

Decomposition and the interaction decoupling mode in electrical systems analysis

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Decomposition is the best method to analyze complicated systems. The whole system is divided into a set of smaller parts, i.e. subsystems. These items are connected to each other by input and output interfaces. Every subsystem could be analyzed using separate algorithms and procedures that are more suitable for solving the local task. In the article, a complicated electrical system is decomposed and the interaction decoupling mode is used to coordinate the local task solution. Each of the first-level subsystems, such as an optimization problem, is defined completely independently from the other problems. The subsystems are instructed to select the local input as well as the local interface output in an optimal fashion. The local task's solution depends not only on the internal subsystems' parameters, but on the value of interfaces too. To achieve the global task solution, local tasks have to be coordinated using an appropriate coordination principle.

KEYWORDS: system complexity, hierarchy, decomposition, coordination, system analysis, algorithm concurrence

1. System complexity

1.1. Introduction

The modern electrical, energetic, or electronic systems are examined as big and very big systems as far as structural complexity and nonlinear internal parameters are concerned. One can find a lot of examples of power supply systems in the modern coal or copper mines.

Depending on component complexity, these systems can be counted as linear or nonlinear systems. We can be required to analyze both stationary and dynamic processes.

In all tasks, a set of equation is built, which can include hundreds or thousands equations. Solving this big dimensionality task is a very time-consuming event for the modern computers. It is very difficult to understand the relations between parts of such complicated systems. In the article, all such systems are known as networks ("electrical network"). A comprehensive look at the big system requires developers to concentrate on the most important components and

the relations between them. The big system "S" can be defined as the following relation set:

$$S = \langle U, \{R\}, Y \rangle \quad (1)$$

where: U – a set of distinguishing input elements, Y – a set of distinguishing output elements, R – relations determined between sets U and Y . A relation can be defined as the connection between the system's components (subsystems).

In [1], the relations between U and Y sets could be depicted in the following two ways:

- An input – output description. The system is defined as a mapping between the input set U and the output set Y . Every element $u \in U$ is assigned to $y \in Y$. If "S" is a function, then the system

$$S: U \rightarrow Y \quad (2)$$

is called a functional system. In practice, the relations between input and output elements are expressed by algebraic or differential equations.

- A particularly convenient way to define a system is by means of a decision problem. The relation is described in an indirect way, as a solving procedure task. System "S" is known as the task solution procedure ("taking a decision"), if the set of task "D" and the solution set "Y" are known. For every element $u \in U$ and $y \in Y$ a couple elements (u, y) belong to the system "S" ($(u, y) \in S$) when and only when "y" is the task solution $D(U)$.

Thus defined electrical system or complex electronic scheme could not be described in a simple and explicit way. Subsequently, the system will be examined as a complex and hierarchical structure, namely with:

- precision and diversity description,
- calculation complexity and decision-making process.

Decomposition and coordination principle will be implemented for linear and nonlinear electric schemes. The reports will be done on the coordination methods. The coordinator algorithm has the most important impact on the quality task solution, including the subtasks, the coordinator and the global task solution.

1.2. Stratification. The levels of description

According to [1, 4], when it comes to complex technical and environmental systems it is impossible to precisely and perfectly describe them using only the terminology of one of the domains. Basically, the dilemma is between the simplicity of description and the need to take into account the aspects of a complex system. A resolution of this dilemma is sought in a hierarchical description. One describes the system as a family of structures, where each one is concerned with the behavior of the system as viewed from a different level of abstraction. In the case of an electrical system, a verbal description of the network's structure is used. In addition to this, a network is expressed by algebraic or differential equations. Practical applications use electronic schemes and the structure of algo-

rithms. To describe an electrical system, various hierarchical models have to be used from different scientific and technical domains (Fig. 1.). For such, a hierarchical description is effective. It is necessary that the functioning on any level be as independent of the functioning on the other level as possible. To distinguish this concept of hierarchy from others, the terms "stratified system" or "stratified description" are used.

