Vibrations in Physical Systems Vol.26 (2014)

Effect of Pad Offset on the Stability of Jeffcott Rotor Operating in Tilting 4-Pad Journal Bearings

Stanislaw STRZELECKI, Zdzisław SOCHA
Research Development Institute of Textile Machinery "POLMATEX-CENARO"
Wolczańska 55/591/15, 90-924 Lodz, Poland
stanislaw.strzelecki@p.lodz.pl, mariusz_krajewski@wp.pl

Zygmunt TOWAREK, Sergiusz ZAKRZEWSKI
Lodz University of Technology, Stefanowskiego 1/15, 90-924 Łódź, Poland
zygtow@p.lodz.pl, sergiusz.zakrzewski@p.lodz.pl

Abstract

Tilting-pad journal bearings are applied in high-speed rotor systems. Excellent stability properties allow obtaining the reliable, vibration free, operation of bearings and rotor. There is still need for better knowledge of such bearings static and dynamic characteristics. In case of tilting-pad journal bearings the stability can be evaluated by the system damping. The system damping gives the damping reserve of the bearings-rotor system and can be obtained by the solution of basic hydrodynamic and rotor motion equations. In this paper the hydrodynamic equations were solved by means of finite difference method. As result, the dynamic characteristics of bearings in form of stiffness and damping coefficients were obtained. These coefficients were the basis for the determination of the system damping.

Keywords: tilting-pad journal bearings, stability of rotor

1. Introduction

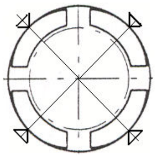
The journal bearing systems of modern high speed rotating machinery apply widely the radial tilting-pad journal bearings [1-6]. For such bearings the determination of both static and dynamic characteristics of system rotor-bearings, critical speeds, response of system on the dynamic load, stability of rotor and system damping, are very essential.

Titling-pad pivot (pad support) can be positioned centrally or shifted from the centre of pad angular length. Angular position of pad pivot has an effect on the bearing static and dynamic characteristics. At assumed load applied to the bearing there are different bearing characteristics for the case of centrally pivoted pad or for the pivot offset, e.g. 0.6 to 0.7. The shifting of pivot from the central position to at least 55 percent position leads to a decrease of the maximum temperature of about 15^0C [1]. Klumpp [3] obtained an increase in the values of oil film pressure at the increase of pivot-offset to 0.7.

The representative of tilting-pad journal bearings is the bearing with 4-pads. Some performances and applications of this bearing are showed in Table 1.

In case of tilting-pad journal bearings there is no stability limit [3-5]. It means that the stability properties of these bearings cannot be evaluated by method used for the multilobe or cylindrical bearings [1]. In case of tilting pad bearing the magnitude of stability reserve for the point of bearing operation in the range of stability is important. The value of damping determines how fast the vibrations decline after the disturbance of static position of operation [1-5].

Table 1 Performances of centrally pivoted, tilting 4-pad (4-PT) journal bearing

Type of bearing	Peripheral speed m/s	Unit load MPa	Sommerfeld number	Stiffness, Damping	Costs	Application
	30…100	0…2,5 (3,0)	0…1,0	oooo, oooo	oooo	Gear trains Steam turbines One shaft compressors

oooo – very high (.) maximum values

Calculation of system damping consists in the application of the characteristic equation of the system rotor-bearings. But quite different as in case of the calculation of limiting speed, the damping is determined from characteristic equation at different angular speeds [1].

The paper presents the results of the theoretical investigation into the stability of Jeffcot rotor (symmetrically supported one mass rotor) operating in tilting 4-pad journal bearings at adiabatic, laminar oil film. The Reynolds', energy, viscosity and geometry equations determine the oil film pressure, temperature distributions, oil film resultant force that are the starting point for the calculations of dynamic characteristics of bearings and stability of rotor. Perturbation method was applied for the calculation of stiffness and damping coefficients of oil film. Stability limit was determined on the basis of system damping [5-8].

