies Eq. (16). When $\lambda_0(t) = c$ where c is a constant, and if $\int_0^{+\infty} A_1 dG_1(A_1) \geq 1$, it is obvious that in equation (16) is invalid, and PM action will increase the maintenance cost with no effect on the state of system. When $\lambda_0(t)$ is monotonous decrease with time, where $\int_0^{+\infty} A_1 dG_1(A_1) \geq 1$ and

$$\lambda_0(t) < \int_{-\infty}^{+\infty} \lambda_0(t + (\alpha_1 - 1)T_{P1}) dF_1(\alpha_1) \quad (17)$$

Then, PM action should also not to be performed. When

$$\lambda_0(t) > \int_{-\infty}^{+\infty} \lambda_0(t + (\alpha_1 - 1)T_{P1}) dF_1(\alpha_1) \quad (18)$$

there may be a suitable T_{P1} to obtain $C_{Total}(1, T_{P1}) < C_{Total}(0, 0)$. If $\lambda_0(t)$ is monotonous increasing and Eq.(18) is satisfied, there may exist some T_{P1} values that could reduce the expected maintenance cost.

It seems that the problem becomes even more complicated with the increase of N , and it is more difficult to obtain the optimum $\bar{T}_P^* = [T_{P1}^*, T_{P2}^*, \dots, T_{PN}^*]$. In the next section, we will use the genetic algorithm (GA) approach to solve the resulting optimization problem.

4. GA Optimization method

Numerous optimization methods have been used to solve the optimization problems and combinatorial optimization problems in reliability engineering. The most popular methods are dynamic programmings and heuristic search algorithms which are strongly problem-oriented. They are designed to solving certain problem and can not adapt to other problem.

The genetic algorithm (GA) is one of the most widely used evolutionary searching methods and it was inspired by the optimization procedure that exists in nature and biological phenomenon. The GA has become the popular universal tool for solving various optimization problems because of its advantage and successful applications of GA to maintenance optimization problems [10, 11]. The GA starts the optimization process from a random generated initial population. The fitness will be calculated for each individual. Then natural selection, crossover and mutation are operated in each population, and terminate criterion is used to determine whether to stop or to continue the GA process.

Solution encoding and decoding procedure must be defined before applying the GA to a specific problem. As we mentioned in section 3, PM action should only be performed in the leisure time such as the weekend, end of month or year. We use a fix length binary string to represent the time table where PM could takes place. The length of the binary string is given by:

$$L = \left\lceil \frac{T_0}{T_{\min}} + 1 \right\rceil \quad (19)$$

where T_o is finite operation/replacement time, T_{min} is minimal leisure interval when PM could be performed, and $\lceil \bullet \rceil$ is the least integer upper bound. If some bits of the string are equal to one, it means PM actions are performed in these leisure times. For example, $T_o = 1 \text{ year}$ and $T_{min} = 1 \text{ week}$ means PM can only perform in weekend and there are only 53 opportunities to do it. Then we use a binary string $s = [0010\dots00]$ with 53 bits to represent the variable to be optimized, where bit 1 means a PM activity should perform in the 2nd weekend after it be installed. Another example is given as: $T_o = 1 \text{ year}$ and $T_{min} = 1 \text{ month}$, and $s = [000100100101]$ represents that PM should be performed at end of the 3rd month, the 6th month, the 9th month and the 11th month. Then $N = 4$ and $\bar{T}_p = [3, 3, 3, 2]$ is a certain solution to maintenance problem.

After encoding the variables, crossover and mutation procedures are used to generate individuals of next population. Then GA continues process until distance of the individuals in each population is less than limit threshold d_{min} or populations are produced N_{rep} times. Finally, the individual with the minimal fitness could be considered as the global optimum result.

5. Case studies

5.1. Case 1: An illustrative example

We consider the failure distribution of a system follows a two-parameter Weibull distribution as:

$$f(t) = \frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta-1} \exp\left(-\left(\frac{t}{\eta}\right)^\beta\right) \quad (20)$$

where $\beta = 1.2$, $\eta = 300$ and its corresponding failure intensity function is given by:

$$\lambda(t) = \frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta-1} \quad (21)$$

where the failure intensity is monotonously increasing with time if there is no PM activity.

