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DEVELOPMENT OF TURBOSHAFT ENGINE ADAPTIVE DYNAMIC MODEL: ANALYSIS OF ESTIMATION ERRORS

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Abstract
One of the most perspective directions of aircraft engine development is related to implementing adaptive automatic electronic control systems (ACS). The significant elements of these systems are algorithms of matching of mathematical models to actual performances of the engine. These adaptive models are used directly in control algorithms and are a combination of static and dynamic sub-models. This work considers the dynamic sub-models formation using the Least Square method (LSM) on a base of the engine parameters that are measured in-flight. While implementing this function in the (ACS), the problem of checking the sufficiency of the used information for ensuring the required precision of the model arises. We must do this checking a priori (to determine a set of operation modes, the shape of the engine test impact and volume of recorded information) and a posteriori. Equations of the engine models are considered. Relations are derived that determine the precision of parameters of these models' estimation depending on the precision of measurement, the composition of the engine power ratings, and durability of observations, at a stepwise change of fuel flow. We present these relations in non-dimensional coordinates that make them universal and ready for application to any turboshaft engine.

Keywords: turbine engine; turboshaft; gas generator; dynamic model; engine time constant; identification; estimation error; design of experiment
Type of the work: research article

1. INTRODUCTION

The mathematical model serves as a foundation of the automatic control system (ACS) during the design, tuning and operation stages, as well as in aircraft engines diagnostics [1]. Also, the gas turbine engine (GTE) as a complicated technical object includes itself with many components with highly individual properties. While ensuring the stable working process, these components also define the working parameters (rotational speeds, temperature, and pressure in gas path cross-sections, fuel and air consumption, thrust, power, etc.) by matching their characteristics, and these parameters are partially measured and then used for control [2].

However, the engine parameters and individual characteristics of the components are not identical for every engine unit, and also vary throughout engine operation. Such a fact imposes an obvious difficulty on the engineering application of high-quality control.

To accommodate this variety in the property and characteristic and provide necessary control quality, the adaption concept is necessary to be introduced into ACS [2–6]. One of the most promising development

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directions for adaptive ACS is to introduce a correction into the engine mathematical model that forms the underlying control algorithms [7–9]. This correction includes the model parameter specification, which best fits the collected experimental data from engine testing and operation. Among them, the automatic adaption available for building a self-adjusting ACS engine is the most efficient way [5,10–12].

The engine static model correction provides obedience with experimental data focusing on steady-state operating modes. Many works that are previously carried out [13–18] investigate such a model and propose algorithms for estimating model coefficients. Researches of various authors [6,11,12,19–22] propose approaches towards dynamic model corrections. But for their engineering realisation in automatic mode, the accuracy of the adaptive model has to be under control. Furthermore, the accuracy is not solely influenced by measuring error and logged data volume, but also due to the intensity of input control actuation.

In this work, we conducted research on the relationship of the error in the estimation of engine dynamic model coefficients regarding the main influencing factors, which are as follows:
– the intensity of input control actuation changes;
– amplitude of input control actuation changes;
– logging frequency;
– measurement errors;
– errors in the prior information used about the model.

The primary goal of this research is to formulate a methodical approach towards the error analysis for estimation of the dynamic model coefficients. Therefore, to develop a schematic approach we choose the simplest research object, a single-spool turbojet engine, whose dynamic property also corresponds with the gas generator of a single-spool turboshaft engine.

2. PROBLEM ANALYSIS

2.1. Mathematical models for turboshaft engine

The base mathematical model of the engine has been formed on characteristics of the engine components and thermodynamic equations that describe conditions of their common operation [23]. It considers the influence of the working fluid composition and the temperature on the fluid thermophysical properties, simulates dynamics of rotors, gas volumes of the gas path, heat exchange and tip clearances variation. The considered tasks of adaptive control can be solved, considering only rotor dynamics. However, even in this simplified case, the model is too complex and labour-intensive to be used in real-time control algorithms.

To develop a high-speed model, the base thermodynamic model is divided into two parts: static and dynamic. The static model calculates the engine parameters for the steady-state modes, and the dynamic model determines the deviations of the parameters from their static values. These deviations correspond to the transient modes. In the previously mentioned nomenclature, the static model has the form

$$\vec{Y}_{st} = f\left(\vec{U}, \vec{F}\right), \quad (1)$$

where $\vec{Y}_{st}$ are values of the parameters determined by the input $\vec{U}$ and $\vec{F}$ for a steady-state mode.

The dynamic model in the state space is expressed as

$$\dot{\vec{X}} = A\Delta\vec{X} + B\Delta\vec{U} + E\Delta\vec{F}, \quad (2)$$

$$\Delta\vec{Y} = C\Delta\vec{X} + D\Delta\vec{U} + G\Delta\vec{F}, \quad (3)$$

where $\dot{\vec{X}}$ are rotation speeds of rotors, $\vec{Y}$ are other engine parameters, $\vec{U}$ are control actions, Δ is a deviation from the static value.

