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MATHEMATICAL MODEL OF TRACTION VEHICLE MOVEMENT

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Abstract – The article presents a mathematical model of electric traction vehicle movement in a given power supply area. Starting from the presentation of the basic features of the 3 kV DC traction power supply system used in Poland, the author presents a simulation model of electric traction vehicles movement, which allows to determine the mobility and current - voltage possibilities on a selected railway line. The obtained simulation results fully confirm the possibility of using the model as an aid in the design, modernization or diagnostics of existing railway lines and train traffic.

Key words – electric traction vehicles, modelling, rail transport.

INTRODUCTION

In the second half of the 20th century, in many countries of the world (e.g. Europe, Asia, North America) huge interest in high speed railways is observed. It boils down to the construction of:

- new railway lines with excellent parameters or modernization of existing lines,
- new and super secure traffic control systems,
- modern and energy-saving electric traction rolling stock,
- strengthening and improving power systems.

The main reason for the development of rail transport are technical considerations, such as: short travel time, punctuality of trains, ride comfort and safety.

Short travel time means high speed trains. High speed requires the use of locomotives or high power train sets. The use of such rolling stock is the consumption of considerable power from the power supply system, which in turn leads to the consumption of large currents and the formation of significant voltage drops in the traction network.

This is a very unfavorable phenomenon, especially in the 3 kV DC power supply system. Therefore, continuous analysis of the operation of the power supply system and the movement of traction vehicles on the route becomes an important issue.

The 3 kV DC system, due to the limitation in power transmission, is used so as to power vehicles with average powers of 6 - 8 MW.

This can be found on the railway lines of Poland, former Czechoslovakia, Belgium, Spain and Italy. This represents about 35% of the length of electrified lines in the world.

It allows trains to run at speeds up to 250 km / h. Due to the low supply voltage (at high vehicle powers), the power supply system has high current loads, which significantly affects the formation of large voltage drops in the traction network. To prevent this, many methods are used, e.g. smaller distances between traction substations (10 - 15 km), substations with higher powers (over 10 MW), replacement of section cabins with substations, single-stage transformation and larger sections of the traction network made of elements with as low resistance as possible. The cross-section of the traction network (2C120-2C-3) most commonly used in Poland is 440 mm^2, and on European lines even 610 mm^2 - in Spain and Italy.

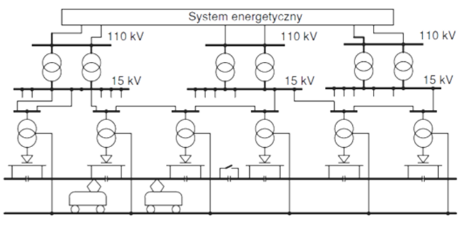

Fig. 1. Diagram of the 3 kV electric traction power supply system. [1]

In order to increase the efficiency of the power supply system, a single-step voltage transformation is used. In this situation, the HV high voltage power lines are directly connected to the traction substation, eliminating indirect voltage transformation (HV / MV). Due to the fact that the 3 kV DC system is obligatory on the PKP network, the built simulation model, calculations and analysis of traction vehicles driving and power supply system operation concern modeling in this system.

I. Simulation Model

The starting point for traction calculations of train traffic is the demand for planned volumes of transport on the basis of which the timetable is determined.

The train timetable for the selected railway line is made graphically (train movement graph) as a graph on the plane s = f (t) (dependence of the road as a function of time) for each train traveling on a given section of the route in a given time. This graph is a set of straight lines (for driving without stopping) or broken lines (for stopping at stations).

It is constructed with all data on: line parameters (horizontal and vertical profile), stations, traffic possibilities on the line (line capacity), speed limits and types of trains, type of locomotives etc. For the selected railway line, the condition for the implementation of the constructed timetable sets strict requirements for the power supply system. The power supply system must be designed so as to provide the required energy efficiency for train movements in accordance with the planned timetable.

These requirements are specified in the guidelines for performing calculations and analyzing the operation of the power supply system.

For this purpose, a simulation model of electric traction vehicle movement was built, enabling the determination of traffic possibilities on a selected railway line for given transport assumptions in accordance with the adopted timetable.

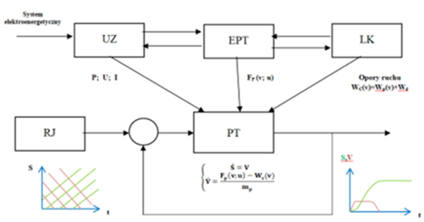

Fig. 2. Simulation model of electric traction vehicles movement

where:

- UZ - block - power supply system
- EPT - block - electric traction vehicle
- LK - block - railway line
- RJ - block - scheduled timetable
- PT - block - theoretical journey (calculation module)

Individual blocks: UZ; EPT; LK, provide all necessary information to the PT calculation block, where, in accordance with the assumed RJ timetable, real-time simulations of theoretical journeys are carried out on a given railway track. [2]

The LK block contains all information regarding the parameters of the railway line (horizontal and vertical profile, tunnels, railway bridges, turns, spaces with high air movements, etc.), allowing to determine the values of basic resistance $W_p\ (v)$ and additional resistance W_d.