2. Oil film pressure and temperature distributions

The oil film pressure, temperature and viscosity distributions have been determined by means geometry, Reynolds, energy and viscosity equations [9-11]. The geometry of tilting 4-pad journal bearings and the system of coordinates show Fig. 1; the pads can be arranged in such a way that the applied load goes between the bottom pads (Fig. 1a) or is directed on the bottom pad (Fig. 1b).

The geometry of lubrication gap besides of the pad relative clearance ψ_s and the angle τ_0 of lobe centre line at stationary state is decided by angular orientation τ_1 centre point of pad. During determination of static equilibrium position of pad, the angle τ_1 should be varied as long as the magnitude of lubricating gap allows obtaining the oil film pressure distribution, which gives the resultant force to go through the support point of pad [3,9].

Geometry of lubricating gap of tilting pad journal bearing determines Eqn. (1).

$$\overline{H}(\varphi) = \psi_s + \frac{\psi_s - 1}{\cos(\tau_1 - \tau_o)} \cdot \cos(\varphi - \tau_1) - \varepsilon \cdot \cos(\varphi - \alpha) \tag{1}$$

where: ψ_s-pad relative clearance, φ - peripheral coordinate, α - attitude angle, ε - relative eccentricity of journal.

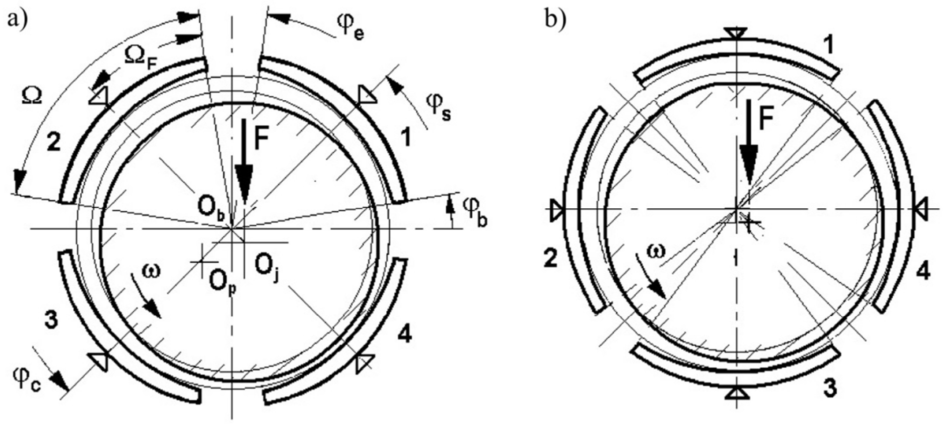


Figure 1. Geometry of tilting 4-pad journal bearings with centrally pivoted pads; a) – load between pads, b) – load on pad; φ_b, φ_c, φ_e, φ_s - angle of pad: begin, centre, end and support, respectively, $\Omega_F\ /\ \Omega$ - pivot offset, $\Omega_F\ /\ \Omega = (\varphi_{s1}-\varphi_{b1})/(\varphi_{e1}-\varphi_{b1})$

Reynold's equation applied in this paper has the form:

$$\frac{\partial}{\partial\varphi}\left(\frac{\overline{H}^3}{\overline{\eta}}\frac{\partial\overline{p}}{\partial\varphi}\right)+\frac{\partial}{\partial\overline{z}}\left(\frac{\overline{H}^3}{\overline{\eta}}\frac{\partial\overline{p}}{\partial\overline{z}}\right)=6\frac{\partial\overline{H}}{\partial\varphi}+12\frac{\partial\overline{H}}{\partial\phi} \tag{2}$$

where: $\overline{H}$ = h/(R-r) - dimensionless oil film thickness, h - oil film thickness (m), $\overline{p}$ - dimensionless oil film pressure, $\overline{p}=p\psi^2/(\eta\ \omega)$, p - oil film pressure (MPa), L - bearing length (m), D-sleeve diameter (m), R, r – sleeve and journal radius (m), $\overline{z}$ – dimensionless axial co-ordinate, $\overline{\eta}$ - dimensionless viscosity, ΔR – radial clearance, ΔR=R-r (m), $\phi=\omega\, t$ - dimensionless time, t – time, ψ - bearing relative clearance , ψ =ΔR/r (‰), ω - angular velocity (sec^{-1}).