Hence, the values of parameters for transient modes are determined as

$$\vec{X} = \vec{X}_{st} + \Delta\vec{X};\ \vec{Y} = \vec{Y}_{st} + \Delta\vec{Y}. \quad (4)$$

As an example, consider a turboshaft engine with a single-spool gas generator, whose scheme is shown in Fig. 1.

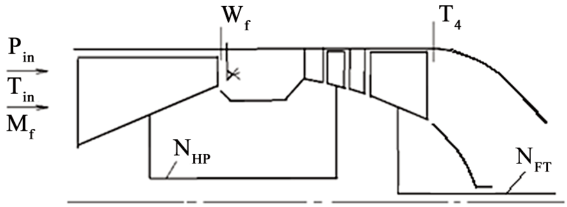

Figure 1. Diagram of turboshaft engine with a single-spool gas generator.

The input parameters are fuel flow W_f and parameters of the air at the intake including pressure P_{in}, temperature T_{in} and Mach number M_f. The output parameters are rotor speeds N_{HP}, N_{FT}, compressor discharge pressure P_3, turbine discharge temperature T_4 and output power Pow. Hence, for this engine

$$\vec{X} = \begin{bmatrix} N_{HP} \\ N_{FT} \end{bmatrix};\ \vec{Y} = \begin{bmatrix} P_3 \\ T_4 \\ Pow_{FT} \end{bmatrix};\ \vec{U} = \begin{bmatrix} W_f \\ Pow \end{bmatrix};\ \vec{F} = \begin{bmatrix} P_{in} \\ T_{in} \\ M_f \end{bmatrix}, \quad (5)$$

where Pow is the power of load (e.g. helicopter's rotor) and Pow_{FT} is the power of free turbine (output power of the engine).

The structure of the mathematical model, which implements Eqs (1)–(5) using the general method presented in [23], is shown in Fig. 2. Here, the following indexes are used: 'HP' – high pressure, 'FT' – free turbine, 'in' – inlet to the engine, 'f' – flight of fuel, 'st' – static, 'cor' – corrected.

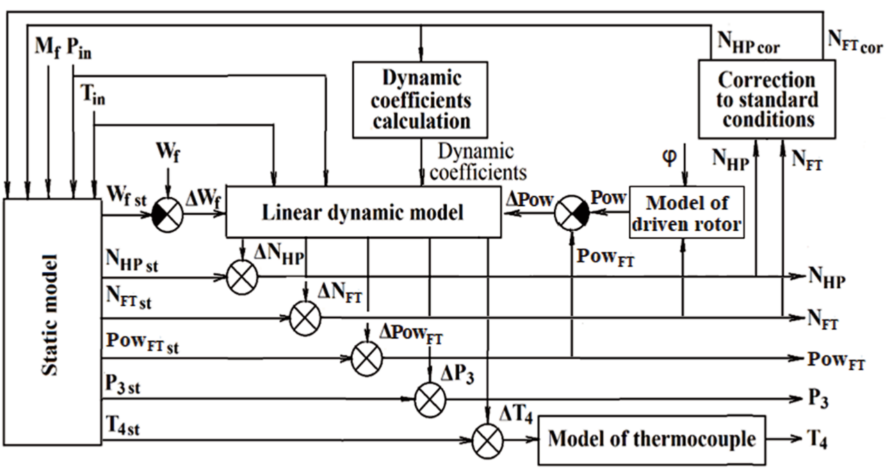

Figure 2. Structure of the engine dynamic model.

2.2. General task formulation of adaptive engine dynamic model

Taking the variables from (5) into consideration, the equations of systems (1) and (2) can be represented as:

$$\begin{bmatrix} \dot{N}_{HP} \\ \dot{N}_{FT} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \Delta N_{HP} \\ \Delta N_{FT} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \Delta W_f \\ \Delta Pow \end{bmatrix}; \tag{6}$$

$$\begin{bmatrix} Pow_{FT} \\ P_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix} \begin{bmatrix} \Delta N_{HP} \\ \Delta N_{FT} \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \\ d_{31} & d_{32} \end{bmatrix} \begin{bmatrix} \Delta W_f \\ \Delta Pow \end{bmatrix}, \tag{7}$$

and the dynamic model correction problem can be formulated as the problem of estimating the parameters of the mathematical model $\vec{\theta} = \left[a_{11} \ldots a_{22} b_{11} \ldots b_{22} c_{11} \ldots c_{32} d_{11} \ldots d_{32} \right]^T$ based on the known engine working process parameters $\vec{Z} = \left[\vec{X} \quad \vec{Y} \right]^T$ that are measured at time $t_1, \ldots, t_n$. Additionally, it is also necessary to solve the problem of experiment planning in general, that is, to determine the optimal control actions $W_f(t)$ and $Pow(t)$ for engine.

2.3. Illustrative example for the adaption problem solving

Consider the task of determining the coefficients of Eq. (1). Then the dynamic model of the engine in the vicinity of the base steady state is (index 'HP' is omitted)

$$\frac{dN}{dt} = a\Delta N + b\Delta W_f, \tag{8}$$

or

$$\tau \frac{dN}{dt} + \Delta N = K\Delta W_f, \tag{9}$$

where τ is time constant of the rotor and K is rotor speed gain over fuel input.

