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Arch. Min. Sci. **65** (2020), 1, 19-33

Electronic version (in color) of this paper is available: http://mining.archives.pl

DOI 10.24425/ams.2020.132703

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STUDY ON THE MECHANICS AND MICRO/MACROECONOMICS OF MULTIPLE STRIP-SHAPED PILLAR RECOVERY

The structural system of a multiple strip-shaped pillar-roof is common in underground mine exploitation, and research on its mechanics and micro/macroeconomics is meaningful for utilizing strip-shaped pillar resources. A general model of the structural system of a multiple strip-shaped pillar-roof was established, the deformation mechanism of the model was analysed by material mechanics, and the deflection curve equations of the model were obtained. Based on the stress strain constitutive relation of the strip pillar and cusp catastrophe theory, the nonlinear dynamic instability mechanism of the structural system of a multiple strip-shaped pillar-roof was analysed, and the expressions of the pillar width for maintaining the stability of different types of structural systems were derived. The benefits of different structural systems were calculated using micro/macroeconomic theory, the type of the structural system was determined, and different recovery schemes were obtained. Theoretical application research was applied to a large manganese mine, and the results demonstrate that no pillar recovery was needed in 2016, a 9-m wide artificial pillar could be built to replace a pillar in 2017, and the construction of 14-m wide artificial pillars can be conducted in 2018.

Keywords: multiple strip-shaped pillars; recovery; mechanics; micro/macroeconomic

1. Introduction

Numerous structural systems consisting of a multiple strip-shaped pillar-roof remain after open stopping is used to exploit underground mineral resources (Wang, 1984). Research on the mechanical stability and economy of the structural system is significant for the safety of stope personnel, the size of the stope and the economic recovery of the pillars.

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In the study of a structural system's stability, Liu et al. (2008) established a mechanical model of the coal pillar roof structural system and the mechanical stability condition of the strip-shaped coal pillar. By combining the beam model in material mechanics and catastrophe theory, Zhang et al. (2011) analysed the mechanical structure of strip-shaped pillars in an underground mine. He et al. (2008) analysed the stability of the structural system of a multiple strip-shaped pillar-roof and its influencing factors. Tan et al. (2016) analysed the mechanical stability of narrow strip coal pillars in the new subtraction mining technology of strip-type Wongawill. The studies above considered the stability of the structural system of the ore pillars. However, a structural system containing both ore pillars and artificial pillars has not been considered.

In the micro/macroeconomic analysis of mining engineering, Yang et al. (2011) analysed the feasibility and economic benefits of replacing ore pillars with artificial pillars under different economic conditions. Yao (2001) proposed the concept of a "safety cost" and argued the economic optimization between safety cost and a safe work environment. Li (1994) analysed different mining economic system models and noted their applicable conditions. Deng et al. (2016) analysed the impact of economic factors on the overall situation of China's mining industry. In the existing research, scholars analysed different mining problems from the perspective of microeconomics and macroeconomics, but conclusions were not obtained that comprehensively combined micro- and macroeconomics.

The general model of a structural system containing a multiple strip-shaped pillar-roof was established, and the minimum width, which could maintain the stability of different types of structural systems, was determined using catastrophe theory. A method for determining the different types of structural systems during different periods was given by combining micro- and macroeconomics. Based on the economic factors of mining during different years, theoretical application research was developed, and different recovery schemes for multiple strip-shaped pillars during mine production were given.

2. General model of the structural system of a multiple strip-shaped pillar-roof and its deformation mechanism

2.1. Selection of the general model

A large manganese mining project was taken as a case study to determine the stability of the structural system, which contains either ore pillars or artificial pillars. Numerous structural systems of multiple strip-shaped pillar-roof are left in an underground mine, and the distribution of the pillars in the structural system is complicated. In order to study the stability of multiple strip-shaped pillar-roof structure system before and after replacing ore pillar with artificial pillar, some assumptions are made: the roof of the vacant area is elastic, stope bottom plate does not deform, the strip-shaped pillar and roof are geometrically regular, continuous and isotropic, and the working face is long. Then, the multiple strip-shaped pillar-roof structure systems are simplified as shown in Fig 1.

After recovering the strip-shaped pillars, there will be a combination of strip-shaped ore pillars and artificial pillars in the structural system. To fully study the interaction between the two adjacent bar pillars and the influence of strip-shaped pillar replacement on the structural system, the area in the virtual line in Fig. 1 was selected as the research object. The two strip-shaped

