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MODIFIED ROOT-MUSIC ALGORITHM AND ITS APPLICATION TO REAL DATA, PASSIVE BEARING ESTIMATION OF SHALLOW WATER ACOUSTIC TARGETS

Sławomir Henclik
R&D Marine Technology Centre
ul. Dickmana 62, 81-109 Gdynia, Poland
e-mail: shen@ctm.gdynia.pl

The main problem addressed in the paper is an idea of modification and generalization of Root-MUSIC method. The algorithm is commonly used for uniform linear arrays. A concept how to generalize this method for non-uniform arrays is presented. An additional parametric test is also proposed for the selection of signal roots. Selected results of shallow water acoustic targets location (in bearing), with passive, linear array of hydrophones is also presented. The data used were acquired with an array situated in littoral waters, made and tested at Marine Technology Centre. A short background of array processing methods, especially subspace-based algorithms is included as well. Some important practical problems are also shortly discussed.

INTRODUCTION

Passive bearing estimation of underwater acoustic targets has many advantages. It involves, however, many complex tasks, when put into practice. Beside the technology, the first is practical implementation of proper algorithms into software, which never is a routine. Many detail problems are also involved with the selection of optimal values for varied processing parameters. There is a place for introducing new, technology and research, concepts as well. A job of manufacturing an underwater acoustic passive array has been taken at Marine Technology Centre. The array, situated in littoral waters is still under tests. A novel research concept and some experimental results based on this job are presented in this paper.

1. SUBSPACE-BASED APPROACH

When an array of N passive sensors receives wideband signal compound of M source signals and white noise it is usually [3,5], for frequency ω and any time t, modeled as:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \qquad (1)$$

In equation (1) **x** is N-dimensional complex vector of sensor signal amplitudes at narrowband frequency ω, **n** is noise part of **x** and **s** is M-dimensional vector of source signals complex amplitudes. The (NxM) steering matrix **A** expresses the phase delays of m^{th} signal at n^{th}

sensor (m=1..M, n=1..N), and has a form (2) of M column steering vectors $\mathbf{a}(\theta_m)$, where θ_m is a parameter that localizes the m[th] source.

$$\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_M)] \quad (2)$$

The steering vector $\mathbf{a}(\theta)$ for a linear array with sensor positions y_n is given by (3).

$$\mathbf{a}(\theta) = [\exp(-jky_1 \sin\theta), \ldots, \exp(-jky_N \sin\theta)]^T \quad (3)$$

k (=ω/c) is the narrowband wavenumber and θ is the bearing, relative to the array broadside, of far field source. The spatial correlation matrix **R** of the array signal **x** is computed as:

$$\mathbf{R} = E\{\mathbf{x}\mathbf{x}^H\} = \mathbf{ASA}^H + \sigma^2\mathbf{I} \quad (4)$$

E{ } stands for expectation value, **I** is the unit matrix and σ^2 is the power of white noise. The signal and noise correlation matrices are defined respectively by (5) and (6).

$$\mathbf{S} = E\{\mathbf{s}\mathbf{s}^H\} \quad (5)$$

$$\mathbf{Q} = E\{\mathbf{n}\mathbf{n}^H\} = \sigma^2\mathbf{I} \quad (6)$$

Exploiting the structure (4) and the fact that $\mathbf{ASA}^H$ is positive definite a standard decomposition of **R** into two ortogonal subspaces can be made. (N-M) eigenvectors (EVs) ortogonal to the columns of **A** have their eigenvalues σ^2, are called noise EVs and span the noise subspace. Others EVs (of the largest eigenvalues) span the signal subspace.