Place the estimation of coefficients in this equation on the measurement results of rotor speed $N(t)$ in a transient process, while the assigned action is a change in fuel consumption $W_f(t)$. Let the model parameter have the following actual values: τ = 1 s, K = 10 rpm/(kg/h).

The initial data are sets of values of the input parameter ΔW_{fj}, $j = 1, \ldots, n$, and the output parameter ΔN_j, $j = 1, \ldots, n$. These values are recorded at moments $t_1, \ldots, t_n$.

Let the fuel consumption be a temporal linear increase at a speed of v = 5 (kg/h)/s till the predetermined deviation W_0 = 10 kg/h, and then maintain as a constant (Fig. 3A):

$$\Delta W_f = \begin{cases} vt, vt < W_0; \\ W_0, vt \geq W_0. \end{cases} \tag{10}$$

Then the reaction of the rotor to this action looks like (Fig. 3B)

$$\Delta N = \begin{cases} Kv\left[t-\tau\left(1-e^{-\frac{t}{\tau}}\right)\right] = -\dfrac{bv}{a}\left[t+\dfrac{1}{a}\left(1-e^{at}\right)\right], vt < W_0; \\ Kv\left[t_0+\tau\left(e^{-\frac{t}{\tau}}-e^{-\frac{t-t_0}{\tau}}\right)\right] = -\dfrac{bv}{a}\left[t_0+\tau\left(e^{at}-e^{a(t-t_0)}\right)\right], vt \ge W_0. \end{cases} \quad (11)$$

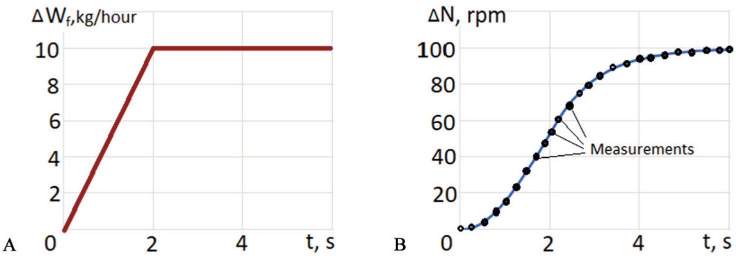

Figure 3. Example of control action (A) and response (B).

The least-square method (LSM) is applied to determine the coefficients [24]. In this way, the value estimated $\hat{\vec{\theta}} = [\hat{a} \quad \hat{b}]^T$ is determined as a result of minimising the LSM function. Due to the non-linear nature of the mathematical model (11), this minimisation problem is also non-linear. To obtain a numerical solution, it is necessary to use the iterative method, which essentially is to linearise the mathematical model with the parameters under searching at each i-th step and perform the correction of the estimate using: $\hat{\vec{\theta}}^i = \hat{\vec{\theta}}^{i-1} + \delta\hat{\vec{\theta}}^i$, where $\delta\hat{\vec{\theta}}^i = \left(H^{(i-1)^T} H^{i-1}\right)^{-1} H^{(i-1)^T}\delta N^i$ is the linear estimate of the corrective add-up; H^{i-1} is a (nx2) sensitivity matrix corresponding to the current estimates $\hat{\vec{\theta}}^{i-1}$; and δN^i is a (nx1) vector comprising deviations of the measured value from the values calculated using the mathematical model:

$$\vec{\theta} = \begin{bmatrix} a \\ b \end{bmatrix}; H^{i-1} = \begin{bmatrix} \left(\dfrac{\partial N}{\partial a}\right)_1^{i-1} & \left(\dfrac{\partial N}{\partial b}\right) \\ \dots & \dots \\ \left(\dfrac{\partial N}{\partial a}\right)_n^{i-1} & \left(\dfrac{\partial N}{\partial b}\right)_n^{i-1} \end{bmatrix}; \delta N^i = \begin{bmatrix} \Delta N_1 - \Delta N_{\text{mod}}\left(\vec{\theta}^{i-1}, t_1\right) \\ \Delta N_n - \Delta N_{\text{mod}}\left(\vec{\theta}^{i-1}, t_n\right) \end{bmatrix}$$

Errors in the estimates are determined by the covariance matrix [16]

$$P\left(\vec{\theta}\right) = \begin{bmatrix} \sigma_a^2 & \text{cov}(a,b) \\ \text{cov}(a,b) & \sigma_b^2 \end{bmatrix} = \sigma_N^2\left(H^T H\right)^{-1}, \quad (12)$$

where

$$\sigma_a^2 = \frac{B}{AB-C^2}; \sigma_b^2 = \frac{A}{AB-C^2}; A = \sum\nolimits_{j=1}^{N}\left(\frac{\partial N}{\partial a}\right)_j^2; B = \sum\nolimits_{j=1}^{n}\left(\frac{\partial N}{\partial b}\right)_j^2; C = \sum\nolimits_{j=1}^{n}\left(\frac{\partial N}{\partial a}\right)_j\left(\frac{\partial N}{\partial b}\right)_j; \quad (13)$$

and σ_N^2 is dispersion of the rotor speed measurement error.