It has been assumed for the pressure region that on the bearing edges the oil film pressure $\overline{p}(\varphi\ ,\ \overline{z})$= 0 and in the regions of negative pressure, $\overline{p}(\ \varphi\ ,\ \overline{z}\ )$ =0. The oil film pressure distribution computed from Eqn. (2) was introduced in the transformed energy equation [9]. Temperature and viscosity distribution were found by the iterative solution of equations (1), (2) and energy one [9-11].

3. Stability of Jeffcot rotor

The equations of motion for the journal and the centre of elastic shaft are given in matrix form by Eqn. (3). All the stiffness and damping coefficients were calculated by means of perturbation method [1,6].

The motion of simple symmetric rotor can be described by the following equation [2]:

$$M \cdot \ddot{x} + B \cdot \dot{x} + C \cdot x = \hat{a}\cos\omega t + \hat{b}\sin\omega t \tag{3}$$

where: M, B, C –matrices of mass, damping and stiffness, $\hat{a}, \hat{b}$ - coefficients of dynamic constraints.

After transformations of equation (3) the real and imaginary part was obtained [2,3]. The stability of elastic rotor-bearing system is analysed based on the following characteristic frequency equation of 6-th order with regard to (λ/ω) [2-6].

$$c_6(\lambda/\omega)^6 + c_5(\lambda/\omega)^5 + c_4(\lambda/\omega)^4 + c_3(\lambda/\omega)^3 + c_2(\lambda/\omega)^2 + c_1(\lambda/\omega) + c_0 = 0 \tag{4}$$

Solution assumption for Eqn. (4) is: $\lambda = -u + iv$, where: $u=1/(\omega T_z)$ and $v=\omega_e/\omega_{cr}$ with ω_e as self-frequency. The coefficients c_0 through c_6 in Eqn. (4) give the set of equations (5):

$$\begin{gathered} c_0 = A_0; \quad c_1 = A_1; \quad c_2 = A_2 + a_0(2A_0 + b_0 A_4); \quad c_3 = a_0(2A_1 + b_0 A_3) \\ c_4 = 2a_0 A_2 + a_0^2(b_0^2 + A_0 + b_0 \cdot A_4); \quad c_5 = a_0^2(A_1 + b_0 \cdot A_3); \quad c_6 = a_0^2 A_2 \end{gathered} \tag{5}$$

where: a_0 – ratio of angular velocity to the angular self-frequency of stiff shaft, $a_0 = \omega^2/\omega_{cr}^2$, b_0 – ratio of Sommerfeld number to the relative elasticity of shaft, $b_0 = So/c_s$, c^* – shaft stiffness, (Nm^{-1}), c_s – relative elasticity of shaft, $c_s = f/\Delta R = g/(\omega_{cr}^2 \cdot \Delta R)$, f- static deflection of shaft, (m), F.- resultant force of oil film (N), F_{stat} – static load of bearing, (N), g – acceleration of gravity, (ms^{-2}), m – mass of the rotor, (kg), So – Sommerfeld number, $So = F \cdot \psi^2 / (L \cdot D \cdot \eta \cdot \omega)$, S_{ok} – critical Sommerfeld number, $S_{ok} = So\ \omega/\omega_{cr}$, ω_{cr} – angular self frequency of stiff rotor, $\omega_{cr} = \sqrt{c^*/m}$.

The terms A_0, A_1, A_2, A_3, A_4, consist the stiffness g_{ik} (i=1,2 and k=1,2) and damping b_{ik} (i=1,2 and k=1,2) coefficients and they have the following meaning:

$$\begin{gathered} A_0 = g_{11} \cdot b_{22} - g_{12} \cdot b_{21}; \quad A_1 = g_{11} \cdot b_{22} + g_{22} \cdot b_{11} - g_{12} \cdot b_{21} - g_{21} \cdot b_{12} \\ A_2 = b_{11} \cdot b_{22} - b_{12} \cdot b_{21}; \quad A_3 = b_{11} + b_{22}; \quad A_4 = g_{11} + g_{22} \end{gathered} \tag{6}$$

where: g_{ik} - dimensionless stiffness coefficients, $g_{ik} = So(\Delta R/F_{stat}) \cdot g'_{ik}$, - stiffness coefficients, (N/m), b_{ik}- dimensionless damping coefficients, $b_{ik} = So(\Delta R/F_{stat})\omega \cdot b'_{ik}$, b'_{ik} - damping coefficients, (N sec/ m),

