Computer Applications in Electrical Engineering
Vol. 13 *2015*

Holonomicity analysis of electromechanical system on the example of simple switched reluctance motor

Mirosław Wciślik, Karol Suchenia
Kielce University of Technology
25-314 Kielce, Al. Tysiąclecia Państwa Polskiego 7
e-mail: m.wcislik@tu.kielce.pl, ksuchenia@tu.kielce.pl

The paper deals with a single-phase reluctance motor used to test the motion equations of an electromechanical system. The identification of the motor electric parameters as a function of the rotation angle of the rotor was carried out using AC 50 Hz voltage source. A mechanical part of the system was designed as a physical pendulum containing the motor rotor and a metal bar mounted on the rotor axis. The parameters of the mechanical part were measured during the pendulum oscillations. The work presents the characteristics and motion equation parameters of the motor dynamics. The reluctance motor motion equation does not fullfil the power balance. The parameters of the motion equations obtained from the experiment and from the second order Lagrange'a equations are compared. The derivation of motion equation, together with a discussion of holonomicity of electromechanical systems is also presented.

KEYWORDS: motion equations, Lagrange function, non-holonomicity, power balance, reluctance motor

1. Introduction

A construction of a reluctance motor (RM) is very simple because of it has not brushes, magnets and rotor windings. Consequently, a reluctance motor is highly reliable and fault-tolerant. The secondary advantage of reluctance motors is that the rate of motor rotation may be easily regulated. As a result, they compete with other widely known motors and are now often used in household devices and in many branches of industry.

In order to analyze the motion equation of the electromechanical system, the reluctance motor with a single pair of stator and rotor poles was chosen – Fig. 1.

The major drawback of the motor is that it starts to operate only for particular angles of the rotor versus stator. However, the electric circuit is described only by means of one equation. Therefore, it facilitates and simplifies the analysis and experimental verification of the mathematical model.

The mathematical model of an electromechanical device consists of electrical and mechanical parts described by two equations. The analysis of systems is based on the measurements of the electric circuit and the mechanical system parameters.

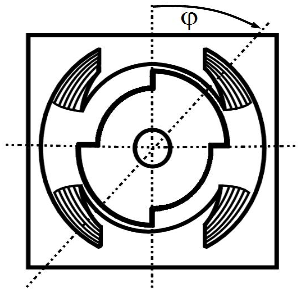

Fig. 1. General diagram of the reluctance motor

The mechanical equation may be formulated as follows [1]:

$$J \cdot \frac{d\omega}{dt} + k \cdot \omega + T_L = T_e \tag{1}$$

where: J - moment of inertia, k - coefficient of friction, T_L – torque of load.

The electromagnetic torque T_e is defined as [2]:

$$T_e = \frac{1}{2} \cdot \frac{\partial L(\varphi)}{\partial \varphi} \cdot i^2 \tag{2}$$

where: i - is current intensity, and $\partial L(\varphi)/\partial\varphi$ is a derivative of inductance versus the rotor rotation angle in relation to stator. The inductance $L(\varphi)$ is a function of the rotation angle φ.

Multiplying the equation (1) by the angular velocity, yields a power balance equation:

$$\frac{1}{2} J \cdot \frac{d\omega^2}{dt} + k \cdot \omega^2 + T_L \cdot \omega = \frac{1}{2} \cdot \frac{\partial L(\varphi)}{\partial \varphi} \cdot \omega \cdot i^2 \tag{3}$$

A term on the right side of the equation (3) represents the power which is transferred from the electrical part of motor. Terms on the left side of equation (3) represent the power, related to the changes of rotor kinetic energy, friction power, and load power.

The equation of the electric system is often written in the form [3-5]:

$$\frac{d}{dt}(L(\varphi)i) + R_s i = U_s \tag{4}$$

where: R_s – stator winding resistance, $L(\varphi)$ – stator winding inductance.

Expansion of the derivative in (4) and multiplying it by current i, yields the following power equation of electric circuit:

$$\frac{1}{2} L(\varphi) \cdot \frac{di^2}{dt} + \frac{\partial L(\varphi)}{\partial \varphi} \cdot \omega \cdot i^2 + R_s \cdot i^2 = U_s \cdot i \tag{5}$$

The first term on the left side of the equation defines the rate of change of magnetic energy which is cumulated in the motor inductance. The second describes the intensity of energy which is transformed to the mechanical system. The last represents the thermal energy loss of stator winding. A term on the right side describes the electric power supplied to the motor. It should be emphasized, that the power transferred to the mechanical system from the electric system in the equation (5) is different than the intensity of energy transferred to the mechanical system in the equation (3).

The explanation of power difference may be found in [6-8]. In these papers power transformed from electric circuit is splitted into two parts:

$$\frac{1}{2}L\frac{di^2}{dt}+\frac{1}{2}i^2\frac{dL}{d\varphi}\omega+R_S i^2=U_S i \tag{6}$$

and then it is written as:

$$\frac{d}{dt}(\frac{1}{2}Li^2)+\frac{1}{2}i^2\frac{dL}{d\varphi}\omega+R_S i^2=U_S i \tag{7}$$

The first term is interpreted as the rate of magnetic energy accumulation, the second one is described as power transformed to the mechanical system. It should be noted, that in steady state the energy stored in the coil has two terms: the first one - $½Ldi^2/dt$, in the steady state equals zero, and the second one - $½i^2\omega dL/d\varphi$, has the same form as the power transferred to the mechanical system. In a steady state the transfer of electric power to mechanical output power is continuous. It means that accumulation rate of magnetic energy is constant and the magnetic energy of motor increases continuously. Is it possible?

2. Identification of parameters

In order to verify the above mentioned relations, the test bench with the reluctance motor was constructed. The stator of the motor had one pair of poles, and a rotor had only one pair of teeth – Fig. 1.

In order to specify the relation between inductance and the rotation angle of the rotor, the motor winding was supplied by from AC 50 Hz voltage source. Due to the symmetry of the motor, the research was carried out for the rotation angle range of 180 degrees set up with a 5 degree step. The instantaneous values of current and the voltage of the motor winding were measured. The measurements were conducted using the National Instruments measurement boards, as well as LabVIEW and Matlab programs. The inductance profile as a function of rotation angle was established using the measured values – Fig. 2.

The intermediate value of a derivative of inductance versus rotation angle for the rising linear section of the characteristic in fig. 2 was calculated. Its value between a 20 degree and a 75 degree rotation angle amounts to:

$$\frac{dL(\varphi)}{d\varphi}\cong 0.0395\frac{H}{rad} \tag{8}$$

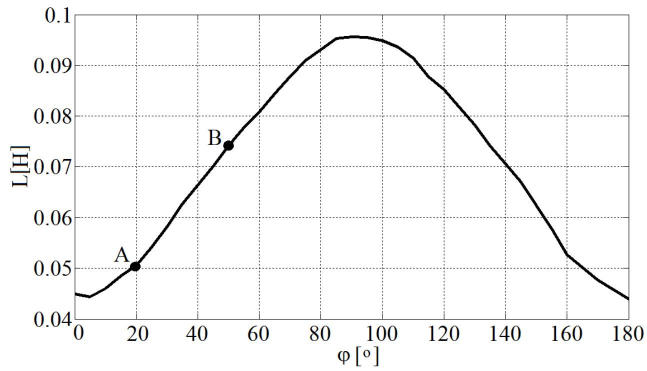

Fig. 2. The inductance profile for the angle of rotation

The measurement of the parameters during the rotation process is quite complex, because the moment of load has to be set and measured. Therefore, in order to analyze the coefficients of the equations (1) and (4), the mechanical system of the motor was modified. A steel rod of 6mm x 6mm x 354mm dimensions and of 100g weight was attached to the rotor axis. The rod and the rotor together formed a physical pendulum – Fig. 3. On a motor shaft there is also a rotary encoder, which is used to measure the inclination angle of the rotor in relation to the stator. When the stator circuit is powered from the stabilized current source, the pendulum moves up by about 30 degree. In the first experiment the rotation angle measurement was related to vertical position of the bar and the direction of earth gravity – point B in fig. 2. The pendulum position was changed manually. After release the oscillations of the steel bar position related to the rotor rotation angle were caused [9]. In the test the changes of inductance were placed on the rising part of the inductance profile – fig. 2. The moment of inertia J is a sum of rotor inertia and pendulum inertia. A resistor R is connected in series with the coil of stator, and it is used to measure current in the stator winding. The values of voltage, current and rotation angle are measured simultaneously using NI 9225 and NI 6216 boards. The boards are serviced by a LabView program installed on the computer where the data are recorded in a text data file *.txt. The data files are loaded into MATLAB and processed using Golay - Savitzky filter to eliminate noise and calculate derivatives. The parameters of the motion equations were calculated using least squares method [10].

In order to check the coefficients of the motion equations (1) and (4), the identification of the equations parameters was carried out. The motor parameters were identified for stabilized current supply, thus derivative of the current in the equation (9) was omitted. The general form of the motion equations of the system is as follows:

$$w1 \cdot i + w2 \cdot \frac{d\varphi}{dt} \cdot i = U_S \tag{9}$$

$$w3 \cdot \frac{d^2\varphi}{dt^2} + w4 \cdot sign\frac{d\varphi}{dt} + w5 \cdot i^2 = \frac{m \cdot g \cdot l \cdot sin(\varphi)}{2} \tag{10}$$

where: $w1$ – stator winding resistance (R_s), $w2$, $w5$ – derivative of inductance versus the rotor rotation angle in relation to stator($\partial L(\varphi)/\partial\varphi$), $w3$ – moment of inertia (J), $w4$ – coefficient of dry friction, φ – the angle of rotor rotation, i – is current intensity, U_s – power supply, m – weight rod, g – gravity acceleration, l – length of rod.

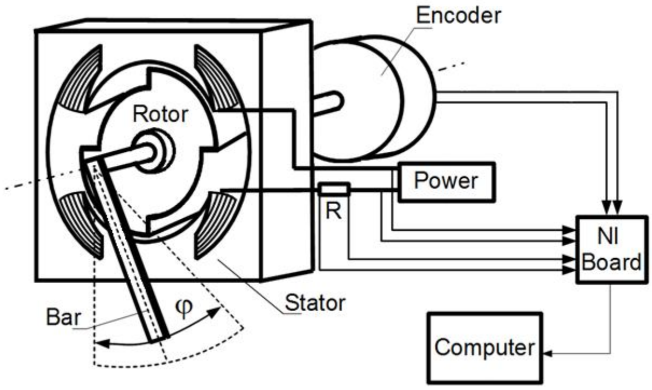

Fig. 3. The measurement diagram of the reluctance motor parameters

The system in Fig. 3 was powered by DC voltage source with a stabilized current value. The current flow affects that the rotor inclines of the pendulum rod by an angle φ. The value of current is selected to match the inclination angle of the pendulum, which should be placed the middle of near the profile positive inclination – at the function of an angle - point B in fig 2. The rotor was placed in such a way, that a vertical position of pendulum could match a 20 degree rotation angle (point A) of the characteristic in fig. 2. The current value in the stator winding was assumed for a 20 degree deflection the pendulum. It corresponds to the point of the equilibrium (point B), with a 50 degree angle – Fig. 2. If the pendulum is deviated from the equilibrium point, it oscillates around it.

The approximate values of coefficients the equations (9) and (10) were presented. Basing on measurement of current i, angle φ, and time derivatives of angle the parameters $w1$,...,$w5$ were identified. Both the voltage induced in the winding, and its approximate calculation from the equation (9) are shown in Fig. 4.

Similarly, time characteristics of load from the equation (10) is displayed in Fig. 5.

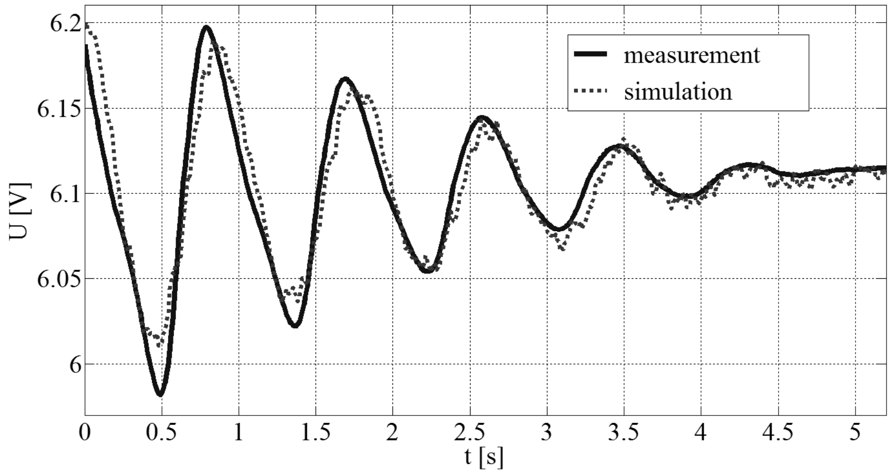

Fig. 4. The waveform of voltage oscillation of the stator windings

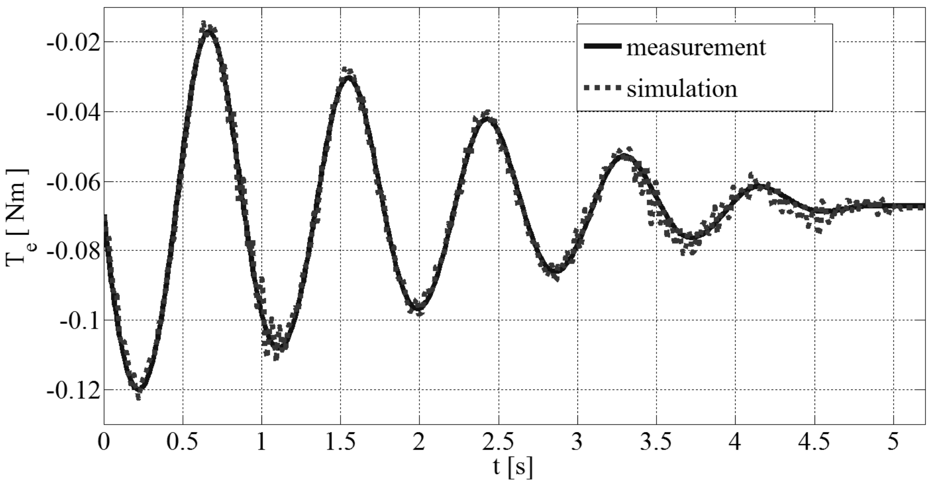

Fig. 5. The process of oscillation for the mechanical equation

On the basis of the measurement, the coefficients of the equation (9) and the coefficients of the equation (10) are:

$$w1 = 3.28\ \Omega \qquad w2 = 0.0199\,\frac{H}{rad} \tag{11}$$

$$w3 = 0.003\ kg \cdot m^2 \qquad w4 = 0.0028\frac{kg \cdot m^2}{s} \qquad w5 = 0.0197\,\frac{H}{rad} \tag{12}$$

It can be seen, that a coefficient *w2* from the equation (11) and a coefficient *w5* from the equation (12) are very close. Both values almost equal a half of the value of the derivative of inductance profile versus the rotation angle (8) as presented in Fig. 2 with the accuracy being one percent. The correlation coefficient between measured and calculated voltages in Fig. 4 is 0.9307. For the mechanical equation the correlation coefficient in fig. 5 is equal to 0.9925.

The experiment showed that in the equation (9), the term which contains the derivative of inductance with respect to rotation angle, should be multiplied by coefficient ½. Therefore, the motion equation of electric circuit of RM should be formulated as follows:

$$U_S = L(\varphi)\frac{di}{dt} + R_S \cdot i + \frac{1}{2}\cdot\frac{dL(\varphi)}{d\varphi}\cdot\omega\cdot i \tag{13}$$

The equation which resulted from the experiment (13) is different from the widely known electric equation of electric motor [3-5] due the presence of the coefficient ½. Therefore, the analysis of relation between power balance and the holonomicity of reluctance motor would be performed.

3. Holonomicity analysis of the reluctance motor

The Lagrange function is most commonly used to describe motion equations of electromechanical systems [11]. It is described by generalized coordinates and velocities as the difference between kinematic and potential energy as follows:

$$fL = T - V \tag{14}$$

where: T –kinematic energy, V – potential energy

The Lagrange function does not take into account a friction and external forces. In general, potential energy is not present in electromechanical systems. The Lagrange function only describes internal energy flows. It describes relationships between particular generalized coordinates and their derivatives. Hence, the holonomicity of the system should be discussed on the basis of Lagrange function of homogeneous and conservative systems. If, electric coordinates (electric charges) q_e and mechanical coordinates (angles) q_m, are discriminated from among generalized coordinated the Lagrange function of the system may be described as:

$$fL = \frac{1}{2}L(q_m)\cdot(\frac{dq_e}{dt})^2 + \frac{1}{2}J\cdot(\frac{dq_m}{dt})^2 \tag{15}$$

Basing on the Lagrange function, d'Alambert - Lagrange equation may be formulated [12]. For the electromechanical system, it is as follows:

$$(\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_e})\delta q_e + (\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_m} - \frac{\partial T}{\partial q_m})\delta q_m = 0 \tag{16}$$

According to [12 p.90], the variations δq_e and δq_m from d'Alambert – Lagrange equation are independent only for the holonomic system, and only for that system, we obtain the Lagrange equation of the second kind in form:

$$\frac{d}{dt}\frac{\partial fL}{\partial \dot{q}_e} = 0 \tag{17}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0 \tag{18}$$

If it is assumed that the variations of generalized coordinates are equal to the time derivatives of the generalized coordinates, the d'Alambert - Lagrange equation (16) becomes the equation of power balance.

By substituting in the equation (16) with Lagrange function (15), and by assuming that $\delta q_e = i =$ const and $\delta q_m = \omega$, the equation of power balance is not fulfilled in steady state:

$$\frac{\partial L(\varphi)}{\partial \varphi}\cdot \omega \cdot i^2 - \frac{1}{2}\cdot\frac{\partial L(\varphi)}{\partial \varphi}\cdot \omega \cdot i^2 \neq 0 \tag{19}$$

The power balance is (19) is different than zero, hence d'Alambert – Lagrange equation for the holonomic system is not satisfied. Consequently, it is apparent, that the analyzed electromechanical system is non-holonomic.

If a dynamic system is described by n generalized coordinates, but its equations contain only $m < n$ of these coordinates and all generalized velocities, then the system is non-holonomic [12 p. 107-108]. The same rule is applicable to the equations of electromechanical systems. Therefore, it may be concluded that the electric equation determines whether the analyzed system is holonomic or non-holonomic. There is no generalized electric coordinate – electric charge in electric equations.

From [13] (pages 145-146) it follows that the same procedure may be applied to both non-holonomic and holonomic systems. However, some correction in the equations is necessary.

"The modifications that have to be made to the Lagrange equations may be found by looking at the problem in a slightly different way, and regarding the constrained system as the unconstrained system acted on by certain external forces, namely those forces which have to be exerted in order to compel the system to obey the constraint. This formulation has the advantage that in it the coordinates q_1, q_2, ...,q_n may be regarded as independent; the constraints now appear as the effect of additional forces, and not as relations between the coordinates. Because the coordinates are independent, Lagrange's equations may be used, and the equations of motion of the constrained systems may be obtained by including the effects of the additional forces in the Lagrangian equations." [13 p.146].

In the analyzed example in electric equation a force Q_N was introduced.

$$\left(\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_e} + Q_N\right)\delta q_e + \left(\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_m} - \frac{\partial T}{\partial q_m}\right)\delta q_m = 0 \tag{20}$$

In [12] a similar method is described. It assumes that, if the system is non-holonomic, we should add the vector of reactions of non-holonomic constraints, as an additional expression (e.g. $Q_N \neq 0$). Then d'Alambert – Lagrange equation may be represented by (20). Basing on the text from [12 p. 92]: "if Q_N is the generalized force of reaction in the process of movement of the non-holonomic system, then the equation (20) describes also the process of movement of some holomonic system, along with kinetic energy T and the generalized force of reaction Q_N". It allows us to analyze the equations of electric and mechanical systems independently.

On the basis of the equation (20) and the energy balance of the conservative system, we may determine a correction Q_N. Assuming that the variations of generalized coordinates are equal to the derivations of these coordinates, and taking into account the correction, mentioned above in steady-state we get a following equation:

$$\frac{\partial L(\varphi)}{\partial \varphi}\cdot\omega\cdot i^2-\frac{1}{2}\cdot\frac{\partial L(\varphi)}{\partial \varphi}\cdot\omega\cdot i^2+Q_N\cdot i=0 \tag{21}$$

Hence an additional force Q_N equals:

$$Q_N=-\frac{1}{2}\frac{\partial L(\varphi)}{\partial \varphi}\cdot\omega\cdot i \tag{22}$$

If the additional force is used in an electric equation and are added dissipative and potential forces, the following relation is obtained:

$$L(\varphi)\cdot\frac{di}{dt}+\frac{1}{2}\frac{\partial L(\varphi)}{\partial \varphi}\cdot\omega\cdot i+R_S\cdot i=U_S \tag{23}$$

It can be seen that the equation (23) is the same as equation (13), established on the basis of measurements. The multiplication of the equation (23) by the current i yields:

$$\frac{1}{2}L(\varphi)\cdot\frac{di^2}{dt}+\frac{1}{2}\frac{\partial L(\varphi)}{\partial \varphi}\cdot\omega\cdot i^2+R_S\cdot i^2=U_S\cdot i \tag{24}$$

The equation is physically interpreted and describes to the power balance of electric equation.

4. Conclusions

It may be concluded that the power balance in the reluctance motor is achieved only after using the additional voltage, which plays a role of "non-holonomic force" in the electric equation of the reluctance motor. Only if, this condition is fulfilled the powers in the given motor model are balanced. It proves that the analyzed motor is a non-holonomic system.

The equation describing an electric circuit for a non-holonomic system should be formulated as (13). Then the power transferred to the mechanical system is equal to the power transferred from the electric equation.

The correction presented in this paper may be also applied in more complex electromagnetic systems e.g. multi phase switching reluctance motor.

References

[1] Tomczewski K.., Symulacja pracy napędu z nowym układem zasilania przełączalnego silnika reluktancyjnego, Przegląd Elektrotechniczny 2009.
[2] Yong-Ho Y., Jae-Moon K., Chung-Yuen W., Byoung-Kuk L., New approach to SRM drive with six-switch converter 2009, Mechatronics 19, 2009, pp. 1321–1333.
[3] R. Krishnan, Switched Reluctance Motor Drives: Modeling, Simulation, Analysis, Design, and Applications, CRC Press LLC, 2001,pp. 39-40.
[4] Sheng-Ming Y., Controlled Dynamic Braking for Switched Reluctance Motor Drives With a Rectifier Front End, IEEE Trans. On Industrial Electronics, vol. 60, no.11, Nov. 2013.
[5] Shun-Chung W., An fully-automated measurement system for identifying magnetization characteristics of switched reluctance motors, Measurement 45, 2012, pp. 1226–1238.
[6] Miller T. J. E., Brushless Permanent-Magnet and Reluctance Motor Drives, Oxford University Press, 1989, pp.168–169.
[7] Miller T. J. E., Electronic Control of Switched Reluctance Machines, Newnes Power Engineering Series, 2001, pp.37–38.
[8] Skvarenina T. L., The Power Electronics Handbook Industrial Electronics Series, CRC Press LLC, 2002, pp.421.
[9] Wciślik M., Suchenia K.: Model i bilans mocy czynnej przełączalnego silnika reluktancyjnego, Przegląd Elektrotechniczny 2014.
[10] Wciślik M., Suchenia K.: Model i bilans mocy czynnej przełączalnego silnika reluktancyjnego, Przegląd Elektrotechniczny 2014.
[11] Landau L. D., Lifszyc J. M., Mechanika PWN Warszawa 2007.
[12] Nejmark J. I., Fufajew N. A., Dynamika układów nieholonomicznych, PWN, 1971.
[13] Bishop R. E. D., Gladwell G. M. L., Michaelson S., Macierzowa analiza drgań, WNT, Warszawa 1972.

(Received: 30. 09. 2015, revised: 1. 12. 2015)