From (11) we obtain the following:

$$\frac{\partial N}{\partial a}=\begin{cases}\frac{bv}{a^3}\left[at\left(1+e^{at}\right)+2\left(1-e^{at}\right)\right], vt<W_0;\\ \frac{bv}{a^3}\left[at_0\left(1+e^{a(t-t_0)}\right)+\left(at-2\right)e^{at}\left(1-e^{-at_0}\right)\right], vt\geq W_0.\end{cases}$$

$$\frac{\partial N}{\partial b}=\begin{cases}-\frac{v}{a}\left[t+\frac{1}{a}\left(1-e^{at}\right)\right], vt<W_0;\\ -\frac{v}{a}\left[t_0-\frac{1}{a}\left(e^{at}-e^{-a(t-t_0)}\right)\right], vt\geq W_0.\end{cases} \quad (14)$$

By applying linearisation to the mathematical model in the vicinity of the actual parameter value for estimation, the expression above allows us to analyse the errors in our estimates.

As an example, we present the analysis of error in the estimation of coefficient a. From (12), we obtain the following:

$$\sigma_a = f(t)\sigma_N, \text{where } f(t)=\frac{1}{\sigma_N}\frac{B}{AB-C^2}.$$

Figure 4 presents the function $f(t)$ of the example under investigation. The figure corresponds to the parameter a estimation under different circumstances while the measuring interval is set to 0.2 s. Curve 1 corresponds to the previously specified engine parameter, which is τ = 1 s, K = 10 rpm/(kg/h); meanwhile, Curve 2 corresponds to the setting that τ = 10 s, K = 10 rpm/(kg/h). In the third case, the engine parameters are τ = 1 s, K = 10 rpm/(kg/h), though the circumstance for estimation is that the value of gain K is known.

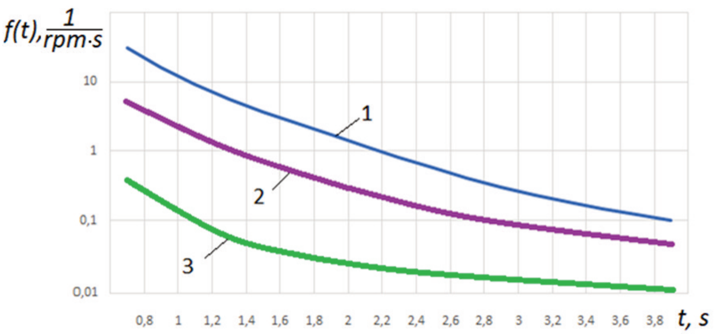

Figure 4. The function $f(t)$, which relates the error in the coefficient estimate a to the error in the rotor speed measurement.

The example shows that errors of estimates significantly depend on many influencing factors: actual values of the coefficients to be found (which according to physical sense depend on operation conditions), errors of measurements (which include not only rotation speed but also fuel flow), the shape of control input (in the above-considered example—amplitude and velocity of fuel flow rate variation), frequency of measurements, errors of the used a priori model and so on.

Therefore, this analysis aims to form a method of the engine dynamic parameters estimation errors prediction, depending on the influencing factors. This method will be key for automatic adaptation algorithm implementation in the engine ACS.

3. METHOD FOR ANALYSING ERRORS IN MATHEMATICAL MODEL PARAMETER ESTIMATION

To formulate the desired method, we investigate a simplified task, which is to determine the engine time constant τ using model (9) with a step change W_0 in the fuel consumption. As shown in the analysis, gain K behaves like a static engine parameter instead of a dynamic one, since it characterises the steepness of the static characteristic; in other words, the dependence of rotor speed on the fuel consumption in the installed operating modes of this engine.

The transient characteristics of the engine under this specified action are

$$\Delta N(t) = KW_0\left(1 - e^{-\frac{t}{\tau}}\right) \tag{15}$$

Factors that influence the total error of time constant estimation are measurement errors: σ_N (rotation speed) and σ_W (fuel flow); errors in the model structure: σ_K (error of parameter K—parametric error) and ΔS (error because of supposition about the constancy of coefficients K and τ; recording time interval Δt.

Furthermore, we suppose that the time interval t is chosen so that it excludes the influence of autocorrelation of the signal, and the step change of fuel flow W_0 is chosen so that it minimises the influence of the model's non-linearity.

Then instrumental error of the estimate is a sum of partial errors, and dispersion of the estimate is a sum of partial dispersions:

$$\Delta\tau = (\Delta\tau)_N + (\Delta\tau)_W + (\Delta\tau)_K\,;\sigma_\tau^2 = \sigma_{\tau N}^2 + \sigma_{\tau W}^2 + \sigma_{\tau K}^2.$$

The least square method can be applied for this case:

$$\Phi(\tau) = \sum_{j=1}^{n}\left[\Delta N_j - KW_0\left(1 - e^{-\frac{t_j}{\tau}}\right)\right]^2 = \sum_{j=1}^{n}(\delta N)_j^2 = \sum_{j=1}^{n}\left(\frac{\partial N}{\partial \tau}\right)_j^2 \Delta\tau^2,$$

where $\Delta\tau^2 = \dfrac{\sum_{j=1}^{n}(\delta N)_j^2}{\sum_{j=1}^{n}\left(\frac{\partial N}{\partial \tau}\right)_j^2}$. Then the covariance of the estimates is $\sigma_{\tau N}^2 = \dfrac{\sigma_N^2}{\sum_{j=1}^{n}\left(\frac{\partial N}{\partial \tau}\right)_j^2}$.