For convenience, we assume that $G_1(A_1), G_2(A_2), \dots, G_N(A_N)$ have the same uniform distribution $G(A)$ which is given by:

$$G(A) = \begin{cases} 0 & A < 0.90 \\ \frac{A - 0.90}{1.20 - 0.90} & 0.90 \leq A < 1.20 \\ 1 & 1.20 \leq A \end{cases} \quad (22)$$

and similarly, $F_1(\alpha_1), F_2(\alpha_2), \dots, F_N(\alpha_N)$ have the same uniform distribution $F(\alpha)$ which is given by:

$$F(\alpha) = \begin{cases} 0 & \alpha < 0.0 \\ \frac{\alpha}{0.6 - 0.0} & 0.0 \leq \alpha < 0.6 \\ 1 & 0.6 \leq \alpha \end{cases} \quad (23)$$

Without lose of generality, we assume $c_r = 400$, $c_p = 10$, $c_r = 200$, where c_r is much larger than c_p because sudden break-

down caused by random failure will lead to the loss of producing and serious delay to original plan. Beside that, it is suggested that the system replacement cycle is $T_o = 1 \text{ year}$, and PM activities should be arranged at the end of each month. Then, there are 12 possible opportunities to perform preventive maintenance. The decision variable s is a 12 bits binary string. We start GA process with initial population selected from feasible region from $[000000000000]$ to $[111111111111]$, two-point crossover method with rate 0.6 and mutation rate 0.05. Finally the optimum variable is $s^* = [000001000100]$ and the optimum PM sequence is $T_p^* = [T_{p1}^*, T_{p2}^*] = [5, 4]$, $N^* = 2$.

Table 1 also presents the sensitivity analysis for various shape parameter β of the failure distribution from 1.0 to 1.5 while the other parameters are fixed. From Table 1, we observe that when β equals to zero which means the failure intensity is constant, PM is not needed. This conclusion is consistent with analysis in section 3. Furthermore, when the β increases, the N^* becomes larger and the first PM activity is more close to launch time. It is because that a larger β denotes the system has more rapidly deterioration process, and the more random failure would happen during replacement cycle while repair cost increases swiftly. PM activity can decrease the large failure intensity back to certain level and reduce the possibility of random failure. Therefore, more PM activities is needed when β increases.

Table 1. Optimum PM sequence and expected maintenance cost when changing β

β	1.0	1.1	1.2	1.3	1.4	1.5
T_p^*	-	[8]	[5, 4]	[4, 3, 3, 1]	[3, 3, 3, 2]	[3, 3, 2, 2, 1]
C_{total}^*	1930.0	2391.0	2863.4	3370.8	3922.7	4539.8

Table 2 illustrates the optimum PM sequences and minimal expected maintenance costs while the scale parameter η of the system failure distribution changes. It shows that with the η decreasing, the less random failure would happen during the replacement cycle, therefore reducing the frequency of PM is necessary. Meanwhile, the expected cost will lower.

Table 2. Optimum PM Sequence and expected maintenance cost when changing η

η	$\eta = 1000.0$	$\eta = 400.0$	$\eta = 150.0$	$\eta = 10.0$
T_p^*	[4, 4]	[3, 3, 3]	[3, 3, 3, 2]	[3, 3, 2, 2, 1]
C_{total}^*	679.6	1970.0	2286.0	2606.4

The optimum T_p^* sequences are listed in Table 3 while changing the ratio of c_p / c_r , where the other parameters are fixed. It is shown that with the c_p / c_r decreasing, PM cost becomes cheaper, and more PM actions could be performed to lower failure intensity without increase too much preventive maintenance cost. When c_p / c_r equal to one, it shows no PM is needed. It can be explained that although PM can reduce the virtual age of system, it increase the slope of failure intensity. When c_p equals c_r , the extra PM cost is much more that random failure repair cost lowered by PM. Then no PM is more economical.

The optimum results by changing the distribution of α_i are listed in Table 4 and shown in Fig. 2. It indicates that more PM actions should be performed while the PM effectiveness increases, and the

Table 3. Different optimum PM Sequence by changing ratio of c_p / c_r

c_p / c_r	1/100	1/25	1/20	1/10	1/5	1/1
T_P^*	[5,4,2]	[5,4]	[5,4]	[5,4]	[7]	-

expected maintenance cost also decreases because of the higher effectual PM actions.

Table 4. T_P^* and C_{total}^* by changing α_i distribution as $U(0, x)$

x	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
T_P^*	[7]	[7]	[6,4]	[6,4]	[5,4]	[5,4]	[4,4,3]	[4,3,3]	[4,3,2,2]	[3,3,2,2,1]
C_{total}^*	2955.9	2940.0	2919.5	2893.6	2863.4	2830.0	2789.2	2736.7	2668.7	2573.6

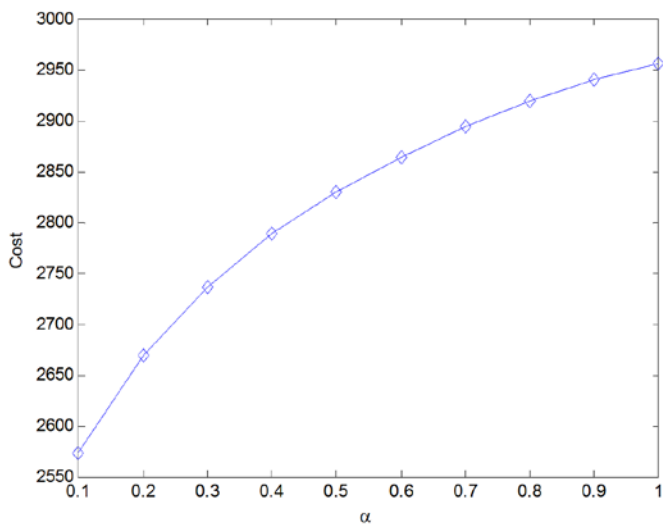


Fig. 2. Optimum cost with different α_i distribution

Table 5 and Table 6 show the optimal expected maintenance cost with periodical interval from 5 to 1 and optimal PM sequential policies and relative cost with fixed PM times respectively, and these comparisons are illustrated in Fig. 3.