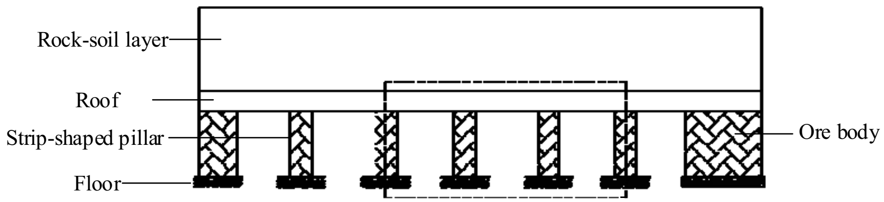

Fig. 1. Structural system of the multiple strip-shaped pillars and roof
1. rock soil layer 2. roof 3. strip-shaped pillar 4. floor 5. ore body

pillars at the boundary of the dotted line were regarded as bearings with an initial deflection of 0. Additionally, the stability of different types of structural systems (2 strip-shaped ore pillars, 2 strip-shaped artificial pillars and 1 ore pillar and 1 artificial pillar) were discussed. The stability of other structural systems in Fig. 1 can be regarded as the development and application of the research object in the dashed line under a changing external stress and initial conditions, so the research object has a certain generality.

2.2. Deformation mechanism of the general model

After selecting the general model, different types of structural systems should be analysed. First, the structural system of a multiple strip-shaped ore pillars-roof was analysed. Assuming that the deformation of the structural system is symmetrically bending, the pillar's support to the roof is regarded as a concentrated force. Its simplified beam model is shown in Fig. 2.

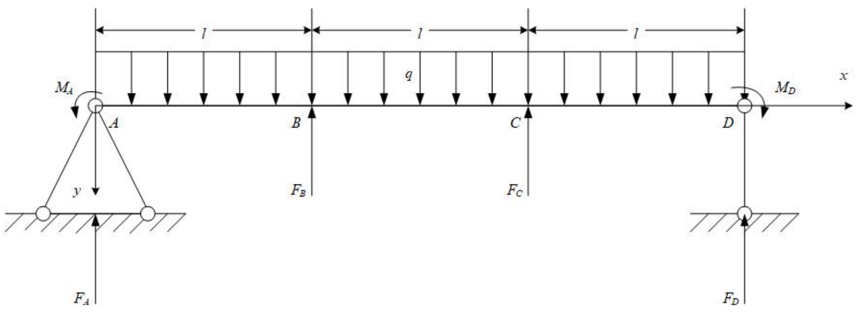

Fig. 2. Simplified beam model of the multiple strip-shaped ore pillars and roof

The basic statically determinate system is selected as a statically determinate simply supported beam, where the left end is a fixed hinge support, and the right end is a movable hinge support. Point "*A*" on the left side of the beam was taken as the origin of the coordinates, the axis of the beam was taken as the x-axis, and the axis moving downward through point "*A*" was taken as the y-axis. F_A, F_B, F_C and F_D are the supporting forces of the beam at both ends and the middle; M_A and M_D are the bending moments provided by the beam edge rock to restrain the torsion of the beam, where $F_A = F_D$, $F_B = F_C$, and $M_A = M_D$. The weight of the overlying strata and the rock beams is simplified as a uniformly distributed load of q, where l is the span of the stope. The following can be obtained using the superposition principle (Sun et al., 2009) and the boundary condition, $\theta_A = 0$.

$$M_A = \frac{3ql^2}{4} - \frac{2F_B l}{3} = M_D \tag{1}$$

According to the actual situation, we can see that $M_A = M_D > 0$, so the bending moment of the beam at "*A*" and "*D*" are the same as the assumed direction. The approximate equations for the deflection curves of each segment need to be separately listed due to the load discontinuity of the beam. The equation of deflection curve of "*AD*" in the beam section was obtained using boundary conditions ($\theta_A = 0$, $z_A = 0$, $\theta_D = 0$ and $z_D = 0$) and continuous conditions ($z_{BC(1)} = z_{AB(1)}$ and $\theta_{AB(1)} = \theta_{BC(1)}$).

$$z(x) = \begin{cases} \frac{1}{EI}\left(\frac{3ql^2}{8}x^2 - \frac{F_B l}{3}x^2 - \frac{ql}{4}x^3 + \frac{F_B}{6}x^3 + \frac{q}{24}x^4\right)(0 \le x \le l) \\ \frac{1}{EI}\left(\frac{F_B l^3}{6} - \frac{F_B l^2}{2}x + \frac{3ql^2}{8}x^2 + \frac{F_B l}{6}x^2 - \frac{ql}{4}x^3 + \frac{q}{24}x^4\right)(l \le x \le 2l) \\ \frac{1}{EI}\left(\frac{3F_B l^3}{2} - \frac{5F_B l^2}{2}x + \frac{3ql^2}{8}x^2 + \frac{7F_B l}{6}x^2 - \frac{F_B}{6}x^3 - \frac{ql}{4}x^3 + \frac{q}{24}x^4\right)(2l \le x \le 3l) \end{cases} \tag{2}$$

The deflections at "*B*" and "*C*" were assumed to be u, so the equation $z(x) = u$ was defined and brought into (2), so the relationship between u, q, and F_B is as follows.