An experimental estimate of **R** is usually computed as the sample covariance matrix expressed by (7). Snapshots $\mathbf{x}_i$ (i=1..K) of array signal are acquired within a time period ΔT.

$$\hat{\mathbf{R}} = \frac{1}{K}\sum_{i=1}^{K} \hat{\mathbf{x}}_i \hat{\mathbf{x}}_i^H \quad (7)$$

The eigenvalue decomposition (EVD) of $\hat{\mathbf{R}}$ is given by (8). **L** is a diagonal matrix of positive eigenvalues sorted in descending order and the column ortonormal EVs form the matrix **U**. Assuming for $\hat{\mathbf{R}}$ the structure (4) the EVs can be split into signal EVs grouped in matrix **F** and noise EVs forming the matrix **G**.

$$\hat{\mathbf{R}} = \mathbf{ULU}^H \quad (8)$$

$$\mathbf{U} = [\mathbf{u}_1, \ldots, \mathbf{u}_M, \mathbf{u}_{M+1}, \ldots, \mathbf{u}_N] = [\mathbf{F}, \mathbf{G}] \quad (9)$$

2. MODIFIED ROOT MUSIC

The basic formula exploited by noise subspace methods, like MUSIC, is the ortogonality (10) between the source steering vector and the noise subspace.

$$\mathbf{G}^H\mathbf{a}(\theta) = \mathbf{0} \quad (10)$$

Assuming at this moment a uniform linear array (ULA) with sensor separation d, the steering vector $\mathbf{a}(\theta)$ can be expressed with the use of complex variable z:

$$\mathbf{a}(\theta) = \mathbf{a}(z) = [1, z, z^2, \ldots, z^{N-1}] \quad (11)$$

$$z = e^{-jkd\sin\theta} \quad (12)$$

Taking the square norm of (10), after some manipulations [2] we get (**J** is the exchange matrix $J_{km}=I_{m,N-k+1}$):

$$\mathbf{a}^T(z)\mathbf{JGG}^H\mathbf{a}(z) = 0 \quad (13)$$

what is in fact the rooting problem of a complex polynomial p(z) of rank r=(2N-2) and coefficients b_k given by (15) and (16), for i=1..N-1.

$$p(z) = \mathbf{a}^T(z)\mathbf{JGG}^H\mathbf{a}(z) = \sum_{k=0}^{2N-2} b_k z^k \quad (14)$$

$$b_{N-1} = \mathrm{Tr}\left(\mathbf{GG}^H\right) = N - M \tag{15}$$

$$b_{N-1+i} = -\sum_{m=1}^{M}\sum_{n=1}^{N-i} U_{nm}U^*_{n+i,m} = \sum_{m=M+1}^{N}\sum_{n=1}^{N-i} U_{nm}U^*_{n+i,m} = b^*_{N-1-i} \tag{16}$$

Half of the roots of p(z) lying inside the unit circle are considered farther (the other half has the form of $1/z^*$). As the signal roots should lie on the unit circle (12), M largest of all the roots (the nearest to the unit circle) are identified as them. The source positions θ_m are determined by the phases ϕ_m of those largest roots.

$$\sin\theta_m = -\frac{\phi_m}{kd} \tag{17}$$

Such a selection is a standard Root-Music procedure [3,4]. Sorting the (N-1) roots z_m with respect to the modulus in descending order, we get the signal roots indexes to be m=1..M. If the M^{th} root has its modulus e.g. $\rho_M=0.92$ and the $(M+1)^{st}$, $\rho_{M+1}=0.81$ such a selection involves no doubts. However, if $\rho_M=0.895$ and $\rho_{M+1}=0.891$ it cannot be guaranteed that the candidate source is located at θ_M rather than at θ_{M+1}. That is why an alternative test is proposed for the selection of signal roots. Let us take into consideration all the roots lying outside the circle of radius ρ and still inside the unit circle. ρ can be kept fixed (e.g. $\rho=0.8$) or varied (e.g. $\rho=0.95\rho_M$). If the number of these roots is L ($M\leq L\leq N-1$) we declare all the roots $z_1...z_L$ as the candidate signal roots. Taking any M-element subset $\{\theta\}_i$ of $\{\theta_1,...,\theta_L\}$ we test it using a function $g(\{\theta\}_i)$. A parametric test [1,3] based on deterministic maximum likelihood - DML (18) or weighted subspace fitting - WSF (19) is proposed. A set of candidate source positions $\{\theta\}_p$ that maximizes the function g() is then claimed as the source locations.

$$g_{DML}(\{\theta\}_i) = \mathrm{Tr}\left(\mathbf{P}_A\hat{\mathbf{R}}\right) \tag{18}$$

$$g_{WSF}(\{\theta\}_i) = \mathrm{Tr}\left(\mathbf{P}_A\mathbf{FWF}^H\right) \tag{19}$$