In case of bearing with the tilting pads interesting is not the absolute stability limiting speed but the magnitude of stability reserve (Fig. 2) for the point of bearing operation in the range of stability. Glienicke [1] investigated into the reserve of stability based on the damping of free vibration and introduced the term “System-damping”. The physical meaning of this term is shown in Fig. 2. The value of damping u determines how fast the vibrations decline after the disturbance of the static position of operation. It gives the magnitude of the stabilizing action of oil film with regard to the destabilizing effect of outside effects.

The damping is determined from characteristic equation (4) at different angular speeds ω. The explanation according to Fig. 2 has such advantage that the system damping u can be obtained as the function of revolutions or as the limiting speed (u = 0).

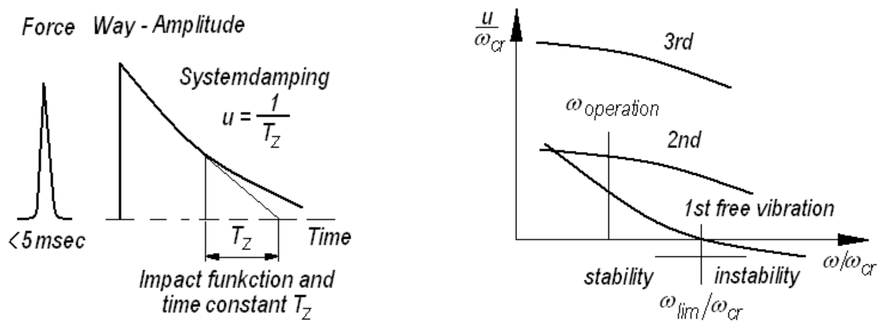


Figure 2. Damping of system according to Glienicke [1]; T_z – time constant

4. Results of calculations

The tilting 4-pad journal bearings of the length to diameter ratio L/D=0.5, the bearing relative clearance ψ =2.0‰ and ψ =1.16‰, relative clearance of the pad ψ_S = 2 and the pivot offset Ω_F/Ω =0.5 and Ω_F/Ω =0.6 (Fig. 1) were taken into consideration. Calculations of dynamic characteristics were carried out for the bearings with the laminar oil film for the range of relative eccentricities ε = 0,1 to ε = 0,8. Vertical direction of load was assumed. The values of thermal coefficients were assumed as K_T = 0,114 and K_T = 0,712 at the oil feeding temperature T_0=30^{0}C and T_0=50^{0}C, respectively and rotational speed 30000 rpm.

Some results of calculations of journal displacements ε versus Sommerfeld number So, the stiffness g_{ik} and damping b_{ik} coefficients as well as the system-damping are shown in Fig. 3 through Fig. 7.

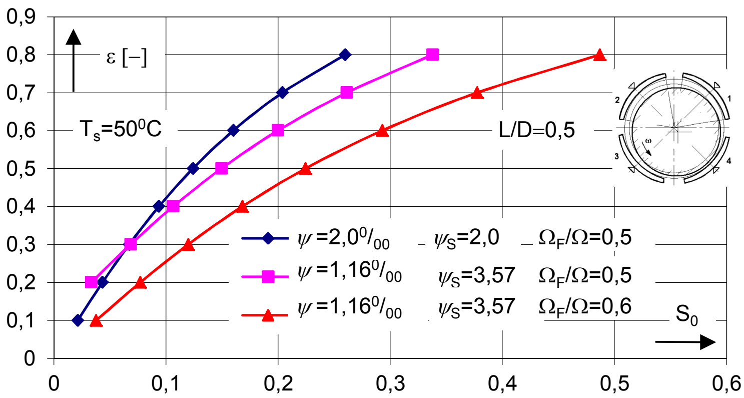


Figure 3. Journal displacement ε versus Sommerfeld number S_0 at different bearing parameters of tilting 4-pad journal bearing

The displacements of journal ε versus the load capacity So (Sommerfeld number) at the length to diameter ratio L/D=0.5, different values of bearing relative clearance ψ, pad relative clearance ψ_s, pad offset Ω_F/Ω and the feeding oil temperature $T_s=50^0C$ is shown in Fig. 3. At assumed Sommerfeld number So and pad offset Ω_F/Ω, and increase in the bearing relative clearance ψ at decreasing pad relative clearance ψ_s causes the increase in the journal displacement ε. However, at assumed bearing relative clearance ψ and pad relative clearance ψ_s the pad offset decreases the journal displacements (e.g. Fig. 3, at So=0.2 and $\Omega_F/\Omega=0.5$ the displacement ε=o.6 but at $\Omega_F/\Omega=0.6$ the displacement is ε =0.45).

Exemplary dynamic characteristics in form of stiffness g_{ik} and damping b_{ik} coefficients determined for the pad pivot offset $\Omega_F/\Omega=0.6$ are shown in Fig. 4 and Fig. 5. Stiffness g_{11} and g_{22} as well as damping b_{11},b_{22} coefficients show the increase at the increase in Sommerfeld number S_0. Among these coefficients the largest values have the stiffness g_{22} and damping b_{22} coefficients particularly at lager values of Sommerfeld numbers (e.g. Figure 4 and Figure 5). The coupled stiffness coefficients $g_{12} = g_{21}$ (Figure 4) and damping ones $b_{11} = b_{22}$ (Figure 5) are equal, respectively. However, the coupled stiffness g_{12} , g_{21} and damping coefficients b_{11} , b_{22} decrease at the increase in Sommerfeld numbers (Figure 4 and Figure 5); this is in agreement with the results of another authors, e.g. Klumpp [3].

The Systemdamping u/ω_{cr} was calculated with the use of the program Mathematica 5.0.

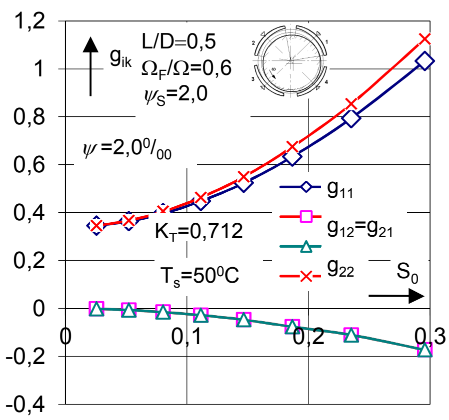


Figure 4 Stiffness coefficients of tilting 4-pad journal bearing

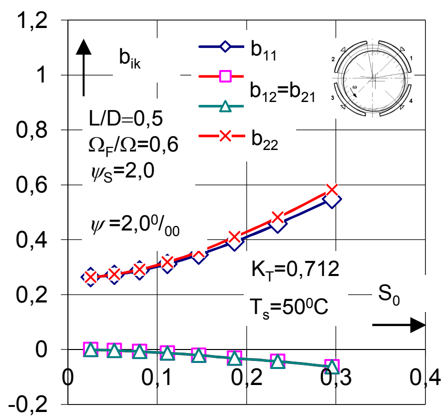


Figure 5 Damping coefficients of tilting 4-pad journal bearing

The system-damping u/ω_{cr} obtained by Klummp [3] and Walle [4] is presented in Fig. 6. Author's results that were obtained for two values of pad offset, i.e. $\Omega_F/\Omega=0.6$ and $\Omega_F/\Omega=0.6$ are showed in Fig. 7; the run of all curves is similar to the runs that were obtained by Klummp [3] and Walle [4] but different in the values of system-damping.

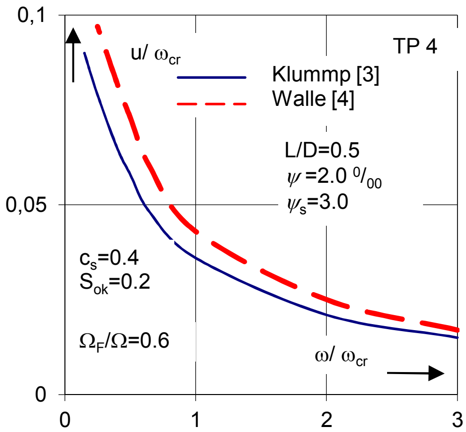


Figure 6. System-damping of tilting 4-pad journal bearing [3,17]

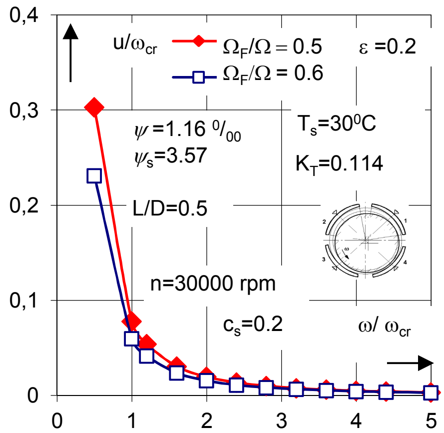


Figure 7. System-damping of tilting 4-pad journal bearing at different pad offset

5. Conclusions

The calculations and analysis of results has allowed drawing the conclusions given below.

1. An increase in pad offset Ω_F/Ω causes the variations in the journal displacements at assumed Sommerfeld number.
2. The stiffness and damping coefficients show changes at the variations of pad offset
3. An increase in pad offset generates the decrease in the system damping.

References

1. J. Glienicke, *Theoretische und experimentelle Ermittlung des Systemdämpung gleitgelagerter Rotoren und ihre Erhöhung durch eine äußere Lagerdämpfung*. Fortschritt Berichte. VDI, Reihe 11 Nr 13. 1972.
2. R. Gasch, H. Pfützner, *Rotordynamik*. Springer -Verlag. 1975
3. R. Klumpp, *Ein Beitrag zur Theorie von Kippsegmentlagern*. Dissertation, Technische Universität Karlsruhe. 1975.
4. F. Walle, *Unwuchtschwingungen und Systemdämpfung gleitgelagerter Rotoren unter praxisnahen Bedingungen*. Ph. D. Thesis. University Karlsruhe. Karlsruhe. 1975
5. D.C. Han, *Statische und dynamische Eigenschaften von Gleitlagern bei hohen Umfangsgeschwindigkeiten und bei Verkantung*. Dissertation. Karlsruhe. 1979.
6. S. Strzelecki, G.C.Jr. Brito, *Dynamics of Tilting Pad Journal Bearings of Large Overall Dimensions Applied in Hydro Generating Units*. Encontro Nacional de

Monitoramento. Proceedings of the III ENAM. 29.10 – 03.11.2006. Foz de Iguasu. PR - Brasil. 1-12.

7. S. Strzelecki, S.M. Ghoneam, Z. Towarek, *Stability of Tilting 5-Pad Journal Bearing*. Proceedings of the ISCORMA-4. Calgary 27-31.08. Canada. 2007.
8. S. Strzelecki, Z. Towarek, *Stability of rotor operating in multilobe or tilting-pad journal bearings*. 10th Conference on DYNAMICAL SYSTEMS. THEORY AND APPLICATIONS. December 7-10, 2009, Łódź, Poland. Proceedings Vol. 1. 473-480.
9. S. Strzelecki, S.M. Ghoneam, *Static characteristics of tilting 3-pad journal bearing*. INTERTRIBO 2009 (October 21-23.2009, Slovak Republik). Proc. of the Conference. 52-55.
10. S. Strzelecki, H. Kapusta, *Effect of Pivot Offset on the Oil Film Temperature Distribution in the Tilting 5-Pad Journal Bearing*. 14th International Colloquium Tribology, January 13-15, 2004. Germany. 971-976
11. S. Strzelecki, *Effect of pivot location on the maximum oil film temperature of tilting 5-pad journal bearing*. IX International Symposium INTERTRIBO 2006. October 11-13, Vysoke Tatry Stara Lesna. 2006.