$$ql = F_B + 6\frac{EI}{l^3}u \tag{3}$$

The following was obtained by bringing (3) into (2).

$$z(x) = \begin{cases} \frac{q}{24EI}x^4 - \left(\frac{u}{l^3} + \frac{ql}{12EI}\right)x^3 + \left(\frac{2u}{l^2} + \frac{ql^2}{24EI}\right)x^2 (0 \le x \le l) \\ \frac{q}{24EI}x^4 - \frac{ql}{4EI}x^3 + \left(-\frac{u}{l^2} + \frac{13ql^2}{24EI}\right)x^2 + \left(\frac{3u}{l} - \frac{ql^3}{2EI}\right)x + \frac{ql^4}{6EI} - u(l \le x \le 2l) \\ \frac{q}{24EI}x^4 + \left(\frac{u}{l^3} - \frac{5ql}{12EI}\right)x^3 + \left(-\frac{7u}{l^2} + \frac{37ql^2}{24EI}\right)x^2 + \left(\frac{15u}{l} - \frac{5ql^3}{2EI}\right)x + \frac{3ql^4}{2EI} - 9u(2l \le x \le 3l) \end{cases} \tag{4}$$

Therefore, the roof moves down to a certain degree before the construction of strip-shaped artificial pillars in a structural system of a multiple strip-shaped artificial pillar-roof. The roof will decrease to the maximum bending deflection, u_0, if there are no strip-shaped ore pillars in the structural system. The deflection was assumed to be u' when the roof was supported by strip-shaped artificial pillars. $u_1 = u_0 - u'$ was used to approximately indicate the compression of a strip-shaped artificial pillar because the roof subsidence was very small, and the maximum deflection was not reached before formation of the artificial pillars. Therefore, the approximate deflection curve equation of the beam, $z_1(x)$, was obtained as follows using material mechanics theory.

$$z_1(x)=\begin{cases}\dfrac{q}{24EI}x^4-\left(\dfrac{u'}{l^3}+\dfrac{ql}{12EI}\right)x^3+\left(\dfrac{2u'}{l^2}+\dfrac{ql^2}{24EI}\right)x^2\left(0\le x\le l\right)\\ \dfrac{q}{24EI}x^4-\dfrac{ql}{4EI}x^3+\left(-\dfrac{u'}{l^2}+\dfrac{13ql^2}{24EI}\right)x^2+\left(\dfrac{3u'}{l}-\dfrac{ql^3}{2EI}\right)x+\dfrac{ql^4}{6EI}-u'\left(l\le x\le 2l\right)\\ \dfrac{q}{24EI}x^4+\left(\dfrac{u'}{l^3}-\dfrac{5ql}{12EI}\right)x^3+\left(-\dfrac{7u'}{l^2}+\dfrac{37ql^2}{24EI}\right)x^2+\left(\dfrac{15u'}{l}-\dfrac{5ql^3}{2EI}\right)x+\dfrac{3ql^4}{2EI}-9u'\left(2l\le x\le 3l\right)\end{cases} \tag{5}$$

In the structural system of multiple types of strip-shaped pillars-roofs, the pillars in the structural system are the combination of a strip-shaped ore pillar and a strip-shaped artificial pillar. The deflections of the ore pillars and artificial pillars were separately assumed to be u_2 and u'_2. The approximate deflection curve equation of the beam, $z_2(x)$, was obtained as follows.

$$z_{2(x)}=\begin{cases}\dfrac{q}{24EI}x^4+\left(-\dfrac{ql}{12EI}-\dfrac{5u_2}{3l^3}+\dfrac{2u_2'}{3l^3}\right)x^3+\left(\dfrac{ql^2}{24EI}+\dfrac{8u_2}{3l^2}-\dfrac{2u_2'}{3l^2}\right)x^2\left(0\le x\le l\right)\\ \dfrac{q}{24EI}x^4+\left(-\dfrac{ql}{4EI}+\dfrac{u_2}{l^3}-\dfrac{u_2'}{l^3}\right)x^3+\left(\dfrac{13ql^2}{24EI}-\dfrac{16u_2}{3l^2}+\dfrac{13u_2'}{3l^2}\right)x^2+\left(-\dfrac{ql^3}{2EI}+\dfrac{8u_2}{l}-\dfrac{5u_2'}{l}\right)x\\ +\dfrac{ql^4}{6EI}-\dfrac{8}{3}u_2+\dfrac{5}{3}u_2'\left(l\le x\le 2l\right)\\ \dfrac{q}{24EI}x^4+\left(-\dfrac{5ql}{12EI}+\dfrac{2u_2}{3l^3}+\dfrac{u_2'}{3l^3}\right)x^3+\left(\dfrac{37ql^2}{24EI}-\dfrac{10u_2}{3l^2}-\dfrac{11u_2'}{3l^2}\right)x^2+\left(-\dfrac{5ql^3}{2EI}+\dfrac{2u_2}{l}+\dfrac{13u_2'}{l}\right)x\\ +\dfrac{3ql^4}{2EI}+6u_2-15u_2'\left(2l<x\le 3l\right)\end{cases} \tag{6}$$

According to the equation (6), for single type multiple strip-shaped pillar-roof structure system, the deflections of B and C are $u_2 = u'_2 = u$. The deflection at the middle point of the beam is the maximum deflection of the beam, that is $z_{\max} = \dfrac{ql^4}{384EI} + \dfrac{5}{4}u$, and the displacement diagram of the analyzed beam is shown in Fig. 3.