The EPT block provides a mathematical description of the vehicle (traction characteristics), giving the value of the tractive force $F_p\ (v;\ u)$ as a function of speed v and determining the dependence on the electrical parameters of the propulsion unit, among others on the parameters and number of engines, gears, wheel diameter etc.

The UZ block contains all information regarding the structure and parameters of the power supply system, i.e. substation and overhead contact line parameters (resistances, voltages, currents, powers, etc.)

The train movement graph, i.e. the set transport task, is the input to the adopted simulation model, determining the mechanical conditions (kinematics and dynamics) of moving vehicles in a given area.

Calculations are made based on vehicle traffic equations, the so-called theoretical journey.

Assuming that a rail vehicle is treated as a material point with the mass concentrated in its center of gravity, the equations of motion describing the changes of distance and speed as a function of time can be presented in the form (1), (2):

$$v(t) = \frac{ds(t)}{dt} \tag{1}$$

$$a(t) = \frac{dv(t)}{dt} = \frac{F_p(v(t),u(t)) - W_p(v(t)) - W_d}{m_p} \tag{2}$$

where:
- s - road [m]
- v - speed [m/s]
- a - vehicle acceleration [m/s²]
- m_p - vehicle mass [t]
- F_p - tractive force [N]
- W_p - traction resistance (basic) [N]
- W_d - traction resistance (additional) [N]

When making calculations, it is convenient to write the equations of motion in the form (3), (4):

$$t = m_p \int_{V_1}^{V_2} \frac{dv(t)}{F_p(v(t),u(t)) - W_p(v(t)) - W_d} \tag{3}$$

$$s = m_p \int_{V_1}^{V_2} \frac{v\,dv(t)}{F_p(v(t),u(t)) - W_p(v(t)) - W_d} \qquad (4)$$

The basic input data for vehicles is the traction characteristics which determine the maximum tractive force as a function of vehicle speed at the rated supply voltage.
The results of these calculations are: distance, speed and acceleration as a function of time.

The next step is to check the energy conditions associated with moving vehicles, starting by determining the value of currents consumed by these vehicles.
The calculated waveforms of currents, together with the previously calculated roads and vehicle speeds as a function of time, are the starting point for calculating other energy parameters, such as voltage drops in the catenary network, consumed currents, power and voltage on traction substation rails and voltages on vehicle pantographs.
An important simplification in the calculation process of the consumed currents by vehicles is the assumption of constant voltage on their pantographs. In fact, this voltage changes, and these changes in turn determine the maximum power of the vehicles, and therefore have a significant impact on their traction characteristics.

II. Simulation Results

Exemplary simulations were carried out in various configurations of train journeys, their results and analysis of simulations of traction vehicle journeys with known parameters and given power supply area. The calculations also took into account the parameters of the Central Railway Main Line. Particular attention was paid to the efficiency of the power supply system and the supply of necessary power to vehicles, taking into account voltage drops in the overhead contact line. The simulation results give the possibility of full visualization of train movement and power supply operation. They allow for forecasting and developing detailed train traffic, and then - for arranging effective timetables.

Digital analysis of rail vehicle driving was carried out on the section 20 km long between two traction substations with a section cabin at a distance of 10 km from substation A (according to figure 3).

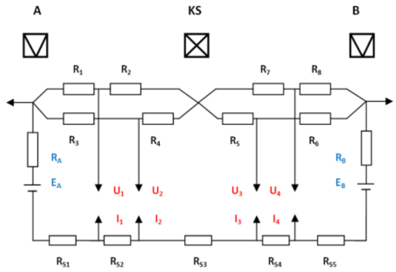

Fig. 3. The basic power supply system for vehicles on a two-track line (two substations with a section cabin)

Traction loads are described for each vehicle as voltage and current on the pantograph:
U_1, I_1; U_2, I_2; U_3, I_3; U_4, I_4.
Source voltages and internal resistances of substation A and B as: E_A, E_B, R_A, R_B.
Upper network resistances: R_1 - R_8, and lower network resistances: R_{S1} - R_{S5}.

The source voltages of substation A and B were assumed to be 3.6 kV; substation internal resistance - 0.12 Ω; unit resistance of the upper network - 0.05 Ω/km and unit resistance of the lower network - 0.01 Ω/km. It was assumed that the vehicles move on a horizontal and straight track (additional resistance $W_d = 0$).

Calculations are made using Mathcad software, and all necessary data are entered into the prepared windows and tables. The results are then exported to an Excel spreadsheet, which allows the presentation of several charts in one coordinate system, on a plane or spatially. The results of simulation driving tests for various cases of vehicle movement on the route are shown in Figures (4 - 9).