Convert the dispersion expression of the estimate to a continuous form, assuming that the measurements are made uniformly over time with an interval t:

$$\sigma_{\tau N}^2 = \frac{\sigma_N^2 \Delta t}{\sum_{j=1}^{n}\left(\frac{\partial N}{\partial \tau}\right)_j^2 \Delta t} \approx \frac{\sigma_N^2 \Delta t}{\int_0^{t_n}\left(\frac{\partial N}{\partial \tau}\right)^2 dt}, \tag{16}$$

where t_n − observation time.

Taking into account (15), we convert (16) into the following form:

$$\frac{\partial N}{\partial \tau} = -KW_0 e^{-\frac{t}{\tau}} \frac{t}{\tau^2}; \quad \int_0^{t_n} \left(\frac{\partial N}{\partial \tau}\right)^2 dt = -\frac{K_2 W_0^2}{2\tau} e^{-\frac{2t_n}{\tau}} \left[\left(\frac{t_n}{\tau}\right)^2 + \frac{t_n}{\tau} + \frac{1}{2}\right];$$

$$\sigma_{\tau N}^2 = \frac{2\sigma_N^2 \tau \Delta t}{K^2 W_0^2} \frac{1}{\frac{1}{2} - e^{-\frac{2t_n}{\tau}} \left[\left(\frac{t_n}{\tau}\right)^2 + \frac{t_n}{\tau} + \frac{1}{2}\right]}. \quad (17)$$

Analytical expression (17) builds a connection from the errors in time constant estimate to the main affecting factors, which are errors in rotor speed measurement, observation time, measuring frequency, as well as the property engine itself. Here, the engine property refers to the time-dependent partial derivative that relates rotor speed over time constant (this derivative also depends on the form of input actuation, which is the characteristics of fuel consumption change).

To get a universal characteristics expression for errors, we introduce the following dimensionless variables:

$$\bar{N} = \frac{N}{K_0 W_0}; \bar{t} = \frac{t}{\tau_0}; \Delta\bar{t} = \frac{\Delta t}{\tau_0},$$

where τ_0 is the actual value of engine time constant and K_0 is the actual value of gain. Therefore, we have

$$\sigma_{\tau N}^2 = \frac{\sigma_{\tau N}^2}{\tau_0} = \sigma_{\bar{N}}^2 \cdot \Delta\bar{t} \cdot f_N\left(\bar{t}_n\right), \text{ where } f_N\left(\bar{t}_n\right) = \frac{2}{\frac{1}{2} - e^{-2\bar{t}_n}\left(\bar{t}_n^2 + \bar{t}_n + \frac{1}{2}\right)} \quad (18)$$

Function $f_N\left(\bar{t}_n\right)$ is drawn in Figure 5. It is obvious that the function value decreases as the time for measurement prolongs, while its value does not converge to zero, but to a finite value equal to 4. This indicates that after the end of the transient process and the stabilisation of rotational speed, the new measurements do not contain information that contributes to determining the actual value of the desired parameter.

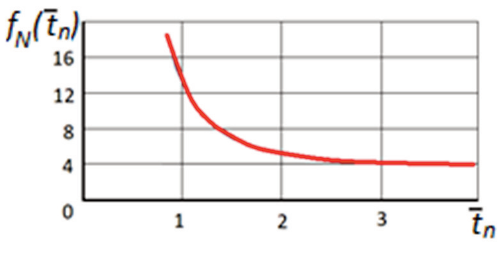

Figure 5. Function, which presents the influence of the error in rotational speed on the engine time constant estimation error.

Using the derived Eq. (18) for the specific dispersion and this diagram, we can estimate the error for any engine, time interval, the intensity of step action and error of measurement. This model of error is also a basis for the design of an experiment for the engine time constant estimation (solving the inverse task of simulation).

This characteristic for the step response may be considered as the minimum possible error, because reaction of rotor on another action will be slower, which increases the error.

4. ANALYSIS OF THE TOTAL ERROR IN THE ENGINE TIME CONSTANT ESTIMATION

We rewrite the LSM expression and substitute into it the model equation, and then linearise regarding all the parameters that contain errors (they are rotor speed, fuel consumption and a priori gain value. The a priori gain value can also be considered as a random quantity, as it is determined on the experimental data basis that originates from the static characteristic of the engine):

$$\Phi(\Delta\tau)=\sum_{j=1}^{n}\left[N_j-KW\left(1-e^{-\frac{t}{\tau}}\right)\right]^2=\sum_{j=1}^{n}\left[\delta N_j-\frac{\partial N}{\partial\tau}\Delta\tau-\frac{\partial N}{\partial W}\delta W-\frac{\partial N}{\partial K}\Delta K\right]^2=$$

$$=\sum_{j=1}^{n}\left[\delta N_j+K_0W_0e^{-\frac{t_j}{\tau_0}}\frac{t_j}{\tau_0^2}\Delta\tau-K\left(1-e^{-\frac{t_j}{\tau_0}}\right)\delta W-W_0\left(1-e^{-\frac{t_j}{\tau_0}}\right)\Delta K\right]^2. \tag{19}$$