$\mathbf{P}_A$ is the projection matrix (20) onto the subspace span(**A**) which is determined by the candidate source positions $\{\theta\}_i$. **W** is a weighting matrix [6,3,1] that can also be set **W=I**.

$$\mathbf{P}_A = \mathbf{A}\left(\mathbf{A}^H\mathbf{A}\right)^{-1}\mathbf{A}^H \tag{20}$$

Now let us assume an arbitrary linear array. Without losing the generality we can claim the sensor locations y_n in equation (3) as rational numbers. In that case there exist such a largest distance d that each sensor position is expressed as an integer multiplication of d. This approach is equivalent to an ULA with gaps (empty nodes - without sensors) and is impractical if d is very small. However real arrays are design as more or less regular and if the number of gaps is not too high comparatively with the number of sensors, proposed approach seems reasonable. The problem is now to compute the polynomial coefficients. The difference lies in the shape of steering vector (11), which has now no element z^{n-1}, if n is the place of a gap. As a consequence in equation (16), in the internal sum, N stands for the number of all the array nodes (sensors and gaps) and the sum goes only through such pairs of nodes (n, n+i) that are sensor-filled. If there are no such pairs a coefficient equals zero. For a polynomial computed in that way typical Root-MUSIC actions are proceeded farther.

3. EXPERIMENTAL RESULTS

At fig.1 a scenario with two shallow water targets located with the use of spectral MUSIC by an ULA of 21 hydrophones is presented. Forward-backward spatial smoothing (FBSS) of rank 5 was applied. The relative processing frequency width was $\gamma=0.04$ (γ is defined as: freq_width*array_length/sound_vel and should be much less than 1), the observation time-length ΔT=5s and the time-band-width-product (without FBSS) K=10. For the same scenario computations were made with the use of Root-MUSIC. Five roots (their sinθ) of the largest magnitude are shown. In bold are typed signal roots chosen with WSF test. The magnitudes of the roots are typed in brackets.

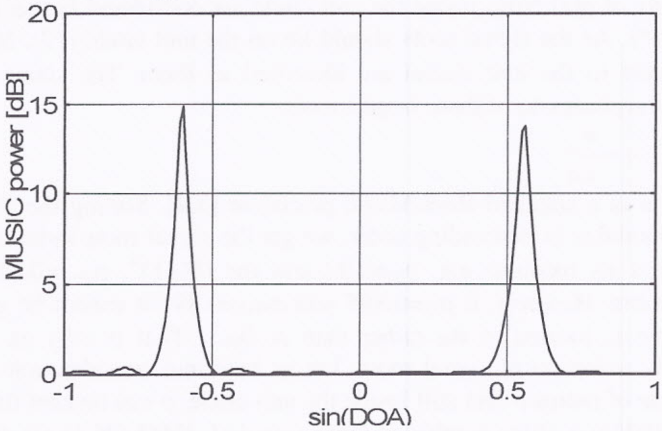

Fig.1. Music spectrum for two far field targets scenario.

1. γ=0.04, ΔT=5s, K=10. Signal roots identified with WSF test → (1,2).
   -0.593 (0.95) **0.553 (0.78)** -0.770 (0.78) -0.416 (0.76) -0.917 (0.74)

Another scenario, for two targets as well, is shown below. Both lines of results concern the same time and frequency, but different processing parameters, which are specified (γ, ΔT, K). One can see that conventional roots selection points out different roots (1,2) each time and parametric test chooses the same source locations.

2. γ=0.04, ΔT=5s, K=10. Signal roots identified with WSF test → (1,5).
   0.636 (0.93) -0.484 (0.88) -0.213 (0.86) 0.571 (0.82) **-0.005 (0.82)**
3. γ=0.08, ΔT=3s, K=12. Signal roots identified with WSF test → (1,2).
   0.010 (0.92) **0.643 (0.92)** 0.464 (0.80) 0.181 (0.77) 0.319 (0.76)

In a number of other tests made it was found that if the signal roots magnitude were close to unity (the data were well fitted to the model) the parametric test led usually to the same results as the conventional selection.

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