Movement of four trains

Assumptions:

Movement of four passenger trains (10 passenger cars) with EP-09 electric locomotives moving: two in the direction from substation A to substation B and two going in the opposite direction. Trains: **No. 2** and **No. 4** (according to Figure 3) start at substations A and B, respectively ($V_2 = 0$; $V_4 = 0$), and trains **No. 1** and **No. 3** start at the section cabin (consuming the current) at speeds V = 30 m/s in opposite directions.

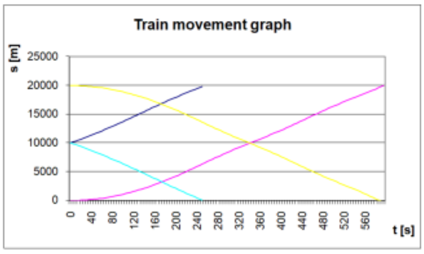

Fig.4. Train movement graph

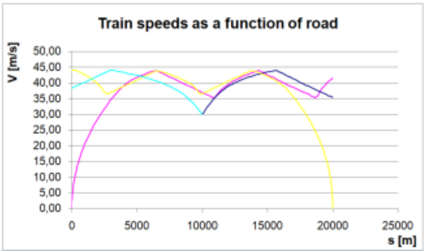

Fig.5. Train speeds as a function of road

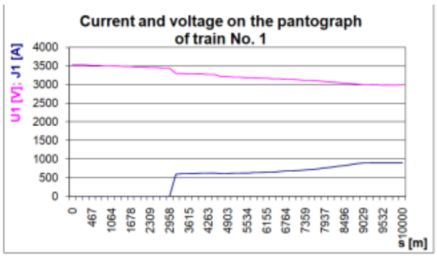

Fig.6. Current and voltage on the pantograph of train No. 1

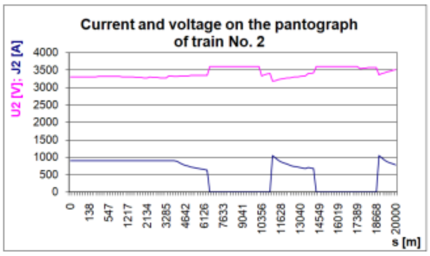

Fig. 7. Current and voltage on the pantograph of train No. 2

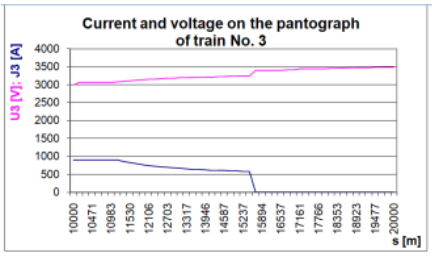

Fig.8. Current and voltage on the pantograph of train No. 3

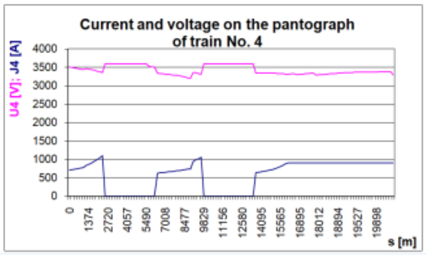

Fig.9. Current and voltage on the pantograph of train No. 4

Figures 4 - 9 show simulation results of four passenger trains moving according to the scheme given in the assumptions.

III. Conclusions

Based on the carried out simulations of train driving, it can be stated that using the presented simulation model it is possible to visualize and analyze in real time, various traffic and current-voltage situations on the route, such as: continuous control of driving parameters of electric traction vehicles, efficiency of the power supply system and supply of the necessary power to vehicles.

The presented model allows to simulate the movement of modern rolling stock with increased power in the area of several power substations, full visualization of parameters (electrical and kinetic) of vehicle movement and supply system parameters. The developed model can be used, for example, to simulate train movement in real conditions (in various configurations), to predict the traction capabilities of railway vehicle traffic, to design or modernize the rail power supply system, to build a train timetable, as well as an aid in the design, modernization or diagnostics of existing railway lines and train traffic.

MODEL MATEMATYCZNY RUCHU POJAZDÓW TRAKCYJNYCH

W artykule przedstawiony jest model matematyczny ruchu elektrycznych pojazdów trakcyjnych w zadanym obszarze zasilania. Wychodząc od przedstawienia podstawowych cech stosowanego w Polsce systemu zasilania trakcji elektrycznej prądu stałego 3 kV, autor zaprezentował model symulacyjny ruchu elektrycznych pojazdów trakcyjnych, pozwalający na określenie możliwości ruchowych i prądowo – napięciowych na wybranej linii kolejowej. Otrzymane wyniki symulacyjne w pełni potwierdzają możliwość wykorzystania modelu, jako środek pomocniczy w projektowaniu, modernizacji lub diagnostyce istniejących linii kolejowych i ruchu pociągów.

Słowa kluczowe: elektryczne pojazdy trakcyjne, modelowanie, transport kolejowy.

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