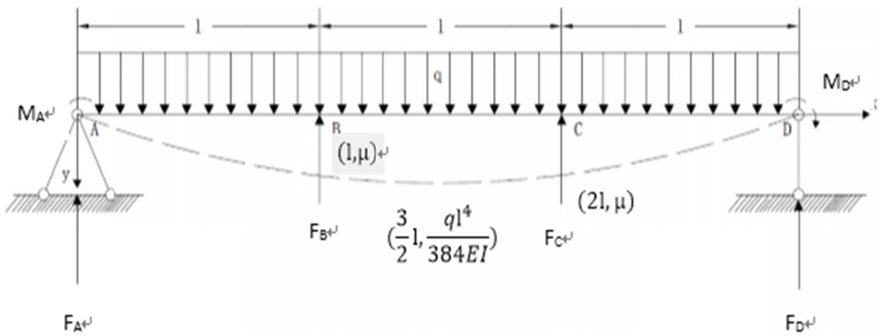

Fig. 3. Displacement diagram of the analyzed beam

3. Catastrophe theory analysis of a structural system of a multiple strip-shaped pillar-roof

3.1. Stress-strain constitutive relation of a strip-shaped pillar

The strip-shaped pillar is characterized by the development of initial defects under external forces because of internal fissures, cementing materials, and other factors (Chen et al., 2019). Under the action of high stress, the stress of the pillar will exceed the ultimate yield strength of the material, and the pillar will change from elastic to plastic. Therefore, the constitutive relationship between the stress and strain of the pillar is nonlinear. To simplify the analysis of the instability of the pillar, the Weibull distribution model can be used as the following (Qin et al., 1999):

$$\sigma = E_1 \varepsilon \exp\left[-(\varepsilon/\varepsilon_0)^m\right] \tag{7}$$

where E_1 is the modulus of elasticity of a pillar in the case of a small deformation; ε is the compression strain, $\varepsilon = u/h$, h is the height of the strip-shaped pillar, and u is the compression displacement of the strip-shaped pillar; ε_0 is the strain of the peak point in the stress-strain curve, $\varepsilon_0 = u_0/h$ and u_0 is the compression displacement of the peak point in the stress-strain curve; and m is a curve-shaped parameter describing the change of local strength, which is an evenness index. The stress-strain constitutive relation of the ore pillar can be better described when $m = 1$. Therefore, the relationship between the force of the ore pillar (whose width is d and length is h) and the amount of compression are as follows:

$$F(u) = kue^{(-u/u_0)} \tag{8}$$

$$k = E_1 d/h \tag{9}$$

where k is the initial stiffness of the strip-shaped ore pillar.

The influence of m on the stress-strain constitutive relation of the strip-shaped artificial pillar was considered because of the different m values of different materials, so the relationship between the force of the artificial pillar and the amount of compression is shown as follows:

$$F(u) = kue^{-(u/u_0)^m} \tag{10}$$

3.2. Cusp catastrophe theory

Catastrophe theory is a nonlinear science with a wide range of applications whose research contents include the sudden change of the state of the structural system caused by the continuous change in the parameters (Saunders, 1980; Zeeman, 1976). By applying the catastrophe theory, Wang (2016) analyzed the instability mechanism of temporary wall structure system and studied the optimization problem of wall thickness. Based on the catastrophe theory, Li (2014) discussed the formation mechanism of the crack plate structure in the roadway surrounding rock. Besides, some researchers have applied catastrophe theory and obtained ideal research results in the field of goaf stability. The above researches show that the catastrophe theory has a good application in the study of nonlinear instability of the structural system formed by underground mining. Therefore, the application of this theory is relatively mature. What's more, for the multiple strip-shaped pillar-roof structure system produced by underground mining, due to the action of ground stress, the energy stored in the structure system will be released to a certain extent, which has a certain paroxysmal. Therefore, the catastrophe theory can be applied to the nonlinear instability study of multiple strip-shaped pillar-roof structure system.

In catastrophe theory, there are 7 primary mutation models with no more than 4 variables (Thom, 1997), with the cusp model widely used. In the cusp model, the standard form of the potential function is the following:

$$V(w)=\frac{1}{4}w^4+\frac{1}{2}pw^2+qw \tag{11}$$

where p and q are control variables and w is the state variable of the structural system.

The equilibrium surface equation of a structural system can be derived from the potential function (the value of the potential function is 0) as follows.

$$V^{'}(w)=\frac{dV(w)}{dw}=w^3+pw+q=0 \tag{12}$$

The cusp catastrophe model is shown in Fig. 4. Fig. 4(a) is the balanced surface; Fig. 4(b) is the projection graph of the balanced surface on the plane pOq, which is referred to as the bifurcate set; Fig. 4(c) is the projection graph of the balanced surface on the plane wOq, and it shows an abrupt jump of the state variable x across the bifurcate set. a and b are the points corresponding to different states on the equilibrium surface; $a^{'}$ and $b^{'}$ are the projection points of a and b on the plane pOq; and c and d are the different state variables when the structural system suddenly changes.

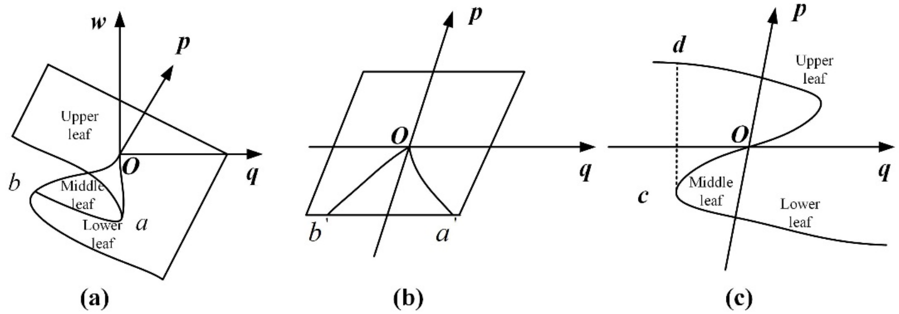

Fig. 4. Cusp catastrophe model

The singularity equation was obtained as follows by a second-order derivation of the potential function.

$$V^{''}(w)=\frac{d^2V(w)}{d^2w}=3w^2+p \tag{13}$$

The bifurcation set equation was obtained as follows by combining equations (12) and (13).

$$\Delta = 4p^3 + 27q^2 \tag{14}$$

If $\Delta < 0$, the structural system is in a state of instability; if $\Delta = 0$, a limit stable state is in the structural system; if $\Delta > 0$, the structural system is in a state of stability.

3.3. Instability mechanism of the structural system

The stability analysis of the three different structural systems of a multiple strip-shaped pillar-roof was performed using catastrophe theory. First, in the structural system of a multiple strip-shaped ore pillar-roof, the compression of the ore pillar, u, was assumed to be deflection of a beam, z. The potential energy of the structural system is as follows:

$$\Pi = U_e + U_s - W \tag{15}$$

where U_e is bending strain energy of the beam, U_s is the compression deformation energy of the ore pillars and W is the work of the uniformly distributed load.

The bending strain energies were separately listed as follows because of the different bending moments in the AB, BC and CD segments.

$$\begin{aligned} U_e &= \frac{1}{2} EI \int_0^{3l} \left[z''(x) \right]^2 dx \\ &= \frac{1}{2} EI \left\{ \int_0^{l} \left[z_1''(x) \right]^2 dx + \int_l^{2l} \left[z_2''(x) \right]^2 dx + \int_{2l}^{3l} \left[z_3''(x) \right]^2 dx \right\} \end{aligned} \tag{16}$$

$$U_s = 2k \int_0^u u e^{-u/u_0} du \tag{17}$$

$$W = \int_0^{3l} qz(x)dx = \int_0^{l} qz_1(x)dx + \int_l^{2l} qz_2(x)dx + \int_{2l}^{3l} qz_3(x)dx \tag{18}$$

Based on equations (4), (15), (16) and (18), the total energy of the structural system was obtained as follows:

$$\Pi = \frac{6EIu^2}{l^3} + \frac{l^5 q^2}{480EI} + 2k \int_0^u u e^{-u/u_0} du - 2qlu - \frac{ql^5}{240EI} \tag{19}$$

The compression u of the left ore pillar in the model was taken as the state variable. The equilibrium surface equation of the structural system can be obtained as follows:

$$\Pi' = \frac{12EIu}{l^3} + 2kue^{-\frac{u}{u_0}} - 2ql \tag{20}$$

According to the smooth properties of a surface in cusp catastrophe theory, the equation of the cusp point is as follows:

$$\Pi''' = e^{-\frac{u}{u_0}} \left(\frac{2ku}{u_0^2} - \frac{4k}{u_0} \right) = 0 \tag{21}$$

$u_1 = 2u_0$ was obtained in equation (21), and the cusp point is located at the inflection point of the constitutive relation curve of the strip-shaped ore pillar.

To obtain a standard form of equilibrium surface equation, equation (20) was expanded by 3rd-order Taylor formula at the cusp as follows.

$$\frac{24EIu_0}{l^3} + 4ku_0e^{-2} - 2ql + \left(\frac{12EI}{l^3} - 2ke^{-2}\right)(u - 2u_0) + \frac{ke^{-2}}{3u_0{}^2}(u - 2u_0)^3 = 0 \tag{22}$$

At the same time, the dimensionless variable y, $y = (u - u_1)/u_1$ is introduced. Therefore, a standard form of the equilibrium surface equation was obtained as follows.

$$y^3 + py + q = 0 \tag{23}$$

where

$$p = \frac{3}{2}(M - 1) \tag{24}$$

$$q = \frac{3}{2}(1 + M + N) \tag{25}$$

$$M = \frac{6EIe^2}{l^3k} = \frac{\frac{12EI}{l^3}}{2ke^{-2}} = \frac{k_1}{k_2} \tag{26}$$

$$N = \frac{qle^2}{2ku_0} \tag{27}$$

M is the stiffness ratio of the stiffness of the roof, k_1, to the equivalent stiffness of the structural system, k_2. N is a geometric parameter and is relevant to the distribution force, q, stope span, l, the initial stiffness of the ore pillar, k, and the deformation value of the inflection point in the constitutive relation of the ore pillar, u_0.

Fig. 4 shows that the structural system is unstable if $p \leq 0$. Thus, $M - 1 \leq 0$ is the necessary condition for the instability of the structural system.

The required width of strip-shaped ore pillars for construction in the structural system of a multiple strip-shaped ore pillar-roof, d_1, was obtained as follows using the necessary condition above.

$$d_1 \geq \frac{6EIh}{e^{-2}E_1l^3} \tag{28}$$

For the structural system of a multiple strip-shaped pillar-roof's stability analysis, the catastrophe theory is similar to the structural system of a multiple strip-shaped ore pillar-roof. Thus, the width of the strip-shaped artificial pillars required for the stability of the structural system, d_2, is as follows.

$$d_2 \geq \frac{6EIh}{\exp\left(-\frac{m+1}{m}\right)E_2l^3} \tag{29}$$

For the structural system of a multiple type strip-shaped pillar-roof, the best supporting effect is obtained when the width of the strip-shaped ore pillar is equal to the width of the strip-shaped artificial pillar. Therefore, the width of the required strip-shaped pillars, d_3, is as follows.

$$d_3 \geq \frac{12EIh}{l^3\left[E_1 e^{-2} + E_2 \exp\left(-\frac{m+1}{m}\right)\right]} \tag{30}$$

4. Micro/macroeconomic analysis of multiple strip-shaped pillar recovery

4.1. Microeconomic analysis of multiple strip-shaped pillar recovery

For a mine, the physical and mechanical properties of the overlying strata, such as γ, c, φ, E, and H, are identical. The physical and mechanical properties of strip-shaped ore pillars, c_1' and ϕ, the required width of the strip-shaped pillars for the stability of different structural systems, d_1, d_2 and d_3, and the density of ore before and after mining, ρ and ρ', respectively, are determined values. The ore recovery rate, K_t, the grade before and after ore mining, α_t and α_t', and the stacking cost of a unit volume of strip-shaped artificial pillar, q_t, were considered as determined values using the combination of motion and stasis. Combined with the price per ton of metal, P_t, the cost of unit quality mining, C_t, over period t and other known conditions of the mine in an econometric model (Li et al., 2001), the econometric model was concretely calculated, with t taken as 1 year for an expedient calculation.

The value of the unit volume ore before and after mining during period t was obtained as follows.

$$v_t = P_t \alpha_t \rho \tag{31}$$

$$v_t' = P_t \alpha_t' \rho' \tag{32}$$

Therefore, the microeconomic income c_1 of the structural system of a multiple strip-shaped ore pillar-roof is as follows:

$$c_1 = -2d_1 hLv_t \tag{33}$$

For the structural system of a multiple strip-shaped artificial pillar-roof, the ore pillars are mined, and artificial pillars are built to support the roof. Thus, the mining cost, construction cost, and other costs are produced (Liu et al., 2019). Therefore, the microeconomic income c_2 of the structural system of a multiple strip-shaped ore pillar-roof is as follows:

$$c_2 = 2d_2 hLv_t' - \left[2d_2 hL\left(C_t \rho + q_t\right) + F_t\right] \tag{34}$$

For the structural system of a multiple type strip-shaped pillar-roof, compared with the structural system of a multiple strip-shaped ore pillar-roof, only one strip-shaped ore pillar is replaced. Therefore, its microeconomic income c_3 is as follows:

$$c_3 = d_3 hLv_t' - d_3 hLv_t - \left[d_3 hL\left(C_t \rho + q_t\right) + F_t'\right] \tag{35}$$

4.2. Macroeconomic analysis of multiple strip-shaped pillar recovery

Macroeconomic evaluation is the analysis of the contribution of mineral resources development to the national economic development goals from the perspective of the national economy, based on financial evaluation. In the macroeconomic analysis of multiple strip-shaped pillars, the economic net present value (*ENPV*) was used, and its expression is as follows (Liu et al., 2009):

$$ENPV = \sum_{t=0}^{n} (E - F)_t (1 + i_s)^{-t} \tag{36}$$

where E is the national economic benefit flow in the process of pillar replacement; F is the national economic cost flow in the process of pillar replacement; $(E - F)_t$ is the net benefit flow of the national economy of pillar replacement; i_s is the social discount rate, which was taken as 8%; and n is the calculation period, which was taken as 1 year.