From the condition of minimising the LSM function, we derive:

$$\frac{\partial\Phi(\Delta\tau)}{\partial\Delta\tau}=2\sum_{j=1}^{n}\left[\delta N_j+K_0W_0e^{-\frac{t_j}{\tau_0}}\frac{t_j}{\tau_0^2}\Delta\tau-\left(1-e^{-\frac{t_j}{\tau_0}}\right)\left(K_0\delta W-W_0\Delta K\right)\right]K_0W_0e^{-\frac{t_j}{\tau_0}}\frac{t_j}{\tau_0^2}=0,$$

Then we obtain the expression for estimates and its variance:

$$\Delta\hat{\tau}=-\frac{\tau_0^2}{K_0W_0}\frac{\sum_{j=1}^{n}\left[\delta N_j-\left(1-e^{-\frac{t_j}{\tau_0}}\right)\left(K_0\delta W-W_0\Delta K\right)\right]t_je^{-\frac{t_j}{\tau_0}}}{\sum_{j=1}^{n}t_j^2e^{-\frac{2t_j}{\tau_0}}}; \tag{20}$$

$$\sigma_\tau^2=\frac{\tau_0^4}{K_0^2W_0^2}\frac{\sum_{j=1}^{n}\left[\sigma_N^2+\left(1-e^{-\frac{t_j}{\tau_0}}\right)\left(K_0^2\sigma_W^2+W_0^2\sigma_K^2\right)\right]t_j^2e^{-\frac{2t_j}{\tau_0}}}{\sum_{j=1}^{n}t_j^2e^{-\frac{2t_j}{\tau_0}}}. \tag{21}$$

The influence of rotor speed measurement is analysed above. Now we analyse the errors in the fuel consumption measurement and the given value of gain.

By converting the sum into integrals in a similar way as (16) and performing integration, we obtain:

$$\sigma_{\tau W}^2=\frac{\tau_0^4K_0^2\sigma_W^2}{K_0^2W_0^2}\frac{\sum_{j=1}^{n}\left(1-e^{-\frac{t_j}{\tau_0}}\right)^2t_j^2e^{-\frac{2t_j}{\tau_0}}}{\left(\sum_{j=1}^{n}t_j^2e^{-\frac{2t_j}{\tau_0}}\right)^2}\approx\frac{\tau_0^4K_0^2\sigma_W^2}{K_0^2W_0^2}\frac{\int_0^{t_n}\left(1-e^{-\frac{t_j}{\tau_0}}\right)^2t_j^2e^{-\frac{2t_j}{\tau_0}}dt}{\left(\int_0^{t_n}t_j^2e^{-\frac{2t_j}{\tau_0}}dt\right)^2}=$$

$$= \frac{\tau_0^2\sigma_W^2\Delta t}{W_0^2}\frac{\frac{115}{216}-\frac{1}{2}e^{-\frac{2t_j}{\tau_0}}\left[\left(\frac{t_n}{\tau}\right)^2+\frac{t_n}{\tau}+\frac{1}{2}\right]+\frac{8}{3}e^{-\frac{3t_j}{\tau_0}}\left[\left(\frac{t_n}{\tau}\right)^2+\frac{2t_n}{3\tau}+\frac{2}{9}\right]-e^{-\frac{4t_j}{\tau_0}}\left[\left(\frac{t_n}{\tau}\right)^2+\frac{t_n}{2\tau}+\frac{1}{8}\right]}{\left\{\frac{1}{2}-e^{-\frac{2t_j}{\tau_0}}\left[\left(\frac{t_n}{\tau}\right)^2+\frac{t_n}{\tau}+\frac{1}{2}\right]\right\}}.$$

The universal error characteristic in dimensionless coordinates is

$$\bar{\sigma}_{\tau W}^2 = \frac{\sigma_{\tau W}^2}{\tau_0^2} = \frac{\sigma_{\bar{W}}^2\Delta\bar{t}}{\tau_0^2}f_1(\bar{t}_n),$$

$$\text{where } f_1(\bar{t}_n) = \frac{\frac{115}{216}-2e^{-2\bar{t}_n}\left[\bar{t}_n^2+\bar{t}_n+\frac{1}{2}\right]+\frac{8}{3}e^{-3\bar{t}_n}\left[\bar{t}_n^2+\frac{2}{3}\bar{t}_n+\frac{2}{9}\right]-e^{-4\bar{t}_n}\left[\bar{t}_n^2+\frac{1}{2}\bar{t}_n+\frac{1}{8}\right]}{\left[\frac{1}{2}-e^{-2\bar{t}}\left(\bar{t}_n^2+\bar{t}_n+\frac{1}{2}\right)\right]^2}. \quad (22)$$

Similarly, using (20), we get an expression for the estimation error because of the error in the gain setting:

$$\sigma_{\tau K}^2 = \frac{\sigma_K^2}{\tau_0^2} = \frac{\sigma_{\bar{K}}^2\Delta\bar{t}}{\tau_0^2}f_1(\bar{t}_n),$$

The function graph $f_1(\bar{t}_n)$ is shown in Fig. 6. From Eq. (22) we observe that, with an increase in the observation time, additional measurements do not decrease the total error, and the function itself tends not to zero, but to a finite value equal to $\frac{115}{216} \approx 0.532$.