As observed in Fig. 3, with the same PM times N , the sequential PM policy is much more economically efficiency than periodic

Table 5. Expected maintenance cost with periodic PM actions

Interval	5	4	3	2	1
T_P^*	[5,5]	[4,4]	[3,3,3]	[2,2,2,2,2]	[1,1,1,1,1,1,1,1,1,1]
C_{total}^*	2888.00	2873.1	2898.6	3002.6	3491.7

Table 6. Optimal PM sequence and expected maintenance cost with fixed PM times N

N	1	2	3	4	5
T_P^*	[7]	[5,4]	[5,4,2]	[4,4,2,1]	[4,3,2,1,1]
C_{total}^*	2886.2	2863.4	2869.7	2891.1	2926.1

one. With N increasing, the economic advantage of sequential policy is becoming dramatically. Therefore, in manufactory production, sequential policies have been widely accepted and applied because it is more reasonable and economical.

5.2. Case 2: Fuel injection pump

The purpose of the fuel injection pump is to deliver an exact metered amount of fuel, under high pressure, at the right time to the injector. It is one of the most important components of diesel engines.

The parameters of the RAM model listed in Table 7 are estimated through the methodology proposed in [5, 15, 25].

According to the system requirements, oil should be refresh every 5000 miles while the fuel injection pump could be censored and do some preventive maintenance.

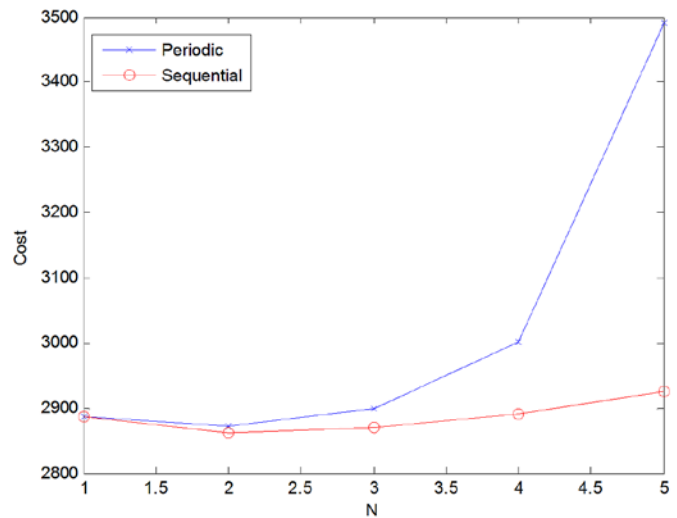


Fig. 3. Comparison between sequential and periodic PM

Table 7. The estimated parameters of fuel injection pump

	Mean	Std
$\hat{\beta}$	2.00	0.15
$\hat{\eta}$	20914.01	121.50
\hat{A}_i	1.05	0.02
$\hat{\alpha}_i$	0.50	0.10

Under the warranty period-50000 miles, the optimal PM policy is obtained based on our proposed models with parameters $\beta = 2.00$ $\eta = 20914.01$ $A_i \sim N(1.05, 0.02)$, $\alpha_i \sim N(0.50, 0.10)$, $c_r = \$18.75$ and $c_p = \$3.75$. The optimal PM sequences are [20000, 15000] miles with the minimal expected maintenance cost equal to \$87.5 .

6. Conclusions

In this paper, we consider the random maintenance features of imperfect PM. This is more reasonable to many practical applications because the efficiency of maintenance action is evaluated from statis-

tical failure data of repairable systems. It could be not precise and always have confidence intervals when estimating the unknown parameters of an imperfect maintenance model. The random degree hybrid imperfect maintenance model is proposed in this paper and a "sequential PM in periodic leisure interval" policy is proposed and solved by using the GA approach. A numerical example and a fuel injection pump are presented to illustrate and implement our proposed model. As in the numerical example, it shows how the expected maintenance cost and PM sequences change with respect to the settings of model parameters. In addition, the periodic and sequential maintenance poli-

cies are compared, and it concludes that a sequential policy is dramatically more economically efficiency than periodic policy with the PM times increasing. In the second case, a practical PM policy in diesel engine is discussed under the proposed models, and it is very useful to manufactories and enterprises to plan optimum maintenance strategy and warranty policy.

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