One can conclude that:

- The terms in which the electrical system is described depend on the developer, his knowledge and his interest in the operation of the system.

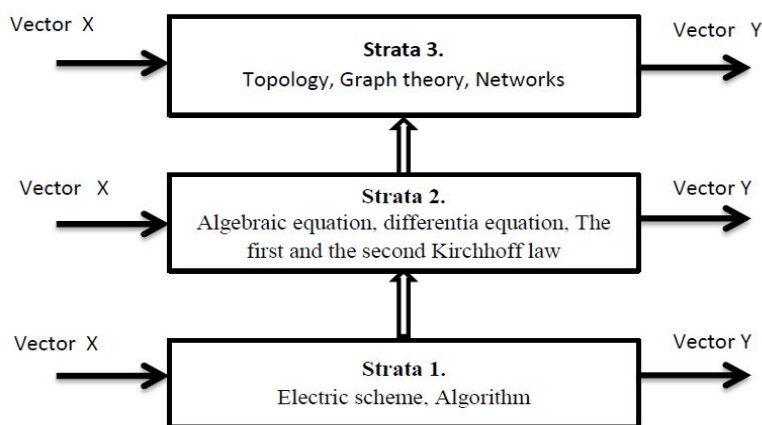


Fig. 1. "A three – strata representation" of an electrical system

- There is an asymmetrical independence between the functionality of the electrical system on different strata.
- Each stratum has its own set of terms, concepts, and principles.
- The understanding of the electrical system increases by crossing the strata. In moving up the hierarchy, one obtains a deeper understanding of its significance.

1.3. Layers. Organization complexity

The complexity of the electrical system topology can be enormous. A scheme can include hundreds of nodes and thousands of edges. For example, analyzing the energy flow from a holistic point of view could be difficult as one would strive to decompose the whole task into a smaller one through building a set of subtasks:

- An external subsystem. Using a graph as a system topology structure, the developer should find the consistent subgraph, including the main edges.

- The set of internal subsystems that are connected with the external subgraph as well as the internal neighboring one.

In this way, the first decomposed layer can be achieved. Using this algorithm, every internal subgraph can be divided into smaller ones. The second decomposed layer can be built. Figure 3 shows three subsystems (subnetworks, subgraphs).

- Master subnetwork S0, which includes the basic system structure. The graph includes two sources of suppliers and the set of edges containing resistors or other electronic passive elements.
- Two local subnetworks S1, S2 that contain only passive elements. Subnetworks are connected with the basic subsystem and are interconnected by interfaces.

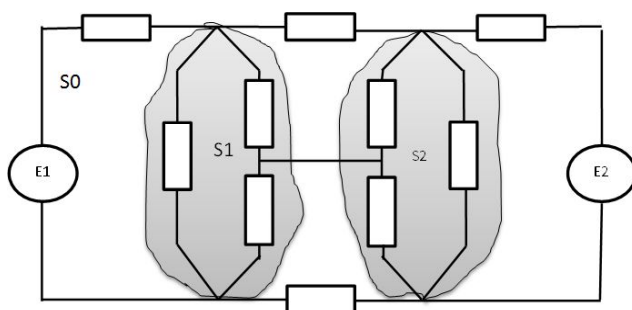


Fig. 2. An electric scheme decomposed into subnetworks

The decomposition process of the whole system structure into the subsystems is described by layers.

- On the first layer, there is a set of autonomous subsystems with their own target functions and local constraints. Every subnetwork could find the solution of its own local task. In electrical systems, the local target function is defined as "optimization task" – looking for the minimum of power dispersed among all of the active and passive elements.
- On the second layer, the master subnetwork also solves its own optimization task.
- Subnetworks are interconnected by both input and output interfaces. An input interface supplies energy to the subsystem. An output interface sends the overflow of energy into the neighboring one.

It should be stressed that every subnetwork has to have, at a minimum, one input and one output interface. A hierarchical subnetwork structure is shown in Fig. 3.