If $ENPV \geq 0$, the pillar replacement is practical; otherwise, it is impractical.

4.3. Determination of the type of different structural systems

In the process of multiple pillar recovery, 3 types of structural systems were identified. After maintaining the minimum width of the 3 types of structural systems, the micro/macroeconomy should be comprehensively considered to determine the remaining type. The 3 remaining types of structural systems were separately considered as scheme 1, scheme 2 and scheme 3. The integration determination method of the mechanics and micro/macroeconomy for the 3 schemes is shown in Fig. 5.

The reservation scheme of the structural systems can be determined in Fig. 5. Based on the minimum widths of different structural systems, different microeconomic incomes (c_1, c_2 and c_3) and different economic net present values ($ENPV_1$, $ENPV_2$ and $ENPV_3$) were obtained. The economic net present values of the 3 structural systems were compared with 0. The schemes whose economic net present value are greater than or equal to 0 can be used as the selected schemes. In schemes whose economy net present values are greater than 0, the scheme with a better microeconomic benefit ($\max\{c_1, c_0\}$, where c_0 is the microeconomic benefit excluding scheme 1) can be selected as the final scheme.

5. Engineering practice

The Daxin Manganese Mine (Dameng mine) is the largest manganese mining enterprise in China, and it is located in Xialei County, in the Guangxi Zhuang Autonomous Region. The structural parameters of the No. 51, 52 and 53 stopes in level +220 m and the mechanical parameters of the rock mass in the Dameng Mine are shown in Table 1. Three different structural systems of a multiple strip-shaped pillar-roof can be reserved, and their corresponding minimum widths are 5.84 m, 13.58 m and 8.16 m. In practical engineering applications, the values above were taken to be 6 m, 14 m and 9 m for convenience of calculation and construction.

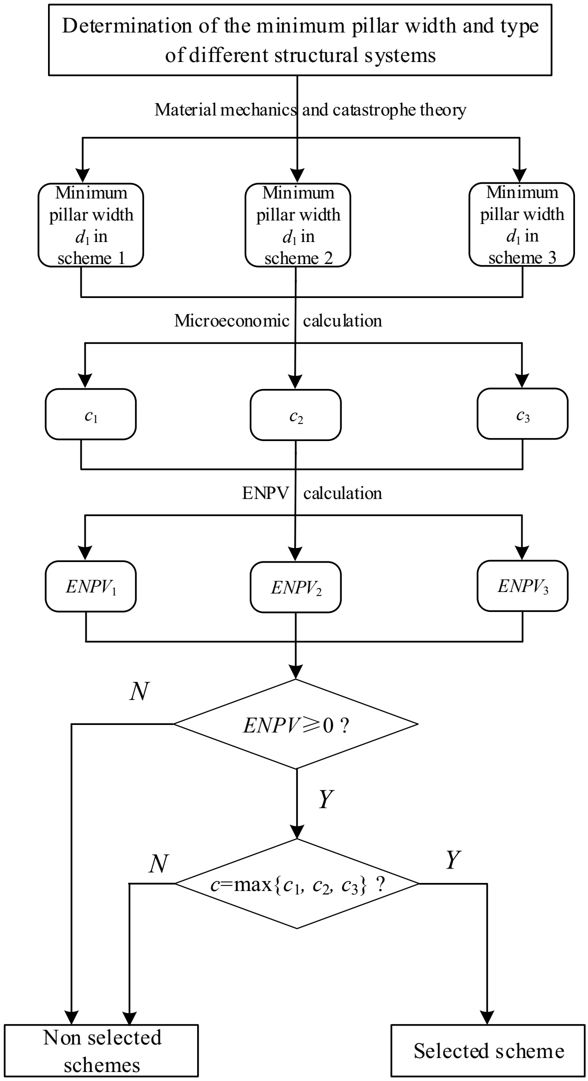

Fig. 5. Mechanical and micro-macroeconomic determination of the different schemes

TABLE 1

Structural parameters in level +220 m and mechanical parameters of the rock mass in the Dameng Mine

Elastic modulus of roof *E* (GPa)	**Roof thickness *h* (m)**	**Roof width *a* (m)**	**Elastic modulus of ore pillar E_1 (GPa)**	**Pillar height *H* (m)**	**Pillar width *d* (m)**	**Single chamber width *l* (m)**	**Length of ore room *L* (m)**
52.44	4	62	84.87	54	8	52	60

Based on the minimum widths, the microeconomic benefits and economic net present values of the different structural systems were calculated. To reflect the dynamic principle, the parameters of the mining economic factors in the last 3 years were selected, as shown in Table 2. The microeconomic benefits corresponding to different years of different structural systems were calculated using equations (31)-(35) and the data in Table 2. The calculation results are shown in Table 3. The economic net present values of the different structural systems during different years were calculated. The $ENPV_1$ of scheme 1 is zero because the macroeconomic benefits are not involved in scheme 1. The economic net present values of scheme 2 and 3 are shown in Table 4.

TABLE 2

Parameters of mining economic factors in the last 3 years

Parameter	**Unit**	**Mining economic factor**		
		2016	**2017**	**2018**
Average density ρ' before mining of pillar	t/m^3	2.6	2.6	2.6
Average density ρ'' after mining of pillar	t/m^3	3.9	3.9	3.9
Mining cost C	yuan/t	1395.3	1185.3	1285.3
Cost of building the artificial pillar q	yuan/m^3	1680	1630	1730
Sales price of manganese metal P	yuan/t	9500	10500	12500
Ore grade α	%	18.64	18.64	18.64
Mined-out ore grade α'	%	15.64	16.64	17.64
Ore recovery rate	%	64	65	70
Other expenses F	Million yuan	14000	10000	9000

TABLE 3

Microeconomic benefits of setting ore pillar and pillar replacement during different years (million yuan)

Calculation year	**Scheme 1**	**Scheme 2**	**Scheme 3**
2016	–159.79	–220.08	–167.88
2017	–197.84	–188.45	–157.63
2018	–235.53	85.09	–149.91

TABLE 4

Cost flow table of the national economic benefit of different schemes during different years (million yuan)

Serial number	**Item**	**Scheme 2**			**Scheme 3**		
		2016	**2017**	**2018**	**2016**	**2017**	**2018**
1	Benefit flow	322.84	428.99	616.10	168.14	218.15	245.53
1.1	Sales of ore	262.84	339.99	546.10	108.14	129.15	175.53
1.2	Indirect flow	60	89	70	60	89	70
2	Cost flow	482.02	428.05	460.81	155.27	137.99	148.59
2.1	Material cost	152.41	147.87	156.95	48.99	47.53	50.45
2.2	Mining cost	329.11	279.58	303.16	105.79	89.86	97.45
2.3	Other expenses	0.5	0.6	0.7	0.5	0.6	0.7
3	Net benefit flow	–159.17	0.94	155.29	12.87	80.16	96.94
4	*ENPV*	–147.38	0.87	143.79	11.91	74.22	89.76
Calculation cycle period n is considered to be 1 year.							

The results were obtained as follows, according to Table 3 and Table 4.

(1) In 2016, the economic net present values of scheme 1 and 3 were not less than 0 and they were all feasible schemes. The microeconomic benefit of scheme 3 was lower due to the mining technology and other management factors. Therefore, the structural system corresponding to scheme 1 could be retained with no ore pillars reclaimed.

(2) In 2017, the economic net present values of all 3 schemes were not less than 0, so they were all feasible schemes. The structural system corresponding to scheme 3 would enable the enterprises to have the least amount of loss, so scheme 3 was adopted. Only a single pillar could be recovered, and a 9-m wide artificial pillar was built simultaneously.

(3) In 2018, the economic net present values of all the 3 schemes were not less than 0, and they were all feasible schemes. The microeconomic benefit of scheme 2 was higher than the other schemes because of the improvement in ore price and mining technology. Scheme 2 could be retained with all the ore pillars reclaimed, and 14-m wide artificial pillars were built simultaneously.

6. Conclusions

(1) The general model of the structural system of a multiple strip-shaped pillar-roof was established. The deformation mechanisms of different structural systems were analysed, and the corresponding deflection curve functions were obtained.

(2) Based on the Weibull distribution model and cusp catastrophe theory, the catastrophic instability mechanisms of different structural systems were analysed, and the minimum widths of the strip-shaped pillars required to maintain the stability of different structural systems were obtained.

(3) Based on the minimum widths, the method of determining the different types of structural systems was given using micro/macroeconomic theories.

(4) The theory was applied to the structural system of the $51^{\#}$, $52^{\#}$ and $53^{\#}$ ore houses in the Dameng Mine. The results showed that to achieve the best micro/macroeconomic benefits, different types of structural systems and pillar recovery schemes should be determined due to the significant changes in the mining economics during various years.

Acknowledgements

This work was financially supported by the National Natural Science Foundation of China (Grant No. 41402306) and the Open Research Fund of the State Key Laboratory of Geomechanics and Geotechnical Engineering, Institute of Rock and Soil Mechanics, Chinese Academy of Sciences (Grant No. Z016015).

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