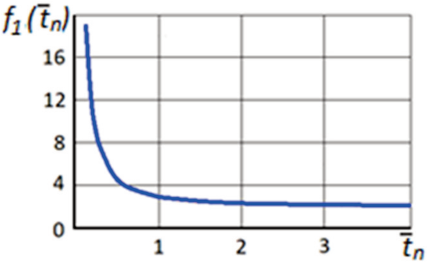

Figure 6. Function that represents the influence of the errors in the fuel consumption measurement and the gain setting on the engine time constant estimation error.

CONCLUSION

The adaptive control systems heavily require to be capable of automatic self-tuning that cannot be achieved without monitoring the static and dynamic parameters of each one engine. To make the adaptation process more accurate, it needs to account for the errors in the measurements and in the estimated parameters. For better estimation of the non-measured parameters, the adaptation method should address the accurate estimation of measurement errors as well as the analysis of a priori information on those errors.

It was proven that parametric estimation errors significantly (many times) depend on multiple factors, such as true values of parameters to be estimated, errors of measurements and a priori information about the model. This is a thing from at least two perspectives. The first one is to keep the error within an acceptable range and the second is from a perspective of the experiment design.

Having researched on the body of the problem, we reached out the generalised approach to the analysis of errors of GTE model adaptation, the main concepts of which are listed below:

1. Linear approximation of the engine dynamic model in the area of true values of parameters to be found.
2. Considering the influence of all measuring errors and parametric errors of the model (measuring errors of rotation speed and fuel flow, error of sensitivity coefficient), shape and parameters of the control action, the time interval between observations.
3. The influencing errors are independent random normally distributed values.
4. Application of specific parameters and the corresponding transformation of equations, which relate estimation error with influencing factors, makes the equations universal, applicable for any engine parameters and any influencing actions.
5. Instrumental errors of the engine time constant estimates have a down limit; this is explained by a finite time of the transient process, after which the output parameters are stabilised and are not sensitive to the engine dynamic parameters (e.g. the time constant). These errors correspond to a durable time interval of recording, when the new data do not give useful information about the time constant, because the transient process has been completed.
6. The results of the step response analysis show the minimum possible error of estimates.

Following this approach, we can determine and save in any convenient view (analytical, table, etc.) the generalised performance of the model parameters estimation, which is the same for any engine, its operating conditions and all influencing factors.

The obtained diagrams may be applied for adjusting the function of the automatic adaptation of the engine on-board model in a composition of the ACS. They may also be used for the design of experiment for the engine dynamic model experimental verification and validation.

The proposed method can be considered as an instrument of engine diagnostic and authors have a plan to make improvements to it based on the following directions:

1. Estimation of dynamic models of other parameters of the engine (temperature, pressure, etc.).
2. The analysis of two-spool and three-spool engines, taking into account correlations of estimates.
3. Analysis of non-linear effects, which in the first order are determined by a dependence of the engine dynamic coefficients on the operation conditions.

The applicability and efficiency of this method are demonstrated using the single-spool turbojet engine as the example.

Thus, this paper proposes and shows a method for predicting errors in estimating the parameters of dynamic models of GTEs. For a single-shaft gas generator, a universal equation is obtained that determines the error in estimating the coefficients of a linear dynamic model as a function of all the main influencing factors: measurement errors and frequency, test exposure intensity, and nominal values of the model coefficients. In particular, at a nominal value of the time constant of 1 s and a fuel supply gain of 10, measurement errors of the rotor speed of 0.2%, fuel consumption of 0.5%, an abrupt change in the fuel supply of 10 kg/s and a registration frequency of 5 Hz, the minimum standard deviation of the estimation error the time constant will be 0.2 s, that is, 20%. This significant level of error requires careful test planning and continuous verification while application of the proposed method as processing is performed.

References

[1] Jaw, L. and Mattingly, J. "*Aircraft Engine Controls: Design, System Analysis, and Health Monitoring.*" American Institute of Aeronautics and Astronautics, Inc., Reston, USA (2009): p. 378

[2] Wang, J., Zhang, W., and Hu, Z. "*Model-Based Nonlinear Control of Aeroengines*." Springer Nature Singapore Pvt. Ltd (2022): p. 238.

[3] Yepifanov, S. "Aircraft Turbine Engine Automatic Control Based on Adaptive Dynamic Models." *Transactions on Aerospace Research* Vol. 4, No. 261 (2020): pp. 61–70.

[4] Ibrahem, I.M.A., Akhrif, O., Moustapha, H., and Staniszewski, M. "Nonlinear Generalized Predictive Controller Based on Ensemble of NARX Models for Industrial Gas Turbine Engine." *Energy* Vol. 230, No. 1 (2021): p. 120700, 14.