Parts of the system are represented as black blocks having input signals U_{ij} and output signals Y_{ij} , where index i defines the subnetwork number and index j defines the next signal number. The state of an internal subnetwork depends not only

on the internal scheme structure (graph structure) and the passive and active electric elements value, but on the input interface signals U as well.

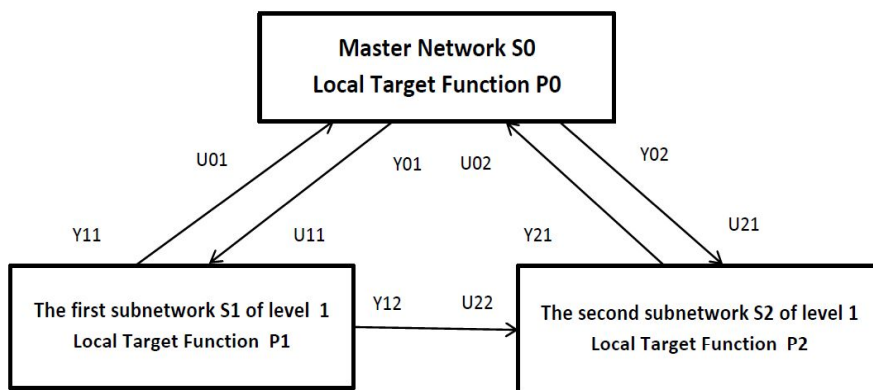


Fig. 3. The hierarchical structure of the decomposed electrical system

The local target function is defined as "subnetwork power" (all passive and active electric scheme items)

$$P_i = \Phi(X, U) \quad (3)$$

All output subnetworks signals depend on both the internal state and the input signal value

$$Y_i = F(X, U) \quad (4)$$

where: X – the state vector, containing current and voltage value analyzed subnetwork, U – the input interface value analyzed subnetwork, Y – output interface value analyzed network.

In practice, subnetworks could have a different number of input and output interfaces. A big system can include tens of interfaces. Every connection can be represented as a vector equation:

$$U_{ij} = Y_{kl} \quad (5)$$

where: i, k – subnetwork numbers, j, l – the interface number.

	Y01	Y02	Y11	Y12	Y21
U01			1		
U02					1
U11	1				
U21		1			
U22				1	

Fig. 4. Matrix connections for a hierarchical system structure

For example, u_{12} indicates input interface 2 for subnetwork number 1. All connections described in equation (6) could be represented in a matrix as a set of all connections.

Up to now, two concepts were implemented – stratification and layers. These concepts help one to decompose a network structure from different points of view and to understand the network's behavior. However, these two aforementioned concepts do not describe the concepts regarding the calculation of the algorithmic structure.

Every algorithm has to calculate all current and voltage values for all passive and active elements. For this task, "echelon" was introduced as the description of a hierarchical algorithm structure.

1.4. Echelon. Calculation algorithm structure

For further consideration, a static electric system will be analyzed. Both current and voltage value have to be calculated for the considered electric scheme. The current value for linear or nonlinear electric circuits has to be calculated by using the principle of the minimum power which disappears in all circuit elements. For this target function a lot of constraints must be added. Constraints are built using Kirchhoff's first law and Kirchhoff's second law. One can formulate three very important properties:

- The circuit analysis task is defined as the task of searching the minimum of the target function value

$$\min P = \min \Phi(X, U) \quad (6)$$

- The target function is additive. This means that the overall power is equal the sum of the subnetworks power

$$P = \sum_{i=1}^{i=N} P_i \quad (7)$$

- Subnetworks are connected through interfaces. This gives the ability to flow power (energy) between subnetworks

$$M \cdot Y - U = 0 \quad (8)$$

For the circuit shown in Figures 2 and 3, the overall (global) target function can be defined as:

$$P = P_0 + P_1 + P_2 \quad (9)$$

With global constraints for the interface value:

$$U - M * Y = 0 \quad (10)$$

where: P_0, P_1, P_2 – target functions for S0, S1, S2 subnetworks; U – the vector of the interface input according to M matrix (Fig. 4); y – the vector of the interface output.

Based on [2, 3], the process of searching for the minimum of the global target function can be realized by using a two-step algorithm. Since the global target function is additive

$$\min P = \min_{X_i U_i} \{ P_1(X_1, U_1, Y_1) + P_2(X_2, U_2, Y_2) + \dots + P_N(X_N, U_N, Y_N) \} \quad (11)$$

with global constraint according to (10) equation, one can state that.

On the first level, the minimum of the local target functions P_i is searched. Only a part of the variables (currents) is used in the calculation process and then determined. One should remember that parts of the external variables, input interfaces U_i , have to be evaluated by the second-level coordinator. They are used by the first-level tasks as parameters which cannot be changed during one iteration. On the second level, the coordinator has to find the optimal value of the interfaces U_i . The full algorithm structure is depicted in Fig. 5.

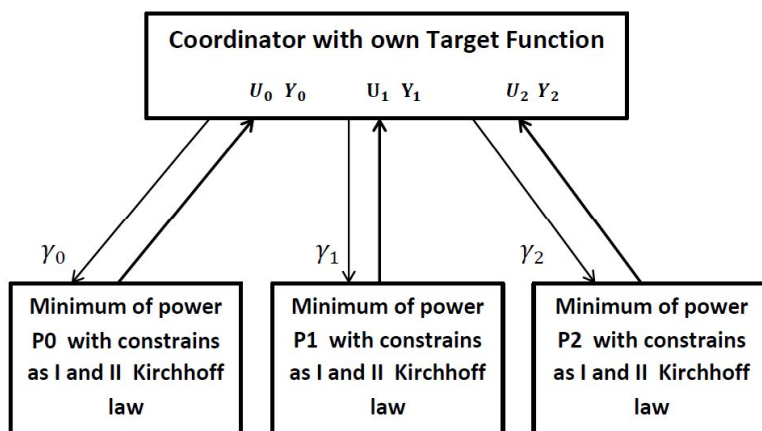


Fig. 5. The scheme of calculation algorithm for an electric circuit

Moreover, to use an explicit decomposition into local subsystems, a new and effective calculation procedure for big electric circuits can be used. A complex algorithm structure contains two iteration processes: the internal, associated with the local solving procedures, and the external, associated with the coordinator task. One should stress that for every iteration, all the first-level tasks have to find their new variables' value, using their own iteration algorithms.

2. Coordination

The coordination task is not simple, which results in the following conditions:

- An electric circuit and the optimization algorithm are usually structurally nonlinear tasks, which are solved by the gradient or non-gradient iteration procedures.
- They are multidimensional tasks. This means that input and output vectors can include tens or hundreds of variables. Also, the graph structure can include hundreds of nodes and edges.

- When decomposing the basic electric circuit structure into strata and defining local target tasks with their own target functions, two kinds of conflict are introduced. The first conflict occurs between the first-level tasks. The second is performed by the coordinator. The coordinator tries to solve its own optimization task, which could be in conflict with the first-level tasks.

The first conflict can be solved if the global target function is a very special kind of function – that of a non-increasing order. Tasks with the structure of an additive function have this kind of ability. The second conflict is solved by a priority action. The coordinator has a higher priority than all the first-level tasks. The coordinator enforces the coordinator value and the first-level tasks have them as parameters. To solve its own task, the coordinator needs information from the all the first-level tasks. So, the first-level tasks are responsible not only for solving their local tasks, but also for calculating the interfaces output. These signals are sent to the coordinator, which has the ability to solve its own task. This process is repeated for as long as the minimum of the coordinator local target function is achieved.

2.1. Coordination principles

To solve its own task, the coordinator can use different coordination principles [1]:

- Interaction prediction principle. The coordinator has the ability to predict (forecast) the interface's value. The coordinator should predict the interface value in such a way that the real interface value calculated by the all the first-level tasks has to be equal to their predicted value. Using this principle, the coordinator can influence the first level. This kind of influence is known as "small scale impact".
- Interaction decoupling. Each of the first-level subnetworks can solve its own optimization task by treating input interfaces as additional decision variables, free to be selected at will. Tasks are completely decoupling. The coordination balance is achieved by the fact that the coordinator uses the local target functions modification. This process is known as "Performance Balance" [1].
- Interaction estimation. Rather than predicting the precise value of the interface input, the coordinator specifies ranges over which they may vary. The coordinator defines more flexible parameters for the first-level tasks.

In this article, the second principle is employed. Interfaces are decoupled and the local target functions are modified. This kind of influence is known as "large scale impact" (by the target function).

2.2. Large-scale impact

As it was stressed before, the structure of the optimization algorithm is based on the minimum of power. This is a very flexible tool to use the additive target function structure. The second problem that should be solved is to write the set of Kirchhoff's

first and second constraints. In this way, "N" local tasks were built. The constraint subset is defined as:

$$\phi_i(K_i^1, K_i^2) = 0 \quad (12)$$

where: $I = 1, 2, \dots, N$ – the number of the first-level tasks, K_i^1 – the set of Kirchhoff's first law, K_i^2 – the set of Kirchhoff's second law.

The set of the global constrain is defined by matrix M (10).

According to "Performance Balance", a set of the local target functions should be modified. To do that, the Lagrange's equation is built:

$$L = \sum_{i=0}^n (P_i(X_i), U_i, Y_i) + \sum_{j=0}^{m_i} \Lambda_i * \Phi_j(K_j^1, K_j^2) + \Gamma * (M * Y - U) \quad (13)$$

where: N – the number of subsets, m_i – the number of constraints for subnetwork "I", Λ_j – the number of the Lagrange multipliers for subnetwork "i", Γ – the multiplier vector for global constraints.

The solution will be searched as follows:

$$\max_{\Lambda} \min_{X, Y, U} L(X, Y, U, \Lambda) \quad (14)$$

"N" of the first-level task can be assigned as:

$$\min_{X_i, U_i, Y_i} [P_i + \sum_{j=0}^{m_i} \lambda_{ij} * \Phi_j(K_j^1, K_j^2) + \sum_{j=0}^{m_i} (M_{ji} * Y_i - \lambda_i * U_i)] \quad (15)$$

where: X_i, U_i, Y_i – the interface parameters and subnetwork technical data, $I = 1, 2, \dots, N$.

The local task coordination depends on Lagrange multipliers. To achieve this, the difference between vectors Y_i and U_i is used:

$$\Gamma_i(n+1) = \Gamma_i(n) - \beta * (M * Y_i - U_i) \quad (16)$$

3. A numerical example

As an example, a quite simple electrical system is examined: with one power supply and four resistors. Fig. 6 shows the final decomposition of the primary electrical circuit into two subnetworks with internal interfaces.

For the first subnetwork, a set of equations can be written according to "power balance", Kirchhoff's first law and Kirchhoff's second law

$$P_1 = e_1 * r_1 - r_1 * i_1^2 - r_2 * i_2^2 \quad (17)$$

$$e_1 - i_1 * r_1 - i_1 * r_2 = 0 \quad (18)$$

$$i_1 - i_2 - y_{11} = 0 \quad (19)$$

$$-i_1 + i_2 + u_{11} = 0 \quad (20)$$

The second subnetwork is simpler as it includes only two passive elements:

$$P_2 = -r_3 * i_3^2 - r_4 * i_4^2 \quad (21)$$

$$u_{21} - i_3 - i_4 = 0 \quad (22)$$

$$i_3 + i_4 - y_{21} = 0 \quad (23)$$

$$i_3 * r_3 - i_4 * r_4 = 0 \quad (24)$$

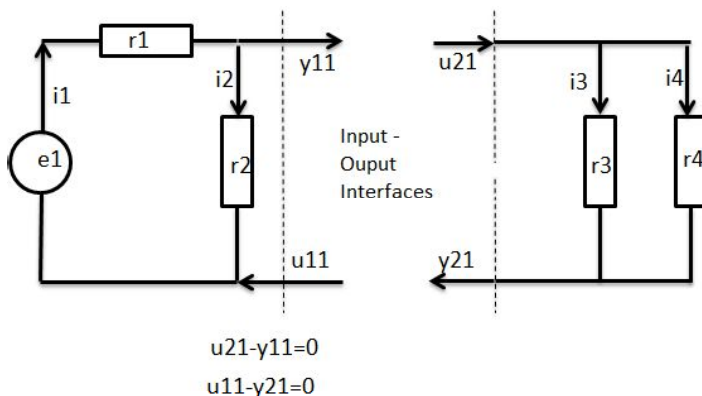


Fig. 6. An electric circuit decomposed into two subnetworks, S1, S2 and interfaces

Finally, interfaces have to be added:

$$u_{11} - y_{21} = 0 * \gamma_1 \quad (25)$$

$$u_{21} - y_{11} = 0 * \gamma_2 \quad (26)$$

By multiplying the first equation by γ_1 , the second by γ_2 and grouping variables according to subnetworks S1, S2, two temporary equations are achieved:

$$\gamma_1 * u_{11} - \gamma_2 * y_{11} \rightarrow \Phi_1 \quad (27)$$

$$\gamma_2 * u_{12} - \gamma_1 * y_{12} \rightarrow \Phi_2 \quad (28)$$

These equations are added to P_1, P_2 respectively achieving two modified local target functions:

$$\Phi_1 = L1 = e_1 * r_1 - r_1 * i_1^2 - r_2 * i_2^2 + \gamma_1 * u_{11} - \gamma_2 * y_{11} \quad (29)$$

$$e_1 - i_1 * r_1 - i_1 * r_2 = 0 \quad (30)$$

$$i_1 - i_2 - y_{11} = 0 \quad (31)$$

$$-i_1 + i_2 + u_{11} = 0 \quad (32)$$

The second subnetwork:

$$\Phi_2 = L2 = -r_3 * i_3^2 - r_4 * i_4^2 + \gamma_2 * u_{12} - \gamma_1 * y_{12} \quad (33)$$

$$u_{21} - i_3 - i_4 = 0 \quad (34)$$

$$i_3 + i_4 - y_{21} = 0 \quad (35)$$

$$i_3 * r_3 - i_4 * r_4 = 0 \quad (36)$$

The next calculation step is to (15). Two independent Lagrange functions $L1$, $L2$ are built (29), (33). Every Lagrange function contains seven variables (three extra variables as Lagrange multipliers).

The algorithm is started by the coordinator. The coordinator has to send into the subnetworks two coordination signals value, γ_1 , γ_2 . After that, two independent subnetworks are responsible for finding the minimum Lagrange functions value $L1$ and $L2$, respectively.

The first subnetwork S1 has to calculate four values i_1 , i_2 , u_{11} , y_{11} . The same is done by the second subnetwork S2.

The subnetworks calculate the interfaces value sent to the coordinator. The coordinator, using formula (16), calculates the value of the new interfaces and sends them to S1 and S2 subnetworks. The process is being repeated for as long as the coordinator achieves its minimum value. This means that interfaces are balanced and the global target function is achieved.

In Figure 7, the coordination signal $\gamma = \gamma_1 - \gamma_2$ is shown in relation to the iteration number.

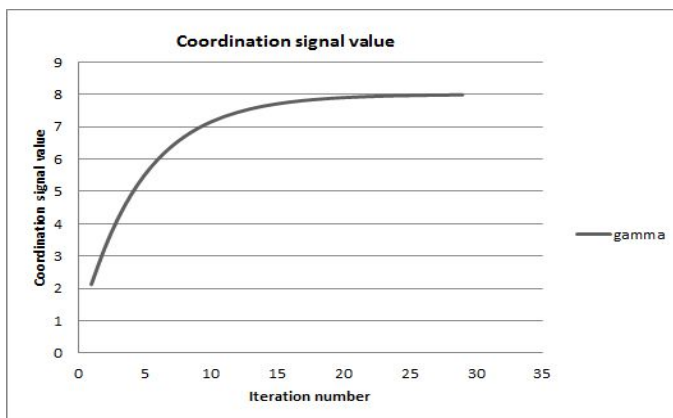


Fig. 7. Coordination signal characteristic

The interfaces play a strategic role in the coordination procedure. There are global signals calculated not only in the first-level tasks, but in the second level ones as well – by the coordinator.

Figure 8 shows the interfaces' dynamic characteristics. In the beginning, their values change dramatically. After that, in an asymptotic way, the balance value is achieved.

In Figure 9 all current characteristics are shown. The dynamic process is the same. This means that the interfaces' impact on all the calculation processes is very big.

Finally, characteristics of two Lagrange functions are shown – $L1$ for subnetwork 1 (Fig. 10) and $L2$ for subnetwork 2 (Fig. 11). For stable state both values are equal zero.

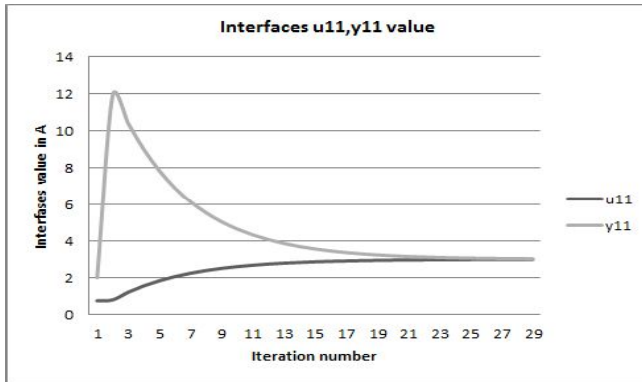


Fig. 8. Interfaces' value for the first subnetwork

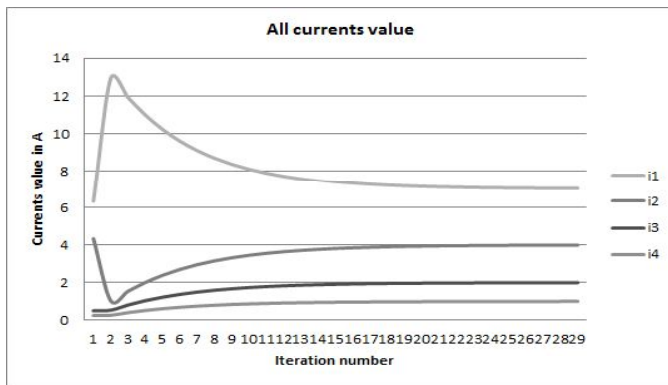


Fig. 9. All the current values in the iteration number

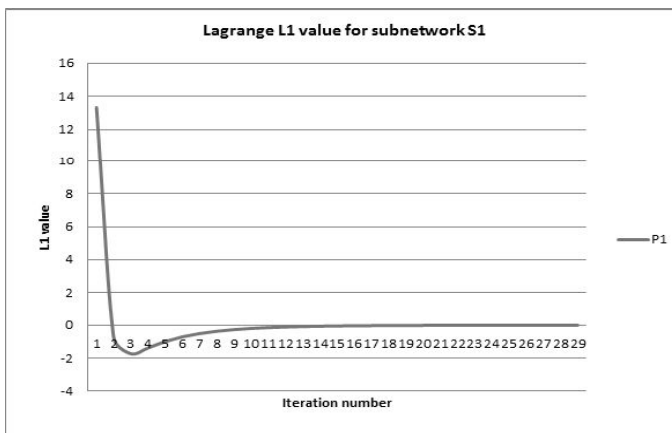


Fig. 10. L1 value in the iteration number

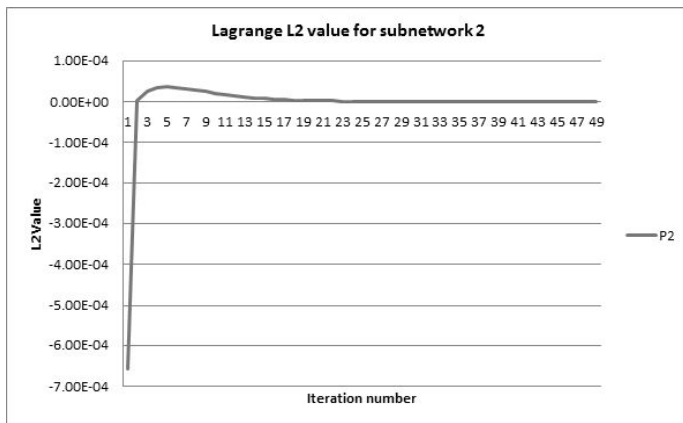


Fig. 11. L2 value in the iteration number

For stable state P1 value for subnetwork 1 one can calculate (17):

$$P_1 = e_1 * r_1 - r_1 * i_1^2 - r_2 * i_2^2 = 24 [W] \quad (37)$$

For subnetwork 2, P2 value (21):

$$P_2 = -r_3 * i_3^2 - r_4 * i_4^2 = -24 [W] \quad (38)$$

As one can expects

$$P = P_1 + P_2 = 0 [W] \quad (39)$$

It is total minimum of power and subnetworks are balanced.

4. Conclusion

In the article, only one coordination principle is described, that is the decoupling mode. This principle is general and can be implemented in different applications. As stressed above, the coordination mode is fulfilled by the target function modification for every subnetwork. Using the primary task decomposition, every subnetwork has a smaller dimensionality and there are fewer variables. In the example, the global task includes 4 variables, i.e. currents. After decomposition, every subnetwork includes 2 variables. Of course, interfaces, as variables, also have to be included into the local task dimensionality.

When the primary task is adequately complicated (the circuit graph is big), the number of interfaces does not have such a significant impact on the subtasks' dimensionality. Thus, the decomposition benefit is obvious.

This is compensated by the coordination structure, which is very simple. The coordinator algorithm uses formula 13. There is one problem that has to be solved by the developer. The coordinator algorithm includes the learning coefficient β . Its value depends on all the subnetworks' structure. Taking too big β value, the calculation pro-

cess can be unstable (oscillations can be seen). For too small β , the calculation process is not optimal. Convergence is slow.

This method can be used in a very effective way when calculating different variants. For example, one or same circuit parameters in one subnetwork are changed. Usually, every disturbance to a local parameter's has a local impact on the state of the new stable system. Therefore, the interfaces values can change slightly and disturbances do not have a significant impact on the other subnetworks. The calculation process is not time-consuming and convergence is fast.

References

- [1] M. Fajter, „Analiza stabilności układu elektrycznego w czasie rzeczywistym”, Poznań University of Technology, Academic Journals, Electrical Engineering, No 81, Poznań 2015.
- [2] Findeisen, W., Wielopoziomowe układy sterowania. PWN, Warszawa 1974.
- [3] Findeisen, W., J. Szymanowski, A. Wierzbicki, Teoria i metody obliczeniowe optymalizacji. PWN, Warszawa 1977.
- [4] Kulikowski, R., Sterowanie w wielkich systemach. PWN, Warszawa 1974.
- [5] Mesarovic, M.D., D. Macko, Y. Takahara, Theory of Hierarchical, Multilevel systems. Academic Press, New York and London 1970.
- [6] Miller, P., M. Wancerz, "Komputerowo wspomagane obliczenia zwarciove w środkowej części rozdzielni", Poznań University of Technology, Academic Journals, Electrical Engineering, No 82, Poznań 2015.

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