[5] Kim, S. "A New Performance Adaptation Method for Aero Gas Turbine Engines Based on Large Amounts of Measured Data." *Energy* Vol. 221 (2021): p. 119863, 15.

[6] Wei, Z., Zhang, S., Jafari, S., and Nikolaidis, T. "Self-Enhancing Model-Based Control for Active Transient Protection and Thrust Response Improvement of Gas Turbine Aero-Engines." *Energy* Vol. 242 (2022): p. 123030, 17.

[7] Lietzau, K. and Kreiner, A. "Model Based Control Concepts for Jet Engines." ASME Paper 2001-GT-0016. (2001): p. 8.

[8] Panov, V. "Model-Based Control and Diagnostic Techniques for Operational Improvements of Gas Turbine Engines." *10th European Turbomachinery Conference*: p. 8. Lappeenranta, Finland (2013).

[9] Guicherd, R. "Distributed Model-Based Control of Turbine Engines." PhD Thesis. University of Sheffield. (2018): p. 212.

[10] Rutkovskii, V.Y., Ilyasov, B.G., and Kabalnov, Y.S. "*Adaptive Control Systems of Aircraft Gas Turbine Engines.*" Moscow Aviation Institute, Moscow (1995): p. 224.

[11] Wei, Z., Zhang, S., Jafari, S., and Nikolaidis, T. "Gas Turbine Aero-Engines Real Time On-Board Modelling: A Review, Research Challenges, and Exploring the Future." *Progress in Aerospace Sciences* Vol. 121 (2020): p. 100693, 16.

[12] Kim, S., Kim, K., and Son, C. "A New Transient Performance Adaptation Method for an Aero Gas Turbine Engine." *Energy* Vol. 193 (2020): p. 116752, 10.

[13] Visser, W.P.J., Kogenhop, O., and Oostveen, M. "A Generic Approach for Gas Turbine Adaptive Modeling". ASME Journal of Engineering for Gas Turbines and Power, vol. 128 GTP-04-1039 (2004).

[14] Mavris, D. and Denney, R. "Optimal Robust Matching of Engine Models to Test Data." Technical Report No. AFRL-SR-AR-TR-09-0119. Aerospace Systems Design Lab., School of Aerospace Eng., Georgia Institute of Technology, Atlanta, GA (2009): p. 52.

[15] Roth, B., Doel, D.L., Mavris, D., and Beeson, D. "High-Accuracy Matching of Engine Performance Models to Test Data." *Proceedings of ASME Turbo Expo 2003: Power for Land, Sea and Air*: GT2003–G38784, Atlanta, Georgia, USA, June 16–19, (2003): p. 9.

[16] Zwingenberg, M., Benra, F.K., Werner, K., and Dobrzynski, B. "Generation of Turbine Maps Using a Fusion of Validated Operational Data and Streamline Curvature Method." *Proceedings of the ASME Turbo Expo 2010: Power for Land, Sea, and Air. Volume 7: Turbomachinery, Parts A, B, and C:* pp. 2757–2768. Glasgow, UK, June 14–18, (2010).

[17] Xu, M., Liu, J., Li, M., Geng, J., Wu, Y., and Song, Z. "Improved Hybrid Modeling Method with Input and Output Self-Tuning for Gas Turbine Engine." *Energy* Vol. 238 (2022): pp. 121672, 19.

[18] Xu, M., Wang, J., Liu, J., Li, M., Geng, J., Wu, Y., and Song, Z. "An Improved Hybrid Modeling Method Based on Extreme Learning Machine for Gas Turbine Engine." *Aerospace Science and Technology* Vol. 107 (2020): p. 106333, 13.

[19] Breikin, T., Arkov, V., and Kulikov, G. "Regularization Approach for Real-time Modelling of Aero Gas Turbines." *Control Engineering Practice* Vol. 12, No. 4: pp. 401–407.

[20] Lyantsev, O., Kazantsev, A., and Abdulnagimov, A. "Identification Method for Nonlinear Dynamic Models of Gas Turbine Engines on Acceleration Mode." *Procedia Engineering* Vol. 176 (2017): pp. 409–415.

[21] Lu, F., Ye, Y., and Huang, J. "Gas Turbine Engine Identification Based on a Bank of Self-Tuning Wiener Models using Fast Kernel Extreme Learning Machine." *Energies* Vol. 10 (2017): p. 1363, 17.

[22] Hulgren, L. "Full-Scale Turbofan-Engine Turbine-Transfer Function Determination using Three Internal

Sensors." Technical Report No. NASA/TM–2012-217252. (2011): p. 13.

[23] Yepifanov, S., Kuznetsov, B., Bogayenko, I. Grabovskii, G., Dyukov, V., Kuzmenko, S., Ryumshyn N., Sametskii, A. "*Synthesis of Turbine Engine Control and Diagnostics Systems.*" Technical Publishing, Kiev, Ukraine (1998): p. 312.

[24] Hansen, C., Pereyra, V., and Scherer, G. "*Least Squares Data Fitting with Applications.*" Johns Hopkins University Press, Baltimore, Maryland, USA (2